CONTENTS

THEORY NOTES

INTRODUCTION TO SCIENCE:

Science is the study of nature and materials scattered in this universe in order to obtain optimum benefits for the mankind and it also provides a dimension to discover "The Truth" of existence of this universe. Physics is the most basic discipline among all sciences as it deals with the basic laws that govern the functions of natural phenomena that occur m our universe. Physics evolves the vision and understanding of the things and happenings both in microscopic and in macroscopic levels. Moreover, physics provides us the foundation upon which other sciences can be better understood, analyzed and manipulated.

MAIN FIELDS OF SCIENCE:

i. The Biological Sciences ii. The Physical Sciences

I. THE BIOLOGICAL SCIENCES:

The sciences which deal with living things and their organisms are called biological sciences and these are further classified into zoology, botany, genetics, physiology, microbiology and anatomy etc.

II. THE PHYSICAL SCIENCES:

The sciences which deal with the composition, properties origin and inter conversion of matter and energy are called physical sciences. These are further divided into number of disciplines such as physics, mathematics, statistics, chemistry, geography, geology, astronomy and electronics etc.

HISTORY OF PHYSICS:

The roots of physics are as deep as the history of mankind. The technological aspect of Physics was even started in primitive ages of man when he knew about the production of fire by rubbing two stones together and when he used wood logs to cross the rivers and lakes. But philosophical aspect of physics took centuries for its evolution.

Egyptians, Greeks, Chinese and Arabs had been recognized as agents of developed centers of science and technology for long periods before Europeans took initiative in this field. As Physics covers the whole universe from macro to micro entity then it has been classified into a number of branches for detailed study of nature. With reference to dual nature of matter i.e. (i) particle nature and (ii) wave nature, there are two fields of Physics:

I. CLASSICAL PHYSICS:

It is based upon Newton's laws of motion, Galileo's relativity, Kepler's laws of planetary motion, Maxwell's laws of Electromagnetism, Kelvin's law of thermodynamics, Faraday's law of electromagnetic induction, Huygen's wave theory of light etc. This field of thoughts has been recognized prior to 1900 AD.

II. QUANTUM OR MODERN PHYSICS:

It is based upon Plank's quantum theory of light and De-Broglie's idea about dual nature of matter which developed revolution in the first quarter of the 20th century. Moreover, it deals with Einstein's relativity, particle physics, cosmology and solid state Physics.

CONTRIBUTION OF MUSLIM SCIENTISTS TOWARDS SCIENCE:

1. IBN-AL- HAITHAM (965-1040 A.D.):

Ibn-al-Haitham was a great scholar of physics and engineer of the Islamic world. His main contribution to physics was the research on optics. He discovered five laws of reflection and refraction of light and he invented pin hole camera. He wrote books on. the human eye and analytical geometry. I-'is most famous book on the light and optics is "Kitabul-Manazir".

2. MOHAMMAD BIN MUSA KHAWARIZMI (780-880 AD.):

Mohammad-bin-Musa Khawarizmi is recognized as one of the greatest mathematician of the Islamic world. He was an important member of the great institution of learning the "Baitul-Hikmat" (House of Wisdom).established by Caliph Mamun-al- Rashid. He was founder of analytical algebra and he wrote first ever book in the world on this subject named Hisabul-Jabr-wal-Muqabla. He developed the term algorithm (Logarithm). He also worked on geometry, geography and astronomy.

3. JABIR-BIN HAYYAN (776-803 AD.):

Jabir-bin-Hayyan is regarded as the greatest chemist of the world. He was first experimental chemist and he developed many physical processes such as crystallization, evaporation, sublimation, distillation, filtration etc. in chemical research He prepared Aqua-Regia which could dissolve gold, he also prepared sulphuric, nitric and tartaric acid. He wrote many books on chemistry.

4. AL-BATANI (858-929 AD.):

Al-Battani was a great a mathematician and astronomer of his time. He developed table for trigonometric functions. He made calculations in connection with solar system, prediction of moon and sun eclipses.

5. AL-BERUNI (973-1048 A.D.):

Al-Beiruni was simultaneously a mathematician, an astronomer, a historian and aphysician. He remain associated with Sultan Mahmood Ghazanavi and he also lived in India during the reign of Mughal King Akbar. He wrote famous book Al-Qanun-ul- Masudi which is considered as an encyclopedia of astronomy.

6 .OMER KHAYYAM (1044-1123 AD.) :

Omer Khayyarn was a versatile philosopher. mathematician and poet of the Islamic world. He developed theorem about binomial expansion and also about two parallel lines in geometry.

7. IBNE-E-SINA (980-1037 A.D.):

Ibn-e-Sina was one of the greatest physician from Islamic world. He was simultaneously a philosopher, mathematician and astronomer but his chief work was in the field of medicine. He wrote many books among which Al-Qanun-Fit-Tibb and Al-Shifa got international recognition. He introduced the use of catheters for extraction of urine in case of kidney failure. He gave intravenous injections by means of a Silver syringe.

8. AL-RAZI (864-930 AD.):

Al-Razi was the most prominent philosopher and physician. He developed chemical techniques for the preparation of drugs. He also worked on kidney stones, small pox and measles. He used anesthesia for operations.

9. YAQUB BIN ISHAQ AL-KINDI (800-873 AD.):

Yaqub bin Ishaq al-Kindi was honored with a title "The first Arab Philosopher". He wrote many research monographs on meteorology. He has about 241 books in his account in the subject of physics, mathematics, astronomy. He also worked on number system and physical geometry.

10. AL-FARABI (870-950 AD.):

Mohammed Al-Farabi was great mathematician,philosopher and musician of his time. He wrote more than 100 books on these topics.

PHYSICAL QUANTITIES:

A physical quantity is a quantity in physics that can be measured. Or a physical quantity is a physical property that can be quantified. Examples of physical quantities are mass, amount of substance, length, time, temperature, electric current, light intensity, force, velocity, density, and many others.

METER:

A unit of length that is one of the seven base SI units and is defined as the length of the path traveled by light in a vacuum during a time interval of 1/299792458 of a second.

OR

Meter is defined as "The distance between the two marks on a Platinum-Iridium bar kept at $0^{O}C$ in the International Bureau of Weight and Measures near Paris."

One meter = 100 cm

One meter = 1000 mm

KILOGRAM:

Kilogram is the unit of mass in S.I. System."Kilogram is defined as the mass of a Platinum-Iridium cylinder placed in the International Bureau of Weight and Measures near Paris."

One kilogram = 1000gram

SECOND:

Second is the unit of time in S.I. System. A second is defined in terms of the time period of Cs-133 atoms. i.e." one second is equal to 9,192,631,770 periods of vibrations of Cs-133 atoms."

OR

Second is the interval which is equal to (1/86,400)th part of a mean solar day.

60 seconds = one minute

3600 seconds = one hour

FUNDAMENTAL UNITS:

The international system of units is based on seven independent units known as Fundamental or Basic Units. These are given here:

DERIVED UNITS:

The units that require two or more basic measurements of same units or different fundamental units for its definition are called derived units.

FUNDAMENTAL UNITS

Quantity	Unit	Symbol
1. Length	meter	m
2. Mass	kilogram	kg
3. Time	second	s
4. Electric Current	ampere	A
5. Temperature	kelvin	K
6. Luminious Intensity	candela	Cd
7. Amount of Substance	mole	mol

DERIVED UNITS

Quantity	Unit	Symbol
1. Area	square meter	m^2
2. Volume	cubic meter	m^3
3. Density	kilogram/ cubic meter	kg/m^3
4. Velocity	meter/second	m/s
5. Angular Velocity	radian/ second	r/s
6. Acceleration	meter/second square	m/s^2
7. Angular Accleration	radian/second square	rad/s^2
8. Frequency	hertz	Hz
9. Force	newton	N
10. Work energy	joule	J

DIMENSION:

Dimension is a philosophical word. In literature it means a line or direction. In mathematics it means an axis but in physics it is used to denote the nature of a physical quantity) which comes from the involvement of fundamental quantities in that particular quantity. So each of the fundamental quantities is called Dimension. Hence the dimensions of physics are mass (M), length (L) and time (T). These are the fundamental quantities on which physics is based upon.

EXAMPLES:

S.No	Physical Quantity	Formula	Dimension	S.I Unit
1	Area	Length x breadth	L^2	m^2
2	Volume	Length x breadth x height	L^3	m^3
3	Density	Mass / Volume	$M\ L^{-3}$	$Kg\ m^{-3}$
4	Speed or Velocity	Distance/ Time	LT^{-1}	$m\ s^{-1}$
5	Acceleration	Velocity/Time	LT^{-2}	$m\ s^{-2}$
6	Force	Mass x acceleration	MLT^{-2}	$kgm\ s^{-2} = N$
7	Pressure	Force/Area	$ML^{-1}T^{-2}$	$kg\ m^{-1}\ s^{-2} = Pa$
8	Momentum	Mass/ Velocity	MLT^{-1}	$kqms^{-1}$
9	Work	Force x Displacement	ML^2T^{-2}	$kg\ m^{-2}\ s^{-2} = J$
10	Energy	Work	ML^2T^{-2}	$kg\ m^{-2}\ s^{-2} = J$
11	Power	Work / Time	ML^2T^{-3}	$kg\ m^{-2}\ s^{-3} = W$
12	Gravitational Constant	Force x Distance^2/Mass^2	$M^{-1}L^3T^{-2}$	$kg^{-1}\ m^{-3}\ s^{-2}$
13	Torque or couple	Force x perpendicular distance	ML^2T^{-2}	N.m
14	Angular Momentum	mass x velocity x radius	ML^2T^{-1}	kgm^2s^{-1}
15	Angle	Arc length \ radius	dimension less	radian

SIGNIFICANT FIGURES:

"All the accurately known digits in a value and the first doubtful digit are known as significant figures." In the measurement of any physical quantity the number of digits about which we are sure are called significant figures All physical measurements involve some degree of inaccuracy due to human error. instrumental error or due to both and therefore the knowledge of precision of a measurement is very important. A significant figure is that which is known to be reasonably reliable. The last figure being reasonably correct guarantees the certainty of the preceding figures.

RULES FOR COUNTING SIGNIFICANT FIGURES:

(i) In whole number values. all the digits except zeros at the right side m recognized as significant figures.

(ii) In decimal number values the zeros at the right side of the number are counted as significant figures but the zeros at the left side are not taken as significant figures.

(iii) Power or exponents to a certain base are not taken as significant figures.

(iv) In addition and subtraction process, the result should be rounded off to contain as many as decimal places as contained in the value of least number of decimal place.

(v) In multiplication and division process the result should be rounded off to contain as many as significant figures as contained in the factor of least significant figures.

FOR EXAMPLE:

S.NO	Value	No. of significant figures
1	0.00045	2(4,5)
2	1.2000	5(1,2,0,0,0)
3	505	3(5,0,5)
4	34000	2(3,4)
5	6.67×10^{32}	3(6,6,7)

M.C.Qs.

1. Physics can be defined as the study of:
(a) Chemical Properties of matter
(b) Physical properties of matter
(c) Relation between matter and energy
(d) Both (b) and (c)

2. Physics can be defined as a branch of science based on a:
(a) Aberration and analysis of facts
(b) Experimental observation and quantitative measurement.
(c) Mathematical calculation and interpretation.
(d) Replication and verification of known facts.

3. The branch of physics deals with the study of production propagation and properties of light:
(a) Magnetics (b) Optics
(c) Statics (d) Acoustics

4. High energy physics deal with the:
(a) Study of electron behavior
(b) Study of electronic charges
(c) Study of mechanics of energetic bodies.
(d) Study of properties and behavior of elementary particles.

5. The significant figures in 0.0064 :
(a) 2 (b) 4 (c) 5 d) 1

6. Archimedes the Greek physicist has made significant contributions in the field of.
(a) High energy physics and electronics
(b) Nuclear and atomic Physics
(c) Mechanics hydraulics and hydrostatics
(d) Special theory of relativity

7. Al-Beruni is famous for finding out the:
(a) Distance of moon from earth
(b) Mass of the earth
(c) Diameter of earth's orbit
(d) Circumference of the earth

8. The book "Kitab-ul-Qanoon-ul Masoodi" was written by:
(a) Iben-e-Sina (b) Al-Razi
(c) Abu-Rehan Al-Beruni (d) Ibn-al-Haitham

9. Dr. Asalam was awarded noble Prize for his work on.

(a) Electronics (b) Radiations
(c) Optics (d) Grand unification theory

10. The first book on analytical "Hisab-ul-jabrwal-Moqabla" was written by:

(a) Al-Khawarzmi (b) Al-Beruni
(c) Al-Razi (d) Ibn-e-sina

11. "Kitab-ul-Manazir" the famous book on optical is written by:

(a) Ibn-e-Sina (b) Al-Khawrzmi
(c) Jabir-bin-Hayan (d) Ibn-ul-Hailham

12. In international system of units, the length mass time electric current temperature, intensity of light and quantity of light and quantity are called:

(a) Derived (b) basic
(c) Fundamental (d) only (b) and (c)

13. Written of the flowing physical quantity will be different units as compared to that of others:

(a) Weight (b) Tension
(c) Buoyant Force (d) Electromotive Force

14. Which one of the following is not of the same quantity?

(a) Horse Power (b) Calorie
(c) Joules (d) BTU

15. The S.I unit of current is:

(a) one volt (b) One ohm
(c) One ampere (d) One ohm-m

16. The famous mathematical and the founder of algebra was.

(a). Al Kindi (b) Al Khwarizmi
(c)Al Beruni (d) Naserudin tusi

17. Light year is a unit of:

(a) Distance (b) Light
(c) Time (d) Pressure

18. Some of the basic S.I. units are:

(a) Second Ampere mole
(b) Kelvin Ampere watt
(c) Candela Mole volt
(d) Meter Second watt

19. 10^{-9} second are equivalent to:

(a) Deci Second (b) Nano Second
(c) Milli second (d) Micro second

20. The S. I unit of temperature is:

(a) Fahrenheit (b) Kelvin
(c) Centigrade (d) Farad

21. One Angstrom equal:

(a) 10^{-8} cm (b) 10^{8} m
(c) 10^{-6} m (d) 10^{8} mm

22. In Physics the term "dimension" represent the:

(a) mechanical nature of a quantity
(b) chemical nature of quantity
(c) Physical nature of quantity
(d) electric nature of quantity

23. Dimension of pressure is:

(a) $ML^{-1}T^{-2}$ (b) $ML^{-2}T^{-3}$
(c) $ML^{-2}T^{-4}$ (d) MLT^{-1}

24. Which one of the following represents the dimension of power?

(a) L^2T^2 (b) MLT^2
(c) ML^2T^{-3} (d) $ML^{-2}T$

25. Which one of the following represent dimension for the unit of torque:

(a) M^2LT^2 (b) ML^2T^{-2}
(c) M^2LT^2 (d) MLT^2

PAST PAPER M.C.Qs.

2022

16 .The first Muslim scientist who invented intravenous injection is.

*Al- beruni
* Ibn ul Haitham
* Yaqub Al kindi
***Ibn e Sina**

20 .The dimensions of ratio between angular momentum and linear momentum is:

*L^{-1}
* ML
*** L**
* L^{-2}

33 .The number of significant figures in 2.0305:

*4
* 3
*** 5**
* 2

2021

1 .The dimensions of Pressure is:

*** $ML^{-1}T^{-2}$**
* $M^1L^{-2}T^{-3}$
* ML^2T^{-1}
* MLT^{-1}

19. The dimensions of G are:

***$M^{-1}L^3T^{-2}$**
*$M^2L^2T^{-2}$
*$M^{-1}L^2T^{-2}$
*MLT^{-2}

31. The dimensions of angular momentum are:

* $M^2L^2T^2$
* M^2L^2T
* ML^2T
*** ML^2T^{-1}**

2019

5.The dimensions of G/g are:

* $M^0L^{-1}T^{-2}$
* $M^1L^2T^{-2}$
* $M^{-1}L^2T^{-2}$
*** $M^{-1}L^2T^0$**

2018

3. The luminous intensity of light is measured in:

*decibel
***candela**
*dioptre
*watt/ m^2

6. The dimensions of angular velocity are :

* ML^0T^{-1}
* ML^0T^{-2}
*** $M^0L^0T^{-1}$**
* $M^0L^0T^{-2}$

2017

5. Light year is the unit of:

*time
***distance**
*velocity
*luminous intensity

15. The number of significant figure in 1.6×10^{-19} is:

***2**
*3
*4
*6

2016

1. The dimensions of G are:

$*M^{-1}L^3T^{-2}$ $*M^2L^2T^{-2}$ $*M^{-1}L^2T^{-2}$ $*MLT^{-2}$

12. KitabulManazir was written by:
***Ibn-Al Haitham** *Al Razi *Abu-Rehan Al Beruni *Jabir bin Hayyan

2015

2. Intravenous injection by means of silver syringe was initially used by:
***Ibn-e-Sina** *Omar Khayyam *Al-Beruni *Jabir bin Hayyan

11. The dimensions of 'G' are
$*ML^{-1}T$ $*ML^{-2}T^3$ **$*M^{-1}L^3T^{-2}$** $*ML^2T^{-2}$

2014

4. The unit of Luminous intensity is:
*Decibel ***Candela** *Dioptre $*w/m^2$

13. The Noble price in Physics was awarded to this Pakistani Scientist:
*Dr. Abdul Qadeer Khan *Dr. Saleem uz Zaman Siddiqui
***Dr. Abdus Salam** *Dr. Samar Mubarak

17. The dimension of Torque is:
$*\ ML^2\ T$ **$*\ ML^2\ T^{-2}$** $*\ ML^2T^2$ $*MLT^2$

2013

13. The dimension of force is:
$*MLT$ $*MLT^{-1}$ $*MLT^2$ **$*\ MLT^{-2}$**

14. Light year is the unit of:
*time ***distance** *velocity *luminous intensity

2012

9. The products of two numbers 5.642 and 4.71 in the prospective significant number are:
*26.57382 * 26.574 *** 26.6** * 26.5738

2011

4. Light year is the unit of:
*time ***distance** *velocity *luminous intensity

16. The dimensions of 'G' are
$*ML^{-1}T$ $*ML^{-2}T^3$ **$*M^{-1}L^3T^{-2}$** $*ML^2T^{-2}$

2010

11. The number of significant figure in 7.050×10^{-2} is:
*2 *3 ***4** *6

12. Kitab ul Manazir is written by:

***Ibn ul Haitham** *Ibn e Sina *Al Razi *Al Beruni

13. The dimensions of angular momentum are:

* $M^2L^2T^2$ * M^2L^2T * ML^2T *** ML^2T^{-1}**

TEXTBOOK NUMERICALS

Q.1: Find the area of a rectangular plate having length (21.3 ± 0.2)cm and width (9.80 ± 0.10)cm.

Data:

Length of Rectangular plate =

$$l = (21.3 \pm 0.2)\text{cm}$$

Width of Rectangular plate =

$$w = (9.80 \pm 0.10)\text{cm}$$

Area of Rectangular Plate = A =?

Solution:

The Area of rectangular plate is given by

$$A = l \times w$$

$$A = (21.3 \pm 0.2) \times (9.80 \pm 0.10)$$

$$A = 208.74 \pm 2.13 \pm 1.96$$

$$\boxed{A = 208.74 \pm 4.09}$$

Result:

The Area of rectangular plate is 208.74 $\pm$ 4.09 cm.

Q.2: Calculate (a) the circumference of a circle of radius 3.5 cm and (b) area of a circle of radius 4.65 cm.

Data:

(a) Radius of Circle = r_1 = 3.5 cm

Circumference of Circle = S =?

(b) Radius of Circle = r_2 = 4.65 cm

Area of Circle = A =?

Solution:

(a) The Circumference of circle is given by

$$S = 2\pi r_1$$

$$S = 2 \times 3.14 \times 3.5$$

$$\boxed{S = 21.98\ cm}$$

(b) The Area of circle is given by

$$A = \pi r_2^2$$

$$A = 3.14 \times (4.65)^2$$

$$\boxed{A = 67.89\ cm^2}$$

Result:

(a) The circumference of circle is 21.98 cm

(b) The area of circle is 67.89 cm^2

Q.3: Show that the expression S = Vit + ½ at² is dimensionally correct, when S is a co-ordinate and has unit of length, Vi is velocity, a is acceleration, and t is time.

Data:

Dimension of S = $[S] = L$

Dimension of V_i = $[V_i] = LT^{-1}$

Dimension of t = $[t] = T$

Dimension of a = $[a] = LT^{-2}$

Dimension of 1/2 = $\left[\frac{1}{2}\right] = No\ Dimension$

Proof:

L.H.S:

L.H.S = $[S]$

$\boxed{L.H.S = L}$ ---(i)

R.H.S:

$$R.H.S = [LT^{-1}][T] + [LT^{-2}][T]^2$$

$$R.H.S = LT^0 + LT^0 \qquad \{T^0 = 1\}$$

$R.H.S = L + L$

$R.H.S = 2L$

Since "2" has no dimension, Therefore

$\boxed{R.H.S = L}$---(ii)

Comparing eq (i) and eq (ii)

$\boxed{L.H.S = R.H.S}$ { Hence Proved}

Q.4: Suppose the displacement of a particle is related to a time according to expression S = ct³ . What are the dimensions of the constant c.

Data:

Dimension of S $= [S] = L$

Dimension of t$= [t] = T$

Dimension of c $= [c] =?$

Solution:

According to the given condition

$S = ct^3$

or $c = \frac{S}{t^3}$

So, $[c] = \frac{L}{T^3}$

$\boxed{[c] = LT^{-3}}$

Result:

The dimension of c is LT^{-3}

Q.5: Estimate the number of litres of gasoline used by all Pakistan's car each year: Given: No. of cars in Pakistan = 500000. Average distance traveled per year by each car = 16000 km gasoline consumption 6km/litre

Data:

Number of litres of gasoline used by all Pakistan's car each year = n = ?

No. of cars in Pakistan = N = 500000

Average distance traveled per year by each car = S = 16000 km

Gasoline consumption = C = 6km/litre.

Solution:

First we calculate the total distance covered by all cars

$S_{total} = S \times N$

$S_{total} = 16000 \times 500000$

$S_{total} = 8000000000\ km$

Now, we will calculate the No. of liters of gasoline by all cars

$n = \frac{S_{total}}{C} = \frac{8000000000}{6}$

$\boxed{n = 1.33 \times 10^9 liters}$

Result: The number of litres of gasoline used by all Pakistan's car each year is $1.33 \times 10^9 liters$

PAST PAPER NUMERICALS

2022

Q.2 ii) Show that any two of the following equations are dimensionally correct:

a) $S=v_it+1/2\ at^2$ b) $T = 2\pi\sqrt{\frac{m}{k}}$ c) $T = 2\pi\sqrt{\frac{l}{g}}$

a) Data:

Dimension of S $= [S] = L$

Dimension of $V_i = [V_i] = LT^{-1}$

Dimension of t $= [t] = T$

Dimension of a $= [a] = LT^{-2}$

Dimension of $1/2 = \left[\frac{1}{2}\right] = No\ Dimension$

Proof:

L.H.S:

L.H.S = $[S]$

$\boxed{L.H.S = L}$ ---(i)

R.H.S:

$R.H.S = [LT^{-1}][T] + [LT^{-2}][T]^2$

$R.H.S = LT^0 + LT^0$ $\{T^0 = 1\}$

$R.H.S = L + L$

$R.H.S = 2L$

Since "2" has no dimension, Therefore

$\boxed{R.H.S = L}$---(ii)

Comparing eq (i) and eq (ii)

$\boxed{L.H.S = R.H.S}$ { Hence Proved}

b)) **Data:**

Dimension of time period $=[T] = T$

Dimension of 2π = No Dimension

Dimension of mass $= [M] = M$

Dimension of k $= [k] = MT^{-2}$

Solution:

$\boxed{L.H.S = [T] = T}$

And

$$R.H.S = \sqrt{\frac{M}{MT^{-2}}}$$

$R.H.S = \sqrt{T^2}$

$\boxed{R.H.S = T}$

$\boxed{L.H.S = R.H.S}$

c)) **Data:**

Dimension of time period $=[T] = T$

Dimension of 2π = No Dimension

Dimension of mass $= [L] = L$

Dimension of k $= [g] = LT^{-2}$

Solution:

$\boxed{L.H.S = [T] = T}$

And

$$R.H.S = \sqrt{\frac{L}{LT^{-2}}}$$

$R.H.S = \sqrt{T^2}$

$\boxed{R.H.S = T}$

$\boxed{L.H.S = R.H.S}$

2017

2(vii) Show that the expression $f = \frac{1}{2l}\sqrt{\frac{F\times l}{m}}$ is dimensionally correct and find the dimension of Kinetic Energy.

Data:

Dimension of frequency$=[f] = T^{-1}$

Dimension of **2=[2]** =NIL

Dimension of Force$=[F] = MLT^{-2}$

Dimension of length$=[l] = L$

Dimension of mass$=[m] = M$

Solution:

$\boxed{L.H.S = [f] = T^{-1}}$

And

$$R.H.S = \frac{1}{L}\sqrt{\frac{MLT^{-2}\times L}{M}}$$

$$R.H.S = \frac{1}{L}\sqrt{L^2T^{-2}}$$

$$R.H.S = \frac{1}{L}\times LT^{-1}$$

$\boxed{R.H.S = T^{-1}}$

$\boxed{L.H.S = R.H.S}$

Now,

$$\boxed{K.E = \frac{1}{2}mv^2 = [M][LT^{-1}]^2 = ML^2T^{-2}}$$

2016

Q.2 (ii) Show that the following formulae are dimensionally correct:

(a) V = fλ. (b) $T = 2\pi\sqrt{\frac{m}{k}}$

(a) Data:

Dimension of speed$=[v] = LT^{-1}$

Dimension of frequency$=[f] = T^{-1}$

Dimension of wavelength $= [\lambda] = L$

Solution:

$L.H.S = [v] = LT^{-1}$

And

$R.H.S = [T^{-1}][L]$

$R.H.S = LT^{-1}$

$L.H.S = R.H.S$

(b) Data:

Dimension of time period $= [T] = T$

Dimension of 2π = No Dimension

Dimension of mass $= [M] = M$

Dimension of k $= [k] = MT^{-2}$

Solution:

$L.H.S = [T] = T$

And

$R.H.S = \sqrt{\frac{M}{MT^{-2}}}$

$R.H.S = \sqrt{T^2}$

$R.H.S = T$

$L.H.S = R.H.S$

2013

Q.2(xv) **Same as 2016 Q.2 (ii)**

2012

Q.2 (ii) Give the dimensions of the following quantities: (a) Torque (b) Angular momentum (c) Pressure (d) K.E

Dimension of Torque $= \tau = Fd = MLT^{-2} \times L = ML^2T^{-2}$

Dimension of Angular Momentum $= l = mvr = M \times LT^{-1} \times L = ML^2T^{-1}$

Dimension of Pressure $= P = \frac{F}{A} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$

Dimension of Kinetic Energy $= K.E = \frac{1}{2}mv^2 = M \times (LT^{-1})^2 = ML^2T^{-2}$

THEORY NOTES

SCALARS

DEFINITION:

"Scalars are those physical quantities which can be specified by a number having appropriate unit"

EXAMPLES: Mass, Temperature, Volume, Work, Energy

PROPERTIES:

a) **DIRECT PREFERENCE:**

It is a measure in which direction is unimportant or meaningless thus, they are quoted as a pure number with a unit

b) **ARITHMETICAL OPERATIONS:**

Scalars can be added, subtracted, multiplied or divide by simple arithmetical rules.

c) **REPRESENTATION:** Scalars are represented by a number with a suitable unit.

d) **EQUIVALENCE:**

Two or more than two scalars (measured in the same system of units) are equal only if they have same magnitude and sign.

VECTORS:

DEFINITION:

"Vectors are those physical quantities which can be specified by magnitude and direction with appropriate"

EXAMPLES: Force, Velocity, Displacement, Torque, Momentum

PROPERTIES:

a) **DIRECT PREFERENCE:**

It is a measure in which direction is important or must be usually be specified. Thus, they are quoted as a number with a unit and a direction.

b) ARITHMETICAL OPERATIONS:

Vectors can't be added, subtracted multiplied or divided by simple Mathematical rules. Addition of vectors must take account of direction. Multiplication of vectors is performed in two ways (i) Scalar Product (ii) Vector Product

c) REPRESENTATION:

Vectors are represented by an arrow-headed line segment.

⟶ Direction

Length = Magnitude

d) EQUIVALENCE:

Two or more Vectors are equal only if they same magnitude and direction.

RESOLUTION OF VECTORS:

Vectors can be resolved in two or more components. Thus, splitting of a vector into components is termed as "Resolution of Vectors".

RECTANGULAR COMPONENTS:

If the components of a vector are perpendicular to each, other then these usually these components are called:

i) The Horizontal component on X-Component
ii) The Vertical component on Y- Component.
iii)

PROCEDURE OF RESOLUTION:

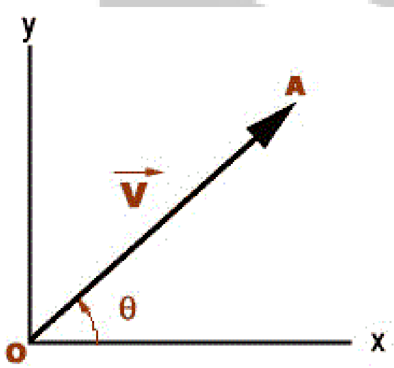


figure 01

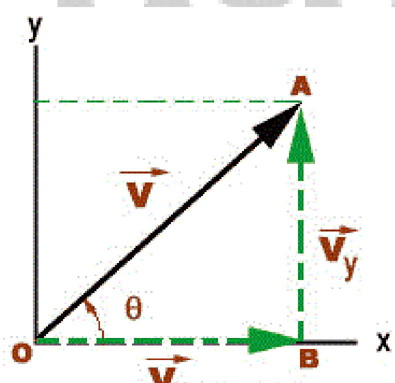


figure 02

Consider a vector $\vec{V}$ acting at a point making an angle **q** with positive X-axis. Vector $\vec{V}$ is represented by a line OA.From point A draw a perpendicular AB on X-axis.Suppose OB and BA represents two vectors.Vector OA is parallel to X-axis and vector BA is parallel to Y-axis.Magnitude of these vectors are V_x and V_y respectively.

By the method of head to tail we notice that the sum of these vectors is equal to vector .Thus Vx and Vy are the rectangular components of vector .

Now,

$\vec{V_x}$ = Horizontal Component
$\vec{V_y}$ = Vertical Component

From figure, $\overrightarrow{OA} = \overrightarrow{OB} + \overrightarrow{AB}$

$$\vec{V} = \vec{V_x} + \vec{V_y}$$

$$\vec{V} = \vec{V_x}\,\hat{i} + \vec{V_y}\,\hat{j}$$

In ΔOAB

$$Cos\theta = \frac{\overrightarrow{OB}}{\overrightarrow{OA}} \quad \text{or } Cos\theta = \frac{\vec{V_x}}{\vec{V}}$$

or

$$\boxed{V_x = VCos\theta} \text{-----------(i)}$$

and

$$Sin\theta = \frac{\overrightarrow{AB}}{\overrightarrow{OA}} \quad \text{or } Sin\theta = \frac{\vec{V_y}}{\vec{V}}$$

or

$$\boxed{V_y = VSin\theta} \text{-----------(ii)}$$

MAGNITUDE:

Squaring eq(i) and eq(ii) and then adding

$$V_x{}^2 + V_y^2 = V^2Cos^2\theta + V^2Sin^2\theta$$

or $$V_x{}^2 + V_y^2 = V^2(Cos^2\theta + Sin^2\theta)$$

Since $Cos^2\theta + Sin^2\theta = 1$

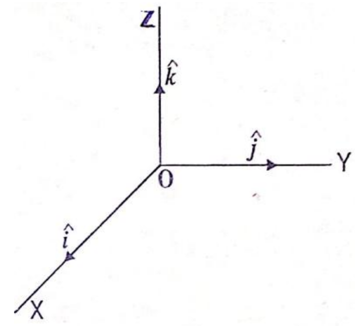
or $$V^2 = V_x{}^2 + V_y^2$$

or

$$\boxed{V = \sqrt{V_x{}^2 + V_y^2}}$$

DIRECTION:

Dividing Eq(i) by Eq(ii)



$$\frac{V_x}{V_y} = \frac{VSin\theta}{VCos\theta}$$

or

$$\frac{V_x}{V_y} = Tan\theta$$

or

$$\boxed{\theta = Tan^{-1}\left(\frac{V_x}{V_y}\right)}$$

UNIT VECTOR:

A vector whose magnitude equals to "one" is called unit vector and it just represents direction of vector. In three dimensional space the unit vectors along x,y and z axis are i, j and k respectively.

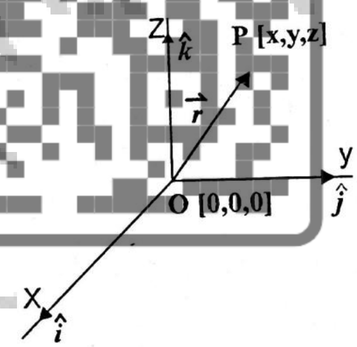
Mathematically a vector quantity $\vec{A}$ is defined through the equation.

$\vec{A} = |\vec{A}|\hat{a}$ or $\vec{A} = A\hat{a}$

Where $|\vec{A}|$ represents magnitude or length of vector, $\hat{a}$ is the unit vector which represents direction of vector $\vec{A}$ e.g. If Force on a body 8 N along x - axis, then $\vec{F} = 8\hat{\imath}\ N$

POSITION VECTOR

A vector which starts from origin or fixed point is called position vector. In three dimensional space it is usually written as $\vec{r}$ and in component form,



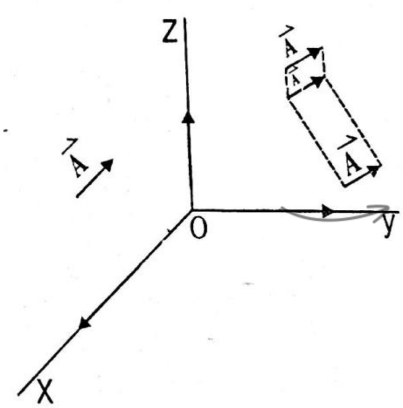
$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

The magnitude of this position vector is represented by

$$r = \sqrt{x^2 + y^2 + z^2}$$

FREE VECTOR :

Such vector which can be displaced anywhere in space parallel to itself is called Free Vector. In this case magnitude and direction remain same. All vectors except positions are free vectors.



NULL VECTOR:

A vector whose magnitude equals to zero and has no direction and it may have any direction is called Null Vector.

Thls vector always appears as resultant of addition of two equal but opposite vectors i.e.

If A and B are equal in magnitudes and parallel but in opposite direction then

$$\vec{A} + \vec{B} = \vec{0}$$

Here $\vec{0}$ is null vector, and it may be written as

$$\vec{0} = 0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$$

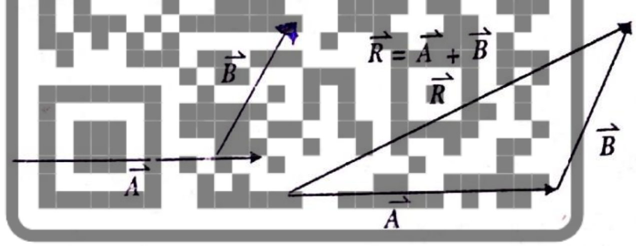
ADDITION OF VECTORS:

A vector can be added in another vector only which results in a new vector of the same kind. This may done in three different ways.

(i) By head - to - tail rule, or graphical method, or triangular law of vector addition.
(ii) Analytical method.
(iii) Addition of vectors by rectangular components method.

(i) GRAPHICAL METHOD (HEAD - TO - TAIL RULE):

In head - to - tail rule, number of vectors can be added by joining tail of successive vectors with the head of previous vector. The resultant vector is obtained by joining the tail of first with the head of last vector.



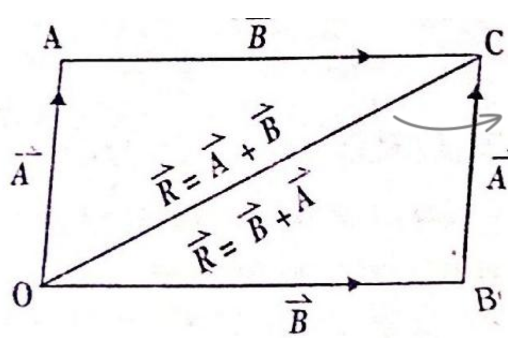
$$\vec{R} = \vec{A} + \vec{B}$$

This equation gives resultant vector and it is also known as “Triangle Law of Vector Addition”.

PROPERTIES OF VECTOR ADDITION:

(a) Commutative law of vector addition:

Suppose two vectors $\vec{A}$ and $\vec{B}$ represent the two adjacent sides of a parallelogram then the diagonal OC represents the resultant vector $\vec{R}$ shown in fig.



Since $\vec{R} = \vec{A} + \vec{B}$

$$\vec{R} = \vec{B} + \vec{A}$$

Therefore

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

This is known as commutative law of vector B addition or parallelogram law of vector addition.

(b) Associative law of vector addition:

Three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are added (suppose using head to tail rule) in two different ways as shown in flg

The resultant vector $\vec{R}$ may be obtained in two different ways i.e.

$$\vec{R_1} = \vec{A} + \vec{B}$$

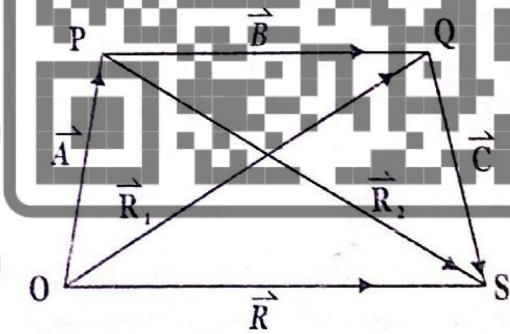
adding $\vec{C}$ on both sides

$$\vec{R_1} + \vec{C} = (\vec{A} + \vec{B}) + \vec{C}$$

$$(\vec{A} + \vec{B}) + \vec{C} = \vec{R} \text{-----(i)}$$

Now,

$$\vec{R_2} = \vec{B} + \vec{C}$$



adding $\vec{A}$ on both sides

$$\vec{A} + \vec{R_1} = \vec{A} + (\vec{B} + \vec{C})$$

$$\vec{A} + (\vec{B} + \vec{C}) = \vec{R} \text{-----(ii)}$$

By comparing eq(i) and eq(ii) we get

$$\boxed{(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})}$$

This property is known as associative law of vector addition.

<u>(ii) Analytical Method of Vector Addition:</u>

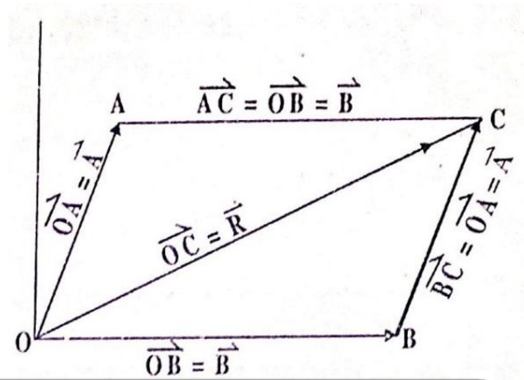
This is a mathematical method of vector addition and it is based upon laws of trigonometry.
According to law of cosines, in any triangle,

$a^2 = b^2 + c^2 - 2bc \, Cos \propto$
$b^2 = a^2 + c^2 - 2ac \, Cos \, \beta$

Then the magnitude of resultant vector will be

$$R^2 = A^2 + B^2 - 2AB \cos < OAC$$

or $R = \sqrt{A^2 + B^2 - 2AB \cos < OAC}$



According to law of sines

$$\frac{a}{sin \propto} = \frac{b}{sin\beta} = \frac{c}{sin\gamma}$$

and in our fig.

$$\frac{A}{sin < ACO} = \frac{B}{sin < AOC} = \frac{R}{sin < OAC}$$

This formula gives direction of resultant vector R. However this method is restricted to the addition of two vectors only.

(iii) Rectangular Components Method:

The way of adding vectors with the help of their rectangular components is called addition of vectors by rectangular components method.

Suppose two Position vectors $\vec{V_1}$, and $\vec{V_2}$ having lengths or magnitudes V_1 and V_2 and making angles θ_1 and θ_2 respectively are to be added.

For this purpose we first adopt head-to-tail rule and then we draw perpendiculars form their heads on x and y axes to get their rectangular components as shown in fig

The rectangular components of V_1 are

$$V_{1x} = V_1 Cos\theta_1$$
$$V_{1y} = V_1 Sin\theta_1$$

The rectangular components of V_2 are

$$V_{2x} = V_2 Cos\theta_2$$
$$V_{2y} = V_2 Sin\theta_2$$

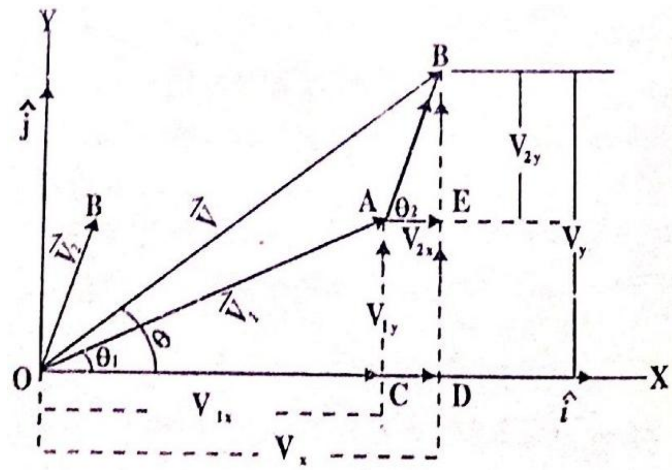
It is clear from the figure that

$$\overrightarrow{V_{1x}} = \overrightarrow{OC}$$
$$\overrightarrow{V_{2x}} = \overrightarrow{AE}$$
$$\overrightarrow{V_x} = \overrightarrow{OD}$$

and

$$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CD}$$

or $$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{AE}$$



$$\overrightarrow{V_x} = \overrightarrow{V_{1x}} + \overrightarrow{V_{2x}}$$
$$\overrightarrow{V_x}\hat{\imath} = \overrightarrow{V_{1x}}\hat{\imath} + \overrightarrow{V_{2x}}\hat{\imath}$$
$$\overrightarrow{V_x}\hat{\imath} = (\overrightarrow{V_{1x}} + \overrightarrow{V_{2x}})\hat{\imath}$$
$$\overrightarrow{V_x} = \overrightarrow{V_{1x}} + \overrightarrow{V_{2x}}$$

or $$\boxed{\overrightarrow{V_x} = V_1 Cos\theta_1 + V_2 Cos\theta_2}$$

Similarly for y component

$$\overrightarrow{V_{1y}} = \overrightarrow{CA}$$
$$\overrightarrow{V_{2y}} = \overrightarrow{EB}$$
$$\overrightarrow{V_y} = \overrightarrow{DB}$$

and

$$\overrightarrow{DB} = \overrightarrow{DE} + \overrightarrow{EB}$$

or $$\overrightarrow{OD} = \overrightarrow{CA} + \overrightarrow{AE}$$

$$\overrightarrow{V_y} = \overrightarrow{V_{1y}} + \overrightarrow{V_{2y}}$$
$$\overrightarrow{V_y}\hat{\jmath} = \overrightarrow{V_{1y}}\hat{\jmath} + \overrightarrow{V_{2y}}\hat{\jmath}$$
$$\overrightarrow{V_y}\hat{\jmath} = (\overrightarrow{V_{1y}} + \overrightarrow{V_{2y}})\hat{\jmath}$$
$$\overrightarrow{V_y} = \overrightarrow{V_{1y}} + \overrightarrow{V_{2y}}$$

or $$\boxed{\overrightarrow{V_y} = V_1 Sin\theta_1 + V_2 Sin\theta_2}$$

Now , the resultant can be calculated by the formula

$$\boxed{V = \sqrt{{V_x}^2 + V_y^2}}$$

PRODUCT OF TWO VECTORS

Vector can be multiplied in two different ways.

i) Scalars product.

ii) Vector product.

SCALAR PRODUCT

DEFINITION:

"The multiplication of two vectors to give a scalar."
Or in other words,
"it involves the multiplication of two vectors in such a way that their product is a scalar quantity".

REPRESENTATION:

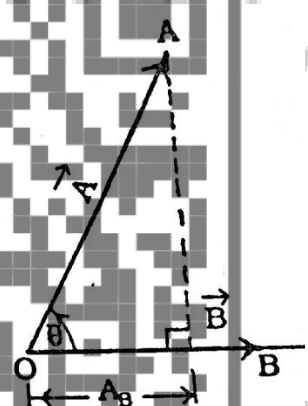
This product is represented by placing a 'DOT' between two vectors.

EXAMPLES:

1) **Work:** Work is a scalar product of force (F) and displacement (d)
2) **Power:** Power is the scalar product of force (F) and d velocity (V)

EXPLANATION:

Scalar product of two vectors A and B is the product of magnitudes of two vectors and the cosine of the angle between them.

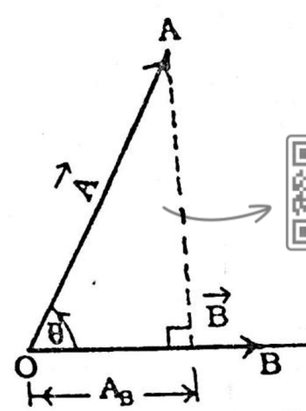


Let vector A and B. Draw a perpendicular form head of B on x axis.

According to the definition of scalar product , it is equal to the product of magnitude of first vector with the length of projection of second vector onto first vector.

Thus,

$$\vec{A}.\vec{B} = ABCos\theta$$



CHARACTERISTICS OF DOT PRODUCT:

1) Commutative Law:

$$\vec{A}.\vec{B} = \vec{B}.\vec{A}$$

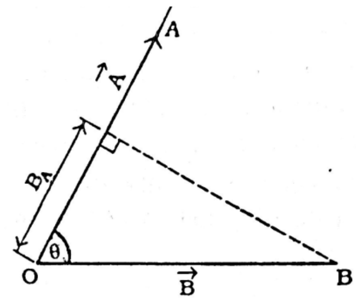
Proof:

Let vector A and B. Draw a perpendicular form head of B on x axis.

According to the definition of scalar product , it is equal to the product of magnitude of first vector with the length of projection of second vector onto first vector.

Thus,

$\vec{A}.\vec{B} \quad = \quad ABCos\theta \quad = \quad A(BCos\theta)$



In the same way, when

ACosθ = Magnitude of component of A onto B,

$\vec{B}.\vec{A} \quad = \quad B(ACos\theta) \quad = \quad BACos\theta$

Hence, it is clear that

$ABCos\theta \quad = \quad BACos\theta$

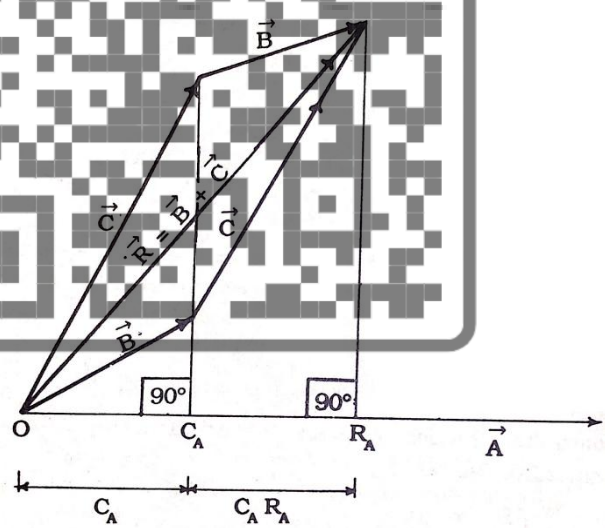
$$\boxed{\vec{A}.\vec{B} \quad = \quad \vec{B}.\vec{A}}$$

2) Distributive Law:

$\vec{A}.(\vec{B}+\vec{C}) \quad = \quad \vec{A}.\vec{B}+\vec{A}.\vec{C}$

Proof:

Let us consider Vector $\vec{A}$, $\vec{B}$ and $\vec{C}$ in different directions.let us first add B and C to get resultant vector $\vec{R}$,i.e.



$\vec{R} = \vec{B} + \vec{C}$

Now

$\vec{A}.\vec{R} = A(R_A)$

$\vec{A}.\vec{R} = A(\overline{ON})$

$\vec{A}.\vec{R} = A(\overline{OM} + \overline{MN})$

$\vec{A}.\vec{R} = A(B_A + C_A)$

$\vec{A}.\vec{R} = AB_A + AC_A$

or

$$\boxed{\vec{A}.(\vec{B}+\vec{C}) \quad = \vec{A}.\vec{B}+\vec{A}.\vec{C}}$$

3. If $\vec{A}$ is parallel to $\vec{B}$ i.e. $\theta = 0°$ then

$$\vec{A}.\vec{B} = AB$$

4. **If $\vec{A} = \vec{B}$ i.e. $\vec{A}$ is parallel and equal to B then**

$$\vec{A}.\vec{A} = (A)\ (A)\ (\cos 0°) = A^2$$

5. **If $\vec{A}$ is perpendicular to $\vec{B}$ i.e. $\theta = 90°$ or one of the vector is null vector then**

$$\vec{A}.\vec{B} = 0$$

6. The unit vectors $\hat{\imath}, \hat{\jmath}$ *and* $\hat{k}$ are perpendicular to each other therefore,

$$\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$$

$$\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = 0$$

VECTOR PRODUCT:

DEFINITION:

It is the multiplication of two Vectors to give vectors
Or,
"It involves the multiplication of two vectors in such a way that the product is also a vector.

REPRESENTATION: It is represented by placing a Cross (X) between the two vectors.

EXPLANATION:

1) **Torque:** It is the vector product of vector r and force F.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

2) **Force:** Force on a particle of change 'q' moving with velocity V in a magnetic field B is given by

$$\vec{F} = q\vec{v} \times \vec{B}$$

EXPLANATION:

Vector product is the product of magnitudes of two vectors and the sine of the angle between than

$$\vec{A} \times \vec{B} = AB \sin\theta\ (\hat{n})$$

DIRECTION OF VECTOR PRODUCT:

Direction of vector product can be determined by right hand rule.

CHARACTERISTICS OF VECTOR PRODUCT:

1. COMMUTATIVE LAW:

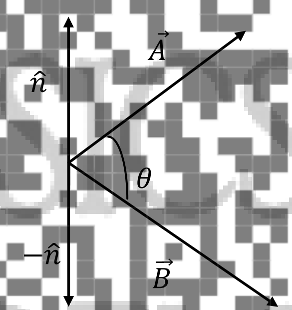
$$\vec{A} \times \vec{B} = -\vec{A} \times \vec{B}$$

It means that vector product is not commutative.

Proof:
By the definition of vector product,

$$\vec{A} \times \vec{B} = AB\,\text{Sin}\theta\,(\hat{n}) \text{------------(i)}$$

where $\hat{n}$ is the unit vector normal pointing outwards the plane of vector $\vec{A}$ and $\vec{B}$.



Similarly,

$$\vec{B} \times \vec{A} = BA\,\text{Sin}(-\theta)\,(\hat{n})$$

or

$$\vec{B} \times \vec{A} = BA\,\text{Sin}(\theta)\,(-\hat{n}) \qquad \text{Since } \text{Sin}(-\theta) = -\text{Sin}\theta$$

or

$$\vec{B} \times \vec{A} = -AB\,\text{Sin}(\theta)\,(\hat{n})$$

Using eq(i) we get,

$$\vec{B} \times \vec{A} = -\vec{A} \times \vec{B}$$

or

$$\boxed{\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}}$$

The above equation shows that vector product is **not** commutative.

2) Distributive Law:

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

or

$$(\vec{A} + \vec{B}) \times \vec{C} = \vec{A} \times \vec{C} + \vec{B} \times \vec{C}$$

3. If $\vec{A}$ is parallel to $\vec{B}$ i.e. $\theta = 0°$ then

$$\vec{A} \times \vec{B} = 0$$

4. If $\vec{A} = \vec{B}$ i.e. $\vec{A}$ is parallel and equal to B then

$$\vec{A} \times \vec{A} = (A)\ (A)\ (\text{Sin}0°) = 0$$

5. If $\vec{A}$ is perpendicular to $\vec{B}$ i.e. $\theta = 90°$ or one of the vector is null vector then

$$\vec{A} \times \vec{B} = AB\ \hat{n} \qquad \text{or} \qquad |\vec{A} \times \vec{B}| = AB$$

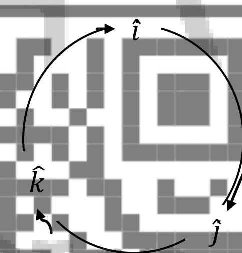
6. The unit vectors $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ are perpendicular to each other therefore,

$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$$

$$\hat{\imath} \times \hat{\jmath} = \hat{k} \quad , \qquad \hat{\jmath} \times \hat{\imath} = = -\hat{k}$$

$$\hat{\jmath} \times \hat{k} = \hat{\imath} \quad , \qquad \hat{k} \times \hat{\jmath} = -\hat{\imath}$$

$$\hat{k} \times \hat{\imath} = \hat{\jmath} \quad , \qquad \hat{\imath} \times \hat{k} = -\hat{\jmath}$$



M.C.Qs.

1. Which of the following is a vector quantity?
(a) Mass (b) Speed
(c) Temperature (d) Acceleration

2. Which one of the following is scalar?
(a) Acceleration (b) Velocity
(c) Force d) Work

3. In contrast to a scalar a vector must have a.
(a) Direction (b) Weight
(c) Quantity (d) None of these

4. Which is the following group of quantities represent the vectors:
(a) Acceleration, Force,Mass
(b) Mass,Displacement,velocity
(c) Acceleration, Electric flux, force
(d) Velocity, Electric field ,momentum

5. The following physical quantities are called vectors:
(a) Time and mass (b) Temperature and density (c) Force and Displacement (d) Length and volume

6. Vectors are physical quantities which are completely specified by:
(a) Magnitude-only (b) Direction only
(c) Magnitude and direction only
(d) None of these

7. Scalar quantities have:

(a) Only magnitudes (b) Only directions
(c) Both magnitude and direction
(d) None of these

8. A unit of a vector $\boldsymbol{A}$ is given by:
(a) $a = \boldsymbol{A}\ A$ (b) $a = \boldsymbol{A}\ /A$
(c) $a = A/\boldsymbol{A}$ (d) $a = \boldsymbol{A} + A$

9. A vector in space has ____________ components.
(a) one (b) Two
(c) Three (d) Four

10. When a vector is multiplied by a negative number its direction.
(a) is reversed (b) remains unchanged
(c) make and angle of 60^o
(d) may be changed or not

11. A vector which can be changed by display parallel to itself and applied at any point is known as:
(a) Parallel vector (b) Null vector
(c) Free vector (d) position vector

12. A vector in any given direction whose magnitude is unity is called:
(a) Normal vector (b) parallel vector
(c) Free vector (d) unit vector

13. The position vector of a point p is a vector that represents its position with respect to:
(a) Another vector
(b) Center of the earth
(c) Any point in space
(d) origin of the coordinate system

14. Negative of a vector has a direction ________that of the original vector.
(a) Same as (b) Perpendicular to
(c) Opposite to (d) Inclined to

15. The sum and difference of two vectors are equal in magnitude. The angle between the vectors is:
(a) 0^o (b) 90^0 (c) 120^o (d) 180^0

16. Two forces act together on an object The magnitude of their resultant is least when the angle between the forces is:
(a) 0° (b) 45° (c) 60° (d) 180°

17. The dot product of $\boldsymbol{i}$ and $\boldsymbol{j}$ is.
(a) more than 1 (b) 1 (c) 0 (d) any value

18. Scalar product obtains when.
(a) A Scalar is multiplied by a scalar.
(b) A scalar is multiplied by a vector
(c) Two vectors are multiplied to give a scalar
(d) Sum of two scalars is taken

19. If dot product of two vectors which are not perpendicular to each other is zero then either of the vector is obtain by adding two or more vectors is called:
(a) A unit vector (b) Opposite to the other
(c) A null vector (d) Position vector

20. The vector obtained by adding two or more vectors is called:
(a) Product Vector (b) Sum vector
(c) Resultant vector (d) Final vector

21. Scalar product of two vectors obeys.
(a) Commutative Law (b) Associate Law
(c) Both "a" and "b" (d) None of the above

22. If the dot product of two non-zero vectors A and B is zero. Their cross product will be of magnitude:
(a) AB Sin θ (b) B cos θ
(c) AB cos θ (d) AB

23. If the angle between the two vectors is zero degree then their:
(a) Dot product is zero
(b) Cross product is zero
(c) Either dot or cross product is zero
(d) Both dot & cross product is zero

24. $\boldsymbol{k}$ x $\boldsymbol{i}$ = ___________.
(a) j (b) $-j$ (c) k
(d) $-k$

25. If a . b = 0 and also a x b = 0 then
(a) a and b are parallel to each other

(b) a and b are perpendicular to each
(c) a and b is a null vector
(d) Either a or b is a null vector

26. The magnitude of vector product is:
(a) Sum of the adjacent side
(b) Area of the parallelogram
(c) Product of the parallelogram
(d) Parameter of the parallelogram

27. If two vectors lie in xy-plane then their cross product lies.
(a) In the same plane (b) Adjacent plane
(c) Alone parallel to that plane
(d) Parallel to the plane

28. Two forces of 8N and 6N are acting simultaneously at right angle the resultant force will be:
(a) 14N (b) 2N (c) 10N (d) 12N

29. Two forces each of magnitude F act perpendicular to each other. The-angle made by the resultant force with the horizontal will be.
(a) 30° (b) 45° (c) 60° (d) 90°

30. When two equal forces F and F makes an angle 180° with each other the magnitude of their resultant is.
(a) F (b) 0 (c) 2F (d) 0.5F

PAST PAPER M.C.Qs.

2022

5. If A .B =0, A ×B =0 and A≠0 then B is:
*equal to A **<u>* Zero</u>** *Perpendicular to A *Anti-parallel to A

10. The dot product of force and velocity is called:
*work **<u>*power</u>** *momentum *energy

18. The magnitude of product i .(k x j):
*zero *1 **<u>*-1</u>** ` *k

24. If a vector has three components each equal to "a" the magnitude of vector will be
*$\sqrt{3a}$ *3a *a^3 ` **<u>*$\sqrt{3}a$</u>**

26. It is not a vector quantity.
*Force *Torque **<u>*Frequency</u>** *Weight

2021

21 .If i, j and k are unit vectors then k. (i x j) is equal to:
*zero **<u>*1</u>** *j *k

29. The y-component of a vector A= 15 units, when it forms an angle of 50 with positive x-axis is

*9.6 units ***11.5units** *-9.6units *-11.5units

2019

6.The magnitude of resultant of two forces of magnitudes 2N and 10N cannot be:

*** 4N** *6N *9N *13N

14.(i x j) . (j x i) is:

***-1** *k *1 *zero

2018

2. Two perpendicular vectors having magnitudes of 4 units and 3 units are added. There resultant has magnitude of :

***5 units** *7 units *12 units *25 units

2017

16. The magnitude of product k.(j x i):

*zero *1 ***-1** ` *k

2016

10.If i, j and k are unit vectors then k. (i x j) is equal to:

*zero ***one** *j *k

16. If A .B =0, A ×B =0 and A≠0 then B is:

*equal to A *** Zero** *Perpendicular to A *Anti-parallel to A

2015

3. The y-components of vector A =15 units when it forms an angle of 50° with positive x-axis is:

*9.6 units * -9.6 units *** 11.5 units** *-11.5 units

2014

7. If A =5i+j and B =2k then A - B is equal to:

*5i+j+2k *5i-j-2k ***5i+j-2k** *-5i-j+2k

16. If A .B =0, A ×B =0 and A≠0 then B is:

*equal to A *** Zero** *Perpendicular to A *Anti-parallel to A

2013

5. Two forces act together on an object; the magnitude of their resultant is minimum when the angle between them is:

*0^0 *45^0 *90^0 ***180^0**

2012

15. If A=a i and B=bj , then A x B is equal to:

***0** * ab k * -ab k * none of these

2011

11. If A .B =0, A ×B =0 and A≠0 then B is:

*equal to A *** Zero** *Perpendicular to A *Anti-parallel to A

2010

10. If A .B =0, A ×B =0 and A≠0 then B is:

*equal to A *** Zero** *Perpendicular to A *Anti-parallel to A

TEXTBOOK NUMERICALS

Q.6: The following forces act on a particle P: $F_1 = 2i + 3j – 5k$, $F_2 = -5i + j + 3k$, $F_3 = i – 2j + 4k$, $F_4 = 4i – 3j – 2k$, measured in newtons Find (a) the resultant of the force (b) the magnitude of the resultant force .

Data:

$F_1 = 2i + 3j – 5k$

$F_2 = -5i + j + 3k$

$F_3 = i – 2j + 4k$

$F_4 = 4i – 3j – 2k$

$\vec{F} = ?$

$|\vec{F}| = ?$

Solution:

$$\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4}$$

$$\vec{F} = 2i + 3j – 5k + (-5i + j + 3k) + i – 2j + 4k + 4i – 3j – 2k$$

$$\vec{F} = (2i - 5i + i + 4i) + (3j + j - 2j - 3j) + (-5k + 3k + 4k - 2k)$$

$$\vec{F} = (2i - j + 0k)$$

$$\boxed{\vec{F} = 2i - j}$$

$$F = \sqrt{x^2 + y^2 + z^2}$$

$$F = \sqrt{(2)^2 + (-1)^2 + (0)^2}$$

$$\boxed{F = \sqrt{5}}$$

Result:

The resultant force is 2i − j and its magnitude is $\sqrt{5}$.

Q.7: If A = 3i – j – 4k, B = -2i + 4j – 3k and C = i + 2j – k, find (a) 2A – B + 3C, (b) │A + B + C│, (c) │3A – 2B + 4C│, (d) a unit vector parallel to 3A – 2B + 4C

Data:

A = 3i – j – 4k

B = -2i + 4j – 3k

C = i + 2j – k

(a) 2A – B + 3C = ?

(b) │A + B + C│= ?

(c) │3A – 2B + 4C│= ?

(d) a unit vector parallel to 3A – 2B + 4C = ?

Solution:

(a) 2A – B + 3C = 2(3i – j – 4k) – (-2i + 4j – 3k) + 3(i + 2j – k)

2A – B + 3C =6i-2j-8k +2i-4j+3k+3i+6j-3k

2A – B + 3C =11i + 0j -8k

$$\boxed{2A - B + 3C = 11i - 8k}$$

(b) A + B + C = 3i – j – 4k +(-2i + 4j – 3k) + i + 2j – k

A + B + C =2 i +5j – 8k

Now, $|A + B + C| = \sqrt{x^2 + y^2 + z^2}$

$$|A + B + C| = \sqrt{(2)^2 + (5)^2 + (-8)^2}$$

$$\boxed{|A + B + C| = \sqrt{93}}$$

(c) 3A – 2B + 4C = 3(3i – j – 4k) – 2(-2i + 4j – 3k) + 4(i + 2j – k)

3A – 2B + 4C = 9i-3j-12k +4i - 8j + 6k+ 4i+8j-4k

$$3A - 2B + 4C = 17\,i - 3\,j - 10\,k$$

Now, $|3A - 2B + 4C| = \sqrt{x^2 + y^2 + z^2}$

$$|3A - 2B + 4C| = \sqrt{(17)^2 + (-3)^2 + (-10)}$$

$$\boxed{|3A - 2B + 4C| = \sqrt{398}}$$

(d) Unit Vector Perpendicular $= \hat{n} = \dfrac{vector}{magnitude}$

$$\boxed{\hat{n} = \frac{3A - 2B + 4C}{|3A - 2B + 4C|} = \frac{17\,i - 3\,j - 10\,k}{\sqrt{398}}}$$

Q.8: Two tugboats are towing a ship. Each exerts a force of 6000N, and the angle between the two ropes is 60º. Calculate the resultant force on the ship.

<u>Data:</u>

$\vec{F_1} = 6000\ N$

$\vec{F_2} = 6000\ N$

$\theta = 60°$

$\vec{F} = ?$

<u>Solution:</u>

According to the Parallelogram Method

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$F =$

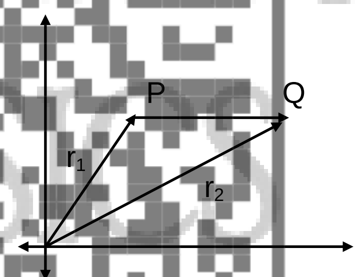
$$\sqrt{(6000)^2 + (6000)^2 + 2(6000)(6000)\cos 60°}$$

$$F = \sqrt{108000000}$$

$$\boxed{F = 10392.3\ N}$$

<u>Result:</u> The resultant force on the ship is 10392.3 N

Q.9: The position vectors of points P and Q are given by r_1 = 2i +3j- k, r_2 = 4i – 3j + 2k. Determine $\overrightarrow{PQ}$ in terms of rectangular unit vector i, j and k and find its magnitude.



<u>Data:</u>

r_1 = 2i + 3j - k

r_2 = 4i – 3j + 2k

$\overrightarrow{PQ} = ?$

$|\overrightarrow{PQ}| = ?$

<u>Solution:</u>

According to the figure

$\overrightarrow{PQ} = r_2 - r_1$

$\overrightarrow{PQ} = 4i - 3j + 2k - (2i + 3j - k)$

$\overrightarrow{PQ} =$ 4i – 3j + 2k – 2i – 3j + k

$$\boxed{\overrightarrow{PQ} = 2i - 6j + 3k}$$

Now, $|\overrightarrow{PQ}| = \sqrt{x^2 + y^2 + z^2}$

$$|\overrightarrow{PQ}| = \sqrt{(2)^2 + (-6)^2 + (3)^2}$$

$$\boxed{|\overrightarrow{PQ}| = 7\ units}$$

<u>Result:</u> $\overrightarrow{PQ} = 2i - 6j + 3k$ and $|\overrightarrow{PQ}| = 7\ units$

Q.10: Prove that the vectors A = 3i + j – 2k, B = -i + 3j + 4k and C = 4i – 2j – 6k can form the sides of a triangle. Find the length of the medians of the triangle.

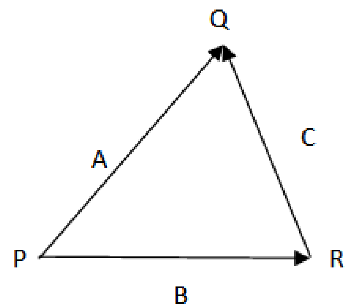
<u>Data:</u>

A = 3i + j – 2k

B = -i + 3j + 4k

C = 4i – 2j – 6k

Δ=?



Length of medians = ?

Solution:

A, B and C can form triangle if

$$\vec{B} + \vec{C} = \vec{A}$$

$-\mathrm{i} + 3\mathrm{j} + 4\mathrm{k} + 4\mathrm{i} - 2\mathrm{j} - 6\mathrm{k} = 3\mathrm{i} + \mathrm{j} - 2\mathrm{k}$

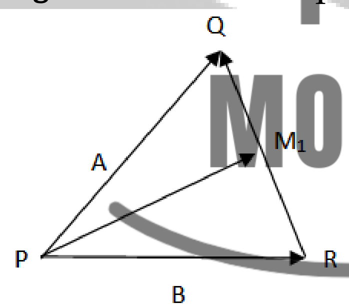
$3\mathrm{i} + \mathrm{j} - 2\mathrm{k} = 3\mathrm{i} + \mathrm{j} - 2\mathrm{k}$

So. L.H.S = R.H.S

Therefore, A, B and C can form triangle.

Now,

Length of 1st Median $\overrightarrow{PM_1}$:



According to the figure

$$\vec{B} + \tfrac{1}{2}\vec{C} = \overrightarrow{PM_1}$$

$$\overrightarrow{PM_1} = -\mathrm{i} + 3\mathrm{j} + 4\mathrm{k} + \tfrac{1}{2}(4\mathrm{i} - 2\mathrm{j} - 6\mathrm{k})$$

$$\overrightarrow{PM_1} = -\mathrm{i} + 3\mathrm{j} + 4\mathrm{k} + 2\mathrm{i} - \mathrm{j} - 3\mathrm{k}$$

$$\boxed{\overrightarrow{PM_1} = \mathrm{i} + 2\mathrm{j} + \mathrm{k}}$$

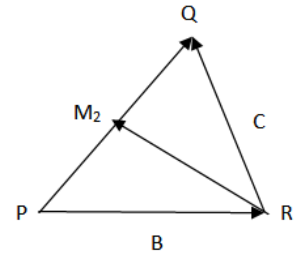
Now,

$$|\overrightarrow{PM_1}| = \sqrt{x^2 + y^2 + z^2}$$

$$|\overrightarrow{PM_1}| = \sqrt{(1)^2 + (2)^2 + (1)^2}$$

$$\boxed{|\overrightarrow{PM_1}| = \sqrt{6}\ units}$$

Length of 2nd Median $\overrightarrow{RM_2}$:



According to the figure

$$\vec{B} + \overrightarrow{RM_2} = \tfrac{1}{2}\vec{A}$$

$$\overrightarrow{RM_2} = -\mathrm{i} + 3\mathrm{j} + 4\mathrm{k} - \tfrac{1}{2}(3\mathrm{i} + \mathrm{j} - 2\mathrm{k})$$

$$\overrightarrow{RM_2} = -\mathrm{i} + 3\mathrm{j} + 4\mathrm{k} - \tfrac{3}{2}\mathrm{i} - \tfrac{1}{2}\mathrm{j} + \mathrm{k}$$

$$\overrightarrow{RM_2} = -\tfrac{5}{2}\mathrm{i} + \tfrac{5}{2}\mathrm{j} + 5\mathrm{k}$$

Now,

$$|\overrightarrow{RM_2}| = \sqrt{x^2 + y^2 + z^2}$$

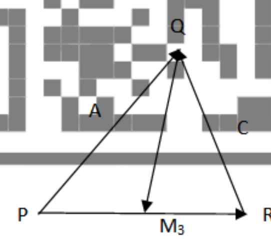
$$|\overrightarrow{RM_2}| = \sqrt{(-\frac{5}{2})^2 + (\frac{5}{2})^2 + (5)^2}$$

$$\boxed{|\overrightarrow{RM_2}| = \frac{1}{2}\sqrt{150}\ units}$$

Length of 3rd Median $\overrightarrow{QM_3}$:

According to the figure

$$\vec{A} + \overrightarrow{QM_3} = \tfrac{1}{2}\vec{B}$$



$$\overrightarrow{QM_3} = \tfrac{1}{2}\vec{B} - \vec{A}$$

$$\overrightarrow{QM_3} = \tfrac{1}{2}(-\mathrm{i} + 3\mathrm{j} + 4\mathrm{k}) - (3\mathrm{i} + \mathrm{j} - 2\mathrm{k})$$

$$\overrightarrow{QM_3} = -\tfrac{1}{2}i + \tfrac{3}{2}j + 2k - 3i - j + 2k$$

$$\overrightarrow{QM_3} = -\tfrac{7}{2}i - \tfrac{1}{2}j + 4k$$

Now,

$$|\overrightarrow{QM_3}| = \sqrt{x^2 + y^2 + z^2}$$

$$|\overrightarrow{QM_3}| = \sqrt{(-\frac{7}{2})^2 + (-\frac{1}{2})^2 + (4)^2}$$

$$\boxed{|\overrightarrow{QM_3}| = \tfrac{1}{2}\sqrt{114}\ units}$$

Q.11: Find the rectangular components of a vector A, 15 unit long when it form an angle with respect to +ve x-axis of (i) 50°, (ii) 130° (iii) 230°, (iv) 310°.

Data:

$|\vec{A}| = 15\ units$

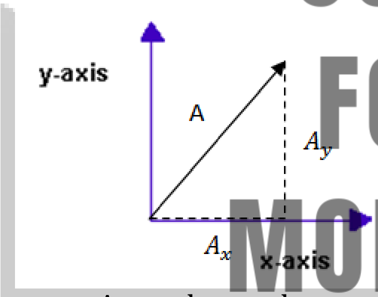
i) $\theta = 50°$, $A_x = ?\ and\ A_y = ?$

i) $\theta = 130°$, $A_x = ?\ and\ A_y = ?$

i) $\theta = 230°$, $A_x = ?\ and\ A_y = ?$

i) $\theta = 310°$, $A_x = ?\ and\ A_y = ?$

Solution:



As we know that

$A_x = A\ cos\theta$

and $A_y = A\ sin\theta$

i) $\theta = 50°$

$A_x = 15\ cos50° = 15 \times 0.642$

$\boxed{A_x = 9.64\ units}$

$A_y = 15\ sin50° = 15 \times 0.766$

$\boxed{A_y = 11.5\ units}$

ii) $\theta = 130°$

$A_x = 15\ cos130° = 15 \times (-0.642)$

$\boxed{A_x = -9.64\ units}$

$A_y = 15\ sin130° = 15 \times 0.766$

$\boxed{A_y = 11.5\ units}$

iii) $\theta = 230°$

$A_x = 15 \cos 230° = 15 \times (-0.642)$

$\boxed{A_x = -9.64\ units}$

$A_y = 15 \sin 230° = 15 \times (-0.766)$

$\boxed{A_y = -11.5\ units}$

iiv) $\theta = 310°$

$A_x = 15 \cos 310° = 15 \times (0.642)$

$\boxed{A_x = 9.64\ units}$

$A_y = 15 \sin 310° = 15 \times (-0.766)$

$\boxed{A_y = -11.5\ units}$

Q12: Two vectors 10 cm and 8 cm long form an angle of (a) 60° (b) 90° and (c) 120°. Find the magnitude of difference and the angle with respect to the larger vector.

Data:

$|\vec{A}| = 10\ cm$

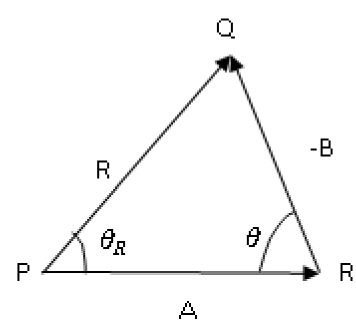
$|\vec{B}| = 8\ cm$

i) $\theta = 60°$, $|\vec{A} - \vec{B}| = ?\ and\ \theta_R = ?$

i) $\theta = 90°$, $|\vec{A} - \vec{B}| = ?\ and\ \theta_R = ?$

i) $\theta = 120°$, $|\vec{A} - \vec{B}| = ?\ and\ \theta_R = ?$

Solution:



i) Using Paralellogram Law

$$|\vec{A}-\vec{B}| = \sqrt{A^2+B^2-2AB\ COS\ \theta}$$

$$|\vec{A}-\vec{B}| = \sqrt{(10)^2+(8)^2-2(10)(8)\ COS\ 60°}$$

$$\boxed{|\vec{A}-\vec{B}| = 9.2\ cm}$$

Using Law of Sines

$$\frac{\sin\theta_R}{B} = \frac{\sin\theta}{|\vec{A}-\vec{B}|}$$

$$\frac{\sin\theta_R}{8} = \frac{\sin 60}{9.2}$$

$$\sin\theta_R = 0.753$$

$$\theta_R = \sin^{-1}(0.753)$$

$$\boxed{\theta_R = 49°}$$

ii) Using Paralellogram Law

$$|\vec{A}-\vec{B}| = \sqrt{A^2+B^2-2AB\ COS\ \theta}$$

$$|\vec{A}-\vec{B}| = \sqrt{(10)^2+(8)^2-2(10)(8)\ COS\ 90°}$$

$$\boxed{|\vec{A}-\vec{B}| = 12.8\ cm}$$

Using Law of Sines

$$\frac{\sin\theta_R}{B} = \frac{\sin\theta}{|\vec{A}-\vec{B}|}$$

$$\frac{\sin\theta_R}{8} = \frac{\sin 90}{12.8}$$

$$\sin\theta_R = 0.625$$

$$\theta_R = \sin^{-1}(0.625)$$

$$\boxed{\theta_R = 38.6°}$$

Or in Minutes:

$$\theta_R = 38° + 0.6 \times 60'$$

$$\boxed{\theta_R = 38°\ 36'}$$

iii) Using Paralellogram Law

$$|\vec{A}-\vec{B}| = \sqrt{A^2+B^2-2AB\ COS\ \theta}$$

$$|\vec{A}-\vec{B}| = \sqrt{(10)^2+(8)^2-2(10)(8)\ COS\ 120°}$$

$$\boxed{|\vec{A}-\vec{B}| = 15.6\ cm}$$

Using Law of Sines

$$\frac{\sin\theta_R}{B} = \frac{\sin\theta}{|\vec{A}-\vec{B}|}$$

$$\frac{\sin\theta_R}{8} = \frac{\sin 120}{15.6}$$

$$\sin\theta_R = 0.444$$

$$\theta_R = \sin^{-1}(0.444)$$

$$\boxed{\theta_R = 26.3°}$$

Or in Minutes:

$$\theta_R = 26° + 0.3 \times 60'$$

$$\boxed{\theta_R = 26°\ 18'}$$

Q.13: The angle between the vector A and B is 60° . Given that |A|=|B| =1, calculate (a) |B - A|; (b) |B + A|

Data:

A|=|B| = 1

(a) |B - A|= ?

(b) |B + A|= ?

Solution:

(a) $$|\vec{B}-\vec{A}| = \sqrt{B^2+A^2-2BA\ COS\ \theta}$$

$$|\vec{B}-\vec{A}| = \sqrt{(1)^2+(1)^2-2(1)(1)\ COS\ 60°}$$

$$|\vec{B}-\vec{A}| = \sqrt{1}$$

$$\boxed{|\vec{B}-\vec{A}| = 1}$$

(b) $$|\vec{B}+\vec{A}| = \sqrt{B^2+A^2+2BA\ COS\ \theta}$$

$$|\vec{B}+\vec{A}| = \sqrt{(1)^2+(1)^2+2(1)(1)\ COS\ 60°}$$

$$\boxed{|\vec{B}+\vec{A}| = \sqrt{3}}$$

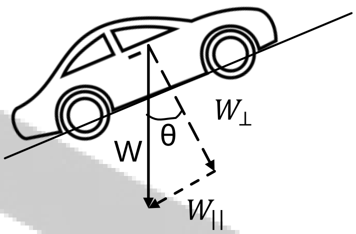
Q.14: A car weighing 10,000 N on a hill which makes an angle of 20° with the horizontal. Find the components of car's weight parallel and perpendicular to the road.

Data:

$|\vec{W}| = 10000\ N$

$\theta = 20°, W_{||} = ?$ and $W_{\perp} = ?$

Solution:



According to the given condition

$$W_{||} = W sin\theta$$

$$W_{||} = 10000 \times \sin 20°$$

$$\boxed{W_{||} = 3420\ N}$$

and

$$W_{\perp} = W cos\theta$$

$$W_{\perp} = 10000 \times \cos 20°$$

$$\boxed{W_{\perp} = 9396.9\ N}$$

Result: The component of car parallel to road is 3420N and perpendicular to road is 9396.9 N

Q.15: Find the angle between A = 2i + 2j – k and B = 6i – 3j + 2k.

Data:

A = 2i + 2j – k

B = 6i – 3j + 2k

$\theta = ?$

Solution:

According to the definition of Dot Product

$$\vec{A}.\vec{B} = ABCos\,\theta$$

$$Cos\,\theta = \frac{A.B}{|A|\,|B|} \text{ --- (i)}$$

$$\vec{A}.\vec{B} = (2\hat{\imath} + 2\hat{\jmath} - \hat{k}).(6\hat{\imath} - 3\hat{\jmath} + 2\hat{k})$$

$$\vec{A}.\vec{B} = 12 - 6 - 2$$

$$\boxed{\vec{A}.\vec{B} = 4}$$

Now, $A = \sqrt{x^2 + y^2 + z^2}$

$$A = \sqrt{(2)^2 + (2)^2 + (-1)^2}$$

$$\boxed{A = 3}$$

and

$$B = \sqrt{x^2 + y^2 + z^2}$$

$$B = \sqrt{(6)^2 + (-3)^2 + (2)^2}$$

$$\boxed{B = 7}$$

Putting values in eq (i)

$$Cos\,\theta = \frac{4}{3\times7}$$

$$\theta = Cos^{-1}(\frac{4}{21})$$

$$\boxed{\theta = 79°}$$

Result:

The angle between given vectors is 79°

Q.16: Find the projection of the vector A = i – 2j + k onto the direction of vector B = 4i – 4j + 7k.

Data:

$\vec{A} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$

$\vec{B} = 4\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$

Projection of A onto B = A_B =?

Solution:

As we know that

$$\vec{A}.\vec{B} = A_B B$$

So $A_B = \frac{\vec{A}.\vec{B}}{B}$ ------- (i)

$$\vec{A}.\vec{B} = (\hat{\imath} - 2\hat{\jmath} + \hat{k}).(4\hat{\imath} - 4\hat{\jmath} + 7\hat{k})$$

$$\vec{A}.\vec{B} = 4 + 8 + 7$$

$$\boxed{\vec{A}.\vec{B} = 19}$$

Now,

$$B = \sqrt{x^2 + y^2 + z^2}$$

$$B = \sqrt{(4)^2 + (-4)^2 + (7)^2}$$

$$\boxed{B = 9}$$

Putting values in eq (i)

$$\boxed{A_B = \frac{19}{9}}$$

Result: The projection of Vector A on Vector B is $\frac{19}{9}$ $units.$

Q.17: Find the angles α, β, γ which the vector A = 3i – 6j + 2k makes with the positive x, y, z axis respectively.

Data:

A = 3i – 6j + 2k

Angle w.r.t. x axis = α = ?

Angle w.r.t. x axis = β = ?

Angle w.r.t. x axis = γ = ?

Solution:

According to the definition of Dot Product

$$Cos\ \theta = \frac{A.B}{|A|\,|B|} \text{ --- (i)}$$

$$A = \sqrt{x^2 + y^2 + z^2}$$

$$A = \sqrt{(3)^2 + (-6)^2 + (2)^2}$$

$$\boxed{A = 7}$$

For α:

$$\vec{A}.\hat{\imath} = (3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}).(\hat{\imath})$$

$$\vec{A}.\hat{\imath} = 3$$

$$\boxed{\vec{A}.\hat{\imath} = 3}$$

Putting values in eq (i)

$$Cos\ \alpha = \frac{3}{7}$$

$$\alpha = Cos^{-1}(\frac{3}{7})$$

$$\boxed{\alpha = 64.6°}$$

For β:

$$\vec{A}.\hat{\jmath} = (3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}).(\hat{\jmath})$$

$$\vec{A}.\hat{\jmath} = -6$$

$$\boxed{\vec{A}.\hat{\jmath} = -6}$$

Putting values in eq (i)

$$Cos\ \beta = -\frac{6}{7}$$

$$\beta = Cos^{-1}(-\frac{6}{7})$$

$$\boxed{\beta = 149°}$$

For γ:

$$\vec{A}.\hat{k} = (3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}).(\hat{k})$$

$$\vec{A}.\hat{k} = 2$$

$$\boxed{\vec{A}.\hat{k} = 2}$$

Putting values in eq (i)

$$Cos\ \gamma = \frac{2}{7}$$

$$\gamma = Cos^{-1}(\frac{2}{7})$$

$$\boxed{\gamma = 73.3°}$$

Result:

Angle w.r.t. x axis = α = 64.6°

Angle w.r.t. x axis = β = 149 °

Angle w.r.t. x axis = γ = 73.3°

Q.18: Find the work done in moving an object along a vector r = 3i + 2j – 5k if the applied force is F = 2i – j – k.

Data:

Displacement = r = 3i + 2j – 5k

Force = F = 2i - j - k

Work = W =?

Solution:

According to the def. of work

$$W = \vec{F}.\vec{S}$$

$$W = (2i - j - k).(3i + 2j - 5k)$$

$$W = 6 - 2 + 5$$

$$\boxed{W = 9\ units}$$

Result:

The work done is 9 units.

Q.19: Find the work done by a force of 30,000 N in moving an o bject through a distance of 45 m when: (a) the force is in the direction of motion: and (b) the force makes an angle of 40° to the direction of motion. Find the rate at which the force is working at a time when the

velocity is 2m/s.

Data:

Force = F = 30000 N

Distance = S = 45 m

a) $\theta = 0^0$ W =? and P = ?

b) $\theta = 40^0$ W =? and P = ?

Velocity = v = 2 m/s

Solution:

Accordint to the definition of work

$W = FS\cos\theta$

a) $W = 30000 \times 45 \times \cos 0°$

$\boxed{W = 1.35 \times 10^6 J}$

$P = FV\cos\theta$

$P = 30000 \times 2 \times \cos 0°$

$\boxed{P = 60000\ W}$

b) $W = 30000 \times 45 \times \cos 40°$

$\boxed{W = 1.03 \times 10^6 J}$

$P = FV\cos\theta$

$P = 30000 \times 2 \times \cos 40°$

$\boxed{P = 45962.6\ W}$

Result:

When $\theta = 0^0$, $W = 1.35 \times 10^6 J$ and $P = 60000\ W$

When $\theta = 40^0$, $W = 1.03 \times 10^6 J$ and $P = 45962.6\ W$

Q.20: Two vectors A and B are such that |A| = 3, |B| = 4, and A.B = -5, find (a) the angle between A and B (b) the length |A + B| and |A - B| (c) the angle between (A + B) and (A – B)

Data:

|A| = 3

|B| = 4

A.B = -5

a) θ = ? (b/w A and B)

b) |A + B| =? and |A - B| = ?

c) θ = ? (b/w A+B and A-B)

Solution:

According to the definition of Dot Product

$\vec{A}.\vec{B} = AB\,Cos\,\theta$

$Cos\,\theta = \frac{A.B}{|A|\,|B|}$ --- (i)

a) $Cos\,\theta = \frac{-5}{3\times4}$

$\theta = \cos^{-1}(-0.416)$

$\boxed{\theta = 114.5°}$

b) $|A + B| = \sqrt{A^2 + B^2 + 2\vec{A}.\vec{B}}$

$|A + B| = \sqrt{(3)^2 + (4)^2 + 2\times(-5)}$

$\boxed{|A + B| = \sqrt{15}}$

and, $|A - B| = \sqrt{A^2 + B^2 - 2\vec{A}.\vec{B}}$

$|A - B| = \sqrt{(3)^2 + (4)^2 - 2\times(-5)}$

$\boxed{|A - B| = \sqrt{35}}$

c) $Cos\,\theta = \frac{(\vec{A}+\vec{B}).(\vec{A}-\vec{B})}{|\vec{A}+\vec{B}||\vec{A}-\vec{B}|}$

$Cos\,\theta = \frac{A^2 - B^2}{\sqrt{15}\times\sqrt{35}} = \frac{(3)^2-(4)^2}{22.91}$

$\theta = \cos^{-1}(-0.305)$

$\boxed{\theta = 107.7°}$

Q.21: If A = 2i – 3j – k, B = i + 4j – 2k. Find (a) A x B (b) B x A (c) (A + B) x (A – B)

Data:

A = 2i – 3j – k

B = i + 4j – 2k

(a) A x B = ?

(b) B x A = ?

(c) (A + B) x (A – B) =?

Solution:

(a) $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -3 & -1 \\ 1 & 4 & -2 \end{vmatrix}$

$\vec{A}\times\vec{B} = \hat{\imath}\begin{vmatrix} -3 & -1 \\ 4 & -2 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & -3 \\ 1 & 4 \end{vmatrix}$

$\vec{A}\times\vec{B} = \hat{\imath}(6+4) - \hat{\jmath}(-4+1) + \hat{k}(8+3)$

$$\boxed{\vec{A}\times\vec{B} = 10\hat{\imath} + 3\hat{\jmath} + 11\hat{k}}$$

b) $\vec{B}\times\vec{A} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 4 & -2 \\ 2 & -3 & -1 \end{vmatrix}$

$\vec{B}\times\vec{A} = \hat{\imath}\begin{vmatrix} 4 & -2 \\ -3 & -1 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 1 & -2 \\ 2 & -1 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 4 \\ 2 & -3 \end{vmatrix}$

$\vec{B}\times\vec{A} = \hat{\imath}(-4-6) - \hat{\jmath}(-1+4) + \hat{k}(-3-8)$

$$\boxed{\vec{B}\times\vec{A} = -10\hat{\imath} - 3\hat{\jmath} - 11\hat{k}}$$

c) $\vec{A}+\vec{B} = 2\hat{\imath} - 3\hat{\jmath} - \hat{k} + (\hat{\imath} + 4\hat{\jmath} - 2\hat{k})$

$$\boxed{\vec{A}+\vec{B} = 3\hat{\imath} + \hat{\jmath} - 3\hat{k}}$$

$\vec{A}-\vec{B} = 2\hat{\imath} - 3\hat{\jmath} - \hat{k} - (\hat{\imath} + 4\hat{\jmath} - 2\hat{k})$

$\vec{A}-\vec{B} = 2\hat{\imath} - 3\hat{\jmath} - \hat{k} - \hat{\imath} - 4\hat{\jmath} + 2\hat{k}$

$$\boxed{\vec{A}+\vec{B} = \hat{\imath} - 7\hat{\jmath} + \hat{k}}$$

Now,

$$(\vec{A}+\vec{B})\times(\vec{A}-\vec{B}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 1 & -3 \\ 1 & -7 & 1 \end{vmatrix}$$

$(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})$

$= \hat{\imath}\begin{vmatrix} 1 & -3 \\ -7 & 1 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 3 & -3 \\ 1 & 1 \end{vmatrix} + \hat{k}\begin{vmatrix} 3 & 1 \\ 1 & -7 \end{vmatrix}$

$(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})$

$= \hat{\imath}(1-21) - \hat{\jmath}(3+3) + \hat{k}(-21 - 1)$

$$\boxed{(\vec{A}+\vec{B})\times(\vec{A}-\vec{B}) = -20\hat{\imath} - 6\hat{\jmath} - 22\hat{k}}$$

Q.22: Determine the unit vector perpendicular to the plane of A = 2i – 6j – 3k and B = 4i + 3j – k.

Data:

$\vec{A} = 2\hat{\imath} - 6\hat{\jmath} - 3\hat{k}$

$\vec{B} = 4\hat{\imath} + 3\hat{\jmath} - \hat{k}$

$\hat{n} = ?$

Solution:

Unit Vector Perpendicular $= \hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$ ------ (i)

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix}$$

$\vec{A}\times\vec{B} = \hat{\imath}\begin{vmatrix} -6 & -3 \\ 3 & -1 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 2 & -3 \\ 4 & -1 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & -6 \\ 4 & 3 \end{vmatrix}$

$\vec{A}\times\vec{B} = \hat{\imath}(6+9) - \hat{\jmath}(-2+12) + \hat{k}(6+24)$

$$\boxed{\vec{A}\times\vec{B} = 15\hat{\imath} - 10\hat{\jmath} + 30\hat{k}}$$

$$|\vec{A}\times\vec{B}| = \sqrt{x^2+y^2+z^2}$$

$$|\vec{A}\times\vec{B}| = \sqrt{(15)^2+(-10)^2+(30)^2}$$

$$\boxed{|\vec{A}\times\vec{B}| = 35}$$

Putting in eq (i)

$$\boxed{\hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|} = \frac{15\hat{\imath}-10\hat{\jmath}+30\hat{k}}{35} = \frac{3}{7}\hat{\imath} - \frac{2}{7}\hat{\jmath} + \frac{6}{7}\hat{k}}$$

Result: The unit vector perpendicular to the plane of A and B is $\frac{3}{7}\hat{\imath} - \frac{2}{7}\hat{\jmath} + \frac{6}{7}\hat{k}$.

Q.23: Using the definition of vector product, prove the law of sines for plane triangles of sides a, b and c.

Proof:

According to the definition of vector product

$$\vec{A}\times\vec{B} = AB\ Sin\theta\ \hat{n}$$

or

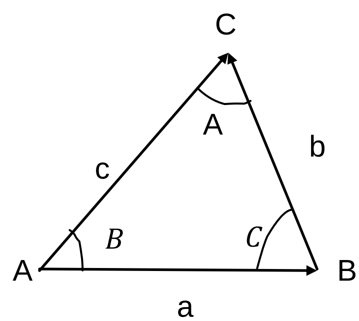
$$|\vec{A}\times\vec{B}| = AB\ SinC \text{ ---(i)}$$

Also,

$$|\vec{B}\times\vec{C}| = BC\ SinA \text{----(ii)}$$

and

$$|\vec{C}\times\vec{A}| = AC\ SinB \text{ ----(iii)}$$



As we know that Area of triangle is given by

$$\Delta = \frac{1}{2}|\vec{A}\times\vec{B}| = \frac{1}{2}|\vec{B}\times\vec{C}| = \frac{1}{2}|\vec{C}\times\vec{A}|$$

Putting values from eq(i) , (ii) and (iii)

$$\frac{1}{2}AB\ SinC\ = \frac{1}{2}BC\ SinA = \frac{1}{2}AC\ SinB$$

$$AB\ SinC\ = BC\ SinA = AC\ SinB$$

Dividing by ABC

$$\frac{AB\ SinC}{ABC} = \frac{BC\ SinA}{ABC} = \frac{AC\ SinB}{ABC}$$

$$\boxed{\frac{SinC}{C} = \frac{SinA}{A} = \frac{SinB}{B}}$$ (Proved)

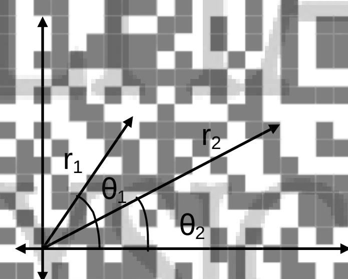
Q.24: If r_1 and r_2 are the position vectors (both lie in xy plane) making angle θ_1 and θ_2 with the positive x-axis measured counter clockwise, find their vector product when

(i) $|r_1| = 4$ cm $\theta_1 = 30^o$ $|r_2| = 3$ cm $\theta_2 = 90^o$

(ii) $|r_1| = 6$ cm $\theta_1 = 220^o$ $|r_2| = 3$ cm $\theta_2 = 40^o$

(iii) $|r_1| = 10$ cm $\theta_1 = 20^o$ $|r_2| = 9$ cm $\theta_2 = 110^o$

<u>Data:</u>



(i) $|r_1| = 4$ cm $\theta_1 = 30^o$ $|r_2| = 3$ cm $\theta_2 = 90^o$

$|\vec{r_1} \times \vec{r_2}| = ?$

(ii) $|r_1| = 6$ cm $\theta_1 = 220^o$ $|r_2| = 3$ cm $\theta_2 = 40^o$

$|\vec{r_1} \times \vec{r_2}| = ?$

(iii) $|r_1| = 10$ cm $\theta_1 = 20^o$ $|r_2| = 9$ cm $\theta_2 = 110^o$

$|\vec{r_1} \times \vec{r_2}| = ?$

<u>Solution:</u>

According to the definition of cross product

$\vec{r_1} \times \vec{r_2} = |r_1||r_2| \cos\theta$ --- (i)

a) $\theta = \theta_2 - \theta_1$

$\theta = 90 - 30$

$\theta = 60°$

Putting values in eq (i)

$|\vec{r_1} \times \vec{r_2}| = 4 \times 3 \times \cos 60°$

$|\vec{r_1} \times \vec{r_2}| = 12 \times \frac{\sqrt{3}}{2}$

$$\boxed{|\vec{r_1} \times \vec{r_2}| = 6\sqrt{3}\ cm^2}$$

b) $\theta = \theta_1 - \theta_2$

$\theta = 220 - 40$

$\theta = 180°$

Putting values in eq (i)

$|\vec{r_1} \times \vec{r_2}| = 6 \times 3 \times \cos 180°$

$|\vec{r_1} \times \vec{r_2}| = 18 \times (-1)$

$$\boxed{|\vec{r_1} \times \vec{r_2}| = -18\ cm^2}$$

c) $\theta = \theta_2 - \theta_1$

$\theta = 110 - 20$

$\theta = 90°$

Putting values in eq (i)

$$|\vec{r_1} \times \vec{r_2}| = 10 \times 9 \times \cos 90°$$

$$|\vec{r_1} \times \vec{r_2}| = 90 \times 0$$

$$\boxed{|\vec{r_1} \times \vec{r_2}| = 0\ cm^2}$$

PAST PAPER NUMERICALS

2021

Q.2 (iv) Determine the unit vector perpendicular in the plane of A= 2i-6j-3k and B =4i+3j-k

Data:

$\vec{A} = 2\hat{\imath} - 6\hat{\jmath} - 3\hat{k}$

$\vec{B} = 4\hat{\imath} + 3\hat{\jmath} - \hat{k}$

$\hat{n} = ?$

Solution:

Unit Vector Perpendicular $= \hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$ ----- (i)

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \hat{\imath}\begin{vmatrix} -6 & -3 \\ 3 & -1 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 2 & -3 \\ 4 & -1 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & -6 \\ 4 & 3 \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \hat{\imath}(6+9) - \hat{\jmath}(-2+12) + \hat{k}(6+24)$$

$$\boxed{\vec{A} \times \vec{B} = 15\hat{\imath} - 10\hat{\jmath} + 30\hat{k}}$$

$$|\vec{A} \times \vec{B}| = \sqrt{x^2 + y^2 + z^2}$$

$$|\vec{A} \times \vec{B}| = \sqrt{(15)^2 + (-10)^2 + (30)^2}$$

$$\boxed{|\vec{A} \times \vec{B}| = 35}$$

Putting in eq (i)

$$\boxed{\hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|} = \frac{15\hat{\imath} - 10\hat{\jmath} + 30\hat{k}}{35} = \frac{3}{7}\hat{\imath} - \frac{2}{7}\hat{\jmath} + \frac{6}{7}\hat{k}}$$

Result: The unit vector perpendicular to the plane of A and B is $\frac{3}{7}\hat{\imath} - \frac{2}{7}\hat{\jmath} + \frac{6}{7}\hat{k}$.

2019

Q.2 (xiii)Two vectors A and B are such that |A|=4, |B|=6 and A.B=13.5.Find the magnitude of difference of vectors and angle between A and B.

Data:

|A|=4

|B|=6

A.B=13.5

Angle $= \theta = ?$

Magnitude of Difference =|A-B|=?

Solution:

$$|A - B| = \sqrt{|A|^2 + |B|^2 - 2A.B}$$

$$|A-B| = \sqrt{(4)^2 + (6)^2 - 2(13.5)}$$

$$|A-B| = \sqrt{25} = 5\ unit$$

$$Cos\ \theta = \frac{A.B}{|A||B|}$$

$$Cos\ \theta = \frac{13.5}{(4)(6)}$$

$$Cos\ \theta = 0.5625$$

$$\boxed{\theta = Cos^{-1}(0.5625) = 55.77^0}$$

Result: The magnitude of difference is 5 $unit$ and angle between them is 55.77^0

2018

Q.2(i) If the vector $\vec{A} = a\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\vec{B} = \hat{\imath} + a\hat{\jmath} + \hat{k}$ are perpendicular to each other then find the value 'a'.

Data:

$\vec{A} = a\hat{\imath} + \hat{\jmath} - 2\hat{k}$

$\vec{B} = \hat{\imath} + a\hat{\jmath} + \hat{k}$

a = ?

Solution:

According to the given condition

$$\vec{A}.\vec{B} = 0 \ (\text{b/c}\ \vec{A} \perp \vec{B})$$

$$(a\hat{\imath} + \hat{\jmath} - 2\hat{k}).(\hat{\imath} + a\hat{\jmath} + \hat{k}) = 0$$

$$a + a - 2 = 0$$

$$2a = 2$$

$$\boxed{a = 1}$$

Result: The value of a for which $\vec{A} \perp \vec{B}$ is 1.

2017

Q.2(i) Textbook Numerical 22

2016

Q.2 (xi) Determine the unit vector perpendicular to the plane of A = 3i + 4j -k and B =4i+3j-2k vectors.

Data:

$\vec{A} = 3\hat{\imath} + 4\hat{\jmath} - \hat{k}$

$\vec{B} = 4\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$

$\hat{n} = ?$

Solution:

Unit Vector Perpendicular $= \hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$ -- (i)

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 4 & -1 \\ 4 & 3 & -2 \end{vmatrix}$$

$$\vec{A}\times\vec{B} = \hat{\imath}\begin{vmatrix} 4 & -1 \\ 3 & -2 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 3 & -1 \\ 4 & -2 \end{vmatrix} + \hat{k}\begin{vmatrix} 3 & 4 \\ 4 & 3 \end{vmatrix}$$

$$\vec{A}\times\vec{B} = \hat{\imath}(-8+3) - \hat{\jmath}(-6+4) + \hat{k}(9-16)$$

$$\boxed{\vec{A}\times\vec{B} = -5\hat{\imath} + 2\hat{\jmath} - 7\hat{k}}$$

$$|\vec{A}\times\vec{B}| = \sqrt{x^2 + y^2 + z^2}$$

$$|\vec{A}\times\vec{B}| = \sqrt{(-5)^2 + (2)^2 + (-7)^2}$$

$$\boxed{|\vec{A}\times\vec{B}| = \sqrt{78}}$$

Putting in eq (i)

$$\boxed{\hat{n} = \frac{\overrightarrow{A\times B}}{|\vec{A}\times\vec{B}|} = \frac{-5\hat{\imath}+2\hat{\jmath}-7\hat{k}}{\sqrt{78}} = -\frac{5}{\sqrt{78}}\hat{\imath} + \frac{2}{\sqrt{78}}\hat{\jmath} - \frac{7}{\sqrt{78}}\hat{k}}$$

Result: The unit vector perpendicular to the plane of A and B is $-\frac{5}{\sqrt{78}}\hat{\imath} + \frac{2}{\sqrt{78}}\hat{\jmath} - \frac{7}{\sqrt{78}}\hat{k}$.

2015

Q.2 xii) Two sides of a triangle are formed by vectors A =3i + 6j -2k and B = 4i - j + 3k . Determine the area of the triangle.

Data:

A =3i + 6j -2k

B = 4i - j + 3k

Area of Triangle = Δ =?

Solution:

Area of Triangle is given by

$\Delta = \frac{1}{2}|\vec{A}\times\vec{B}|$ ---- (i)

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 3 & 6 & -2\\ 4 & -1 & 3\end{vmatrix}$$

$$\vec{A}\times\vec{B}=\hat{\imath}\begin{vmatrix}6 & -2\\ -1 & 3\end{vmatrix}-\hat{\jmath}\begin{vmatrix}3 & -2\\ 4 & 3\end{vmatrix}+\hat{k}\begin{vmatrix}3 & 6\\ 4 & -1\end{vmatrix}$$

$$\vec{A}\times\vec{B}=\hat{\imath}(18-2)-\hat{\jmath}(9+8)+\hat{k}(-3-24)$$

$$\boxed{\vec{A}\times\vec{B}=16\hat{\imath}-17\hat{\jmath}-27\hat{k}}$$

Now,

$$|\vec{A}\times\vec{B}|=\sqrt{x^2+y^2+z^2}$$

$$|\vec{A}\times\vec{B}|=\sqrt{(16)^2+(-17)^2+(-27)^2}=35.70$$

Putting in eq (i)

$$\boxed{\Delta=\frac{1}{2}(35.70)=17.85\ sq.units}$$

Result: The area formed by these vectors is 17.85 sq.units

2014

(x) Determine the unit vector perpendicular to the plane containing A and B.

$\vec{A}=2\hat{\imath}-3\hat{\jmath}-\hat{k}$ **and** $B=\hat{\imath}+4\hat{\jmath}-\widehat{2k}$

Data:

$\vec{A}=2\hat{\imath}-3\hat{\jmath}-\hat{k}$

$\vec{B}=\hat{\imath}+4\hat{\jmath}-2\hat{k}$

$\hat{n}=?$

Solution:

Unit Vector Perpendicular $=\hat{n}=\frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$ -- (i)

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 2 & -3 & -1\\ 1 & 4 & -2\end{vmatrix}$$

$$\vec{A}\times\vec{B}=\hat{\imath}\begin{vmatrix}-3 & -1\\ 4 & -2\end{vmatrix}-\hat{\jmath}\begin{vmatrix}2 & -1\\ 1 & -2\end{vmatrix}+\hat{k}\begin{vmatrix}2 & -3\\ 1 & 4\end{vmatrix}$$

$$\vec{A}\times\vec{B}=\hat{\imath}(6+4)-\hat{\jmath}(-4+1)+\hat{k}(8+3)$$

$$\boxed{\vec{A}\times\vec{B}=10\hat{\imath}+3\hat{\jmath}+11\hat{k}}$$

$$|\vec{A}\times\vec{B}|=\sqrt{x^2+y^2+z^2}$$

$$|\vec{A}\times\vec{B}|=\sqrt{(10)^2+(3)^2+(11)^2}$$

$$\boxed{|\vec{A}\times\vec{B}|=\sqrt{230}}$$

Putting in eq (i)

$$\boxed{\hat{n}=\frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}=\frac{10\hat{\imath}+3\hat{\jmath}+11\hat{k}}{\sqrt{230}}}$$

Result: The unit vector perpendicular to the plane of A and B is $\frac{10\hat{\imath}+3\hat{\jmath}+11\hat{k}}{\sqrt{230}}$.

2013

Q.2 (i) If $\vec{A}=3\hat{\imath}+\hat{\jmath}-2\hat{k}$ **and** $\vec{B}=-\hat{\imath}+3\hat{\jmath}+4\hat{k}$. **Find the projection of A on to B.**

Data:

$\vec{A}=3\hat{\imath}+\hat{\jmath}-2\hat{k}$

$\vec{B}=-\hat{\imath}+3\hat{\jmath}+4\hat{k}$

Projection of A onto B = A_B =?

Solution:

As we know that

$$\vec{A}.\vec{B}=A_B B$$

So $A_B=\frac{\vec{A}.\vec{B}}{B}$ ------- (i)

$$\vec{A}.\vec{B}=(3\hat{\imath}+\hat{\jmath}-2\hat{k}).(-\hat{\imath}+3\hat{\jmath}+4\hat{k})$$

$$\vec{A}.\vec{B}=-3+3-8$$

$$\boxed{\vec{A}.\vec{B}=-8}$$

Now,

$$B=\sqrt{x^2+y^2+z^2}$$

$$B=\sqrt{(-1)^2+(3)^2+(4)^2}$$

$$\boxed{B=\sqrt{26}}$$

Putting values in eq (i)

$$\boxed{A_B=-\frac{8}{\sqrt{26}}}$$

Result: The projection of Vector A on Vector B is $-\frac{8}{\sqrt{26}}$ *units.*

Q.2 (iv) Prove that $|\vec{A}\times\vec{B}|^2+(\vec{A}.\vec{B})^2=A^2B^2$

Proof:

Taking L.H.S

L.H.S $= |\vec{A}\times\vec{B}|^2 + (\vec{A}.\vec{B})^2$ ---- (i)

Since

$$|\vec{A}\times\vec{B}| = ABSin\theta$$

And

$$\vec{A}.\vec{B} = ABCos\theta$$

Putting values in eq (i)

L.H.S $=(ABSin\theta)^2 + (ABCos\theta)^2$

L.H.S $=A^2B^2(Sin^2\theta + Cos^2\theta)$

Since $\quad Sin^2\theta + Cos^2\theta = 1$

Therefore $\quad$ L.H.S $=A^2B^2(1)$

Or $\quad$ L.H.S $=A^2B^2$

Or $\quad \boxed{L.H.S = R.H.S}$

2012

Q.2 (iii) Two vectors A and B are such that A=4 , B=6 and |A-B|=5. Find |A+B|

Data:

Magnitude of $\vec{A} = |\vec{A}| = 4$

Magnitude of $\vec{B} = |\vec{B}| = 6$

Magnitude of $|\vec{A}-\vec{B}| = 5$

Magnitude of $|\vec{A}+\vec{B}| = ?$

Solution:

As we know that

$$|\vec{A}-\vec{B}| = \sqrt{A^2 + B^2 - 2AB\ cos\theta}$$

$$5 = \sqrt{(4)^2 + (6)^2 - 2(4)(6)\ cos\theta}$$

S.O.B.S

$$25 = 16 + 36 - 48\ cos\theta$$

$$-27 = -48 \times cos\theta$$

$$cos\theta = \frac{27}{48}$$

$$\theta = \cos^{-1}\left(\frac{27}{48}\right)$$

$$\theta = 55.7^0$$

Now,

$$|\vec{A}+\vec{B}| = \sqrt{A^2 + B^2 + 2AB\ cos\theta}$$

$$|\vec{A}+\vec{B}| = \sqrt{(4)^2 + (6)^2 + 2(4)(6)\cos(55.7)}$$

$$|\vec{A}+\vec{B}| = \sqrt{79.5}$$

Result: The magnitude of $\vec{A}+\vec{B}$ is $\sqrt{79.5}$ units.

2011

Q.2(vi) Textbook Numerical 22

2010

Q.2(viii) If one of the rectangular components of force 50 N is 25N; find the value of the other.

Data:

Magnitude of Force $= F = 50\ N$

One component of Force $= F_1 = 25\ N$

Second component of Force $= F_2 = ?$

Solution:

The magnitude of force is given by

$$F = \sqrt{F_1^2 + F_2^2}$$

$$50 = \sqrt{(25)^2 + F_2^2}$$

$$50 = \sqrt{625 + F_2^2}$$

S.O.B.S.

$$(50)^2 = 625 + F_2^2$$

$$F_2^2 = 2500 - 625$$

$$F_2^2 = 1875$$

Taking Square root O.B.S.

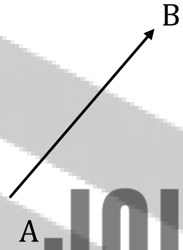
$$\boxed{F_2 = 43.3\ N}$$

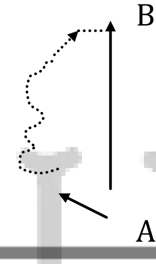
Result: The second component of force is 43.3 N.

THEORY NOTES

DISPLACEMENT:

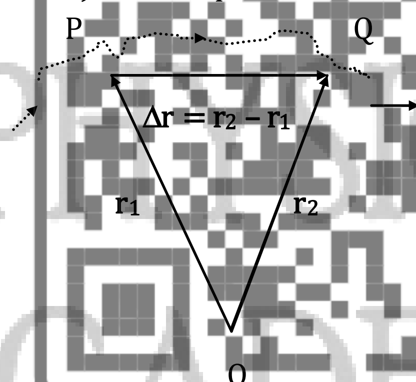
"The change of position of a body in a particular direction is called displacement" .It is the maximum distance between two points. It is a vector quantity.





VELOCITY:

DEFINITION "it is the change in position (or displacement) with respect to time."



EXPLANATION:

From above definition,

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

Consider a body moves along the path AC. Let $\overrightarrow{r1}$ and $\overrightarrow{r2}$ position vectors from origin to the points 'P' ad 'Q'

As the body moves from 'P' to 'Q" in time $\Delta t = t_2 - t_1$ undergoes a change in position $\overrightarrow{\Delta r} = \overrightarrow{r2} - \overrightarrow{r1}$. The average velocity is given by

$$\overrightarrow{V_{av}} = \frac{\overrightarrow{\Delta r}}{\Delta t} = \frac{\text{Displacement}}{\text{Time}}$$

Hence rate of change of position of a body in the direction of displacement is called 'velocity' If time is very small such that $\Delta t \rightarrow 0$, the velocity is called 'instantaneous velocity".

$$\overrightarrow{V_{ins}} = \lim_{\Delta t \rightarrow 0} \frac{\overrightarrow{\Delta r}}{\Delta t}$$

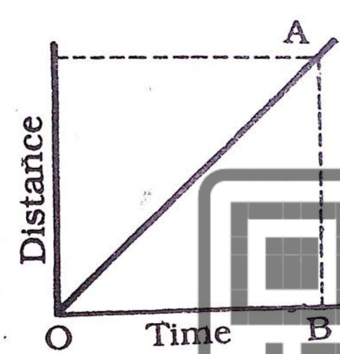
VELOCITY FROM DISTANCE – TIME GRAPH:

Form figure ,

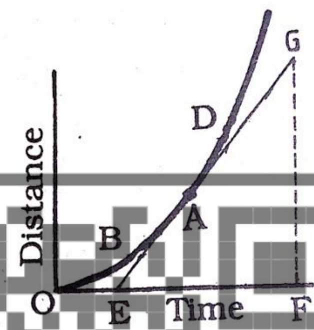
Velocity = Distance/ Time

$$= \frac{\overrightarrow{AB}}{\overrightarrow{OB}}$$

This graph represents uniform velocity.



For variable velocity,



Velocity at A $= \frac{\overrightarrow{GF}}{\overrightarrow{EF}}$

ACCELERATION:

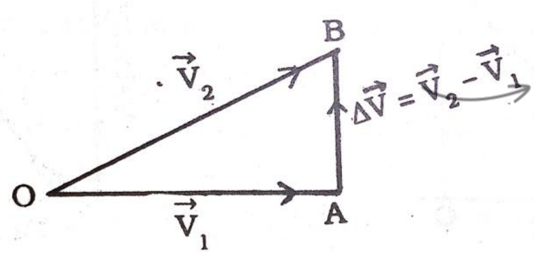
DEFINITION: The rate of change of velocity of a body is called acceleration.

EXPLANATION:
It is a vector quantity and its direction is parallel to the direction of velocity.

ACCELERATION	**=**	**CHANGE OF VELOCITY / TIME**

Consider a body in motion let Vi be it velocity instant "t_1" and V_2 at instant t_2,The average acceleration during this internal is given by

$$\vec{a}_{av} = \frac{\vec{V}_2 - \vec{V}_1}{t_2 - t_1} = \frac{\overrightarrow{\Delta V}}{\Delta t}$$



If Δt is very small then at $\Delta t \rightarrow 0$,

$$\vec{a}_{ins} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta V}}{\Delta t}$$

This acceleration is called instantaneous acceleration.

Unit: The S.I unit of acceleration is m/sec^2

ACCELERATION FROM VELOCITY – TIME GRAPH;

i) When the body moves with uniform acceleration the graph between velocity and time is a straight line.

Acceleration = $\overrightarrow{PQ} / \overrightarrow{OQ}$

ii) When the body moves with variable acceleration, the graph is a curve.

Acceleration = $\overrightarrow{MN} / \overrightarrow{LN}$

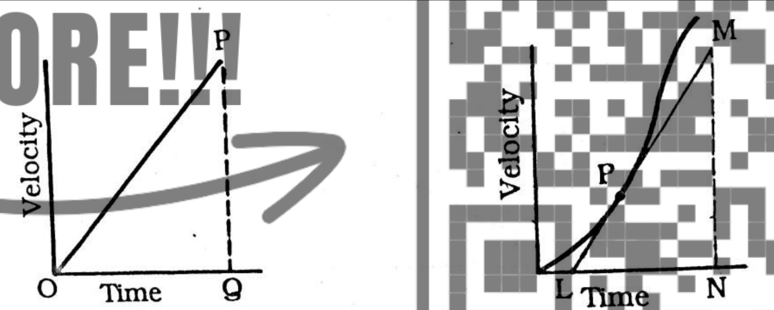


Fig. 3.5 (a) uniform acceleration *(b) Non Uniform acceleration*

EQUATIONS OF UNIFORMLY ACCELERATED RECTILINEAR MOTION:

There are three fundamental equations about uniformly accelerated rectilinear motion,

1. $V_f = V_i + at$
2. $S = V_i t + \frac{1}{2} at^2$
3. $V_f^2 = V_i^2 + 2aS$

In case of motion under gravity with nearly constant acceleration we just replace 'a' with 'g' i.e. acceleration due to gravity in equations of motion, as weight is always directed downwards.

1. $V_f = V_i + gt$
2. $h = V_i t + \frac{1}{2} gt^2$
3. $V_f^2 = V_i^2 + 2gh$

Where g = 9.8 m/s^2
or =980 cm/s^2
=32ft/s^2

The most common example of motion with nearly constant acceleration is that of a body falling towards the earth. This acceleration is due to pull of earth (gravity).If the body moves towards earth, neglecting resistance and small changes in the acceleration with altitude, the body is referred to as free falling body and this motion is called Free Fall. Such type of vertical motion under the action of gravity is a good example of uniformly accelerated motion.

NEWTON'S LAWS OF MOTION

(1) Newton's First Law of Motion:

Newton's first law of motion consists of two parts.

(i) The first part states that a body cannot change its state of rest or uniform motion in straight line itself unless it is acted upon by some unbalanced force to change its state. It can also be stated that a moving body when not acted upon by some net force would have free motion, that is uniform motion in straight line.

(ii) The second part states that force is an agent which changes or tends to change the state of rest or uniform motion i.e. it produces acceleration in the body. The first law of motion is also known as the law of inertia.

Inertia:

Everybody in this universe has a property that it always offers some resistance to the change of its state. This property is known as Inertia and it is because of the mass of body. Therefore we need force to overcome inertia for the change of its state, either rest or motion. Hence Newton's first law of motion is also known as inertia.

(2) Newton's Second Law of Motion:

Newton's second Law states that : "when a force acts upon a certain body, the acceleration produced is proportional to the force and it is in the direction of the force."

$$\vec{F} \propto \vec{a}$$

$$\vec{F} = m\vec{a}$$

where
F = Net force on the body
m = mass of the body
a = acceleration in the body

It is clear from the above equation that the acceleration for certain force on the body is inversely proportional to the mass of the body.

(3) Newton's Third Law of Motion:

Newton's third law can be stated as "To every action there is a an equal but opposite reaction".

The statement means that in every interaction, there is a pair of forces acting on the two interacting objects. The size of the forces on the first object equals the size of the force on the second object. The direction of the force on the first object is opposite to the direction of the force on the second object. Forces always come in pairs - equal and opposite action-reaction force pairs.

TENSION:

"Tension is the reaction force produced in a string when a mass is, suspended from it.
Motion of bodies connected by a string:

Case I: When both objective move vertically:

Consider two objects of masses 'm_1' and 'm_2' connected by a string which passes over a frictionless pulley, as shown in figure.

Since $m_1 > m_2$

Therefore 'A' moves in downward direction with acceleration 'a' while 'B' moves in upward direction.

i) Downward motion of Body 'A'

Since body 'A' is moving downwards, thus,

Net force – weight = Tension

$F_1 = W_1 - T$

$m_1a = m_1g \quad T \longrightarrow (1)$

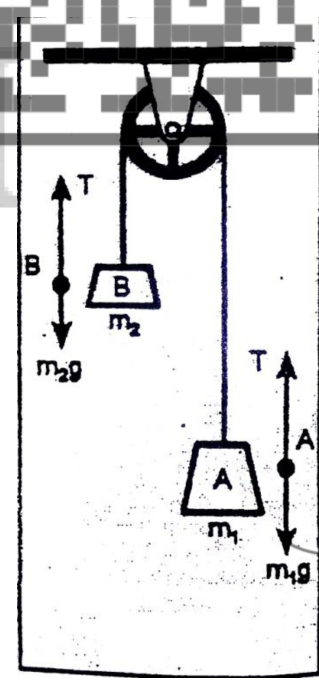
ii) Upward motion of Body 'B'

Since body 'B' is moving upwards direction thus,

Net force = Tension – weight

$F_2 = W_2 - T$

$m_2a = m_2g \quad T \longrightarrow (2)$



For Accelaration:

Adding eq (!) and eq (2)

$m_1a = m_1g - T$

$+ m_2a = T - m_2g$

$m_1a + m_2a = m_1g - m_2g$

$a(m_1 - m_2) = (m_1 - m_2)g$

$$a = \frac{(m_1 - m_2)g}{(m_1 - m_2)}$$

For Tension:

putting value of 'a 'in eq (2),

$m_2a = T - m_2g \longrightarrow (2)$

$$m_2 \frac{(m_1 - m_2)}{(m_1 + m_2)} g = T - m_2g$$

$$m_2 \frac{(m_1 - m_2)}{(m_1 + m_2)}g + m_2g = T$$

$$T = m_2g\left(\frac{m_1 - m_2}{m_1 + m_2} + 1\right)$$

$$T = m_2g\left(\frac{m_1 - m_2 + m_1 + m_2}{m_1 + m_2}\right)$$

$$\mathbf{T = \frac{2\, m_1\, m_2 g}{m_1 + m_2}}$$

Case II: **When one object moves vertically and other moves on a horizontal surface:**

Consider two object of masses 'm1' and 'm2' connected by a string which pass over a frictionless pully. The body B moves on a smooth horizontal surface towards the pully, while 'A' moves vertically.

i) DOWNWARD MOTION OF BODY 'A"

Since body 'A' moves in downward direction, Net force = weight – Tension

$F_1 = W_1 - T$

$m_1a = m_1g - T \longrightarrow (1)$

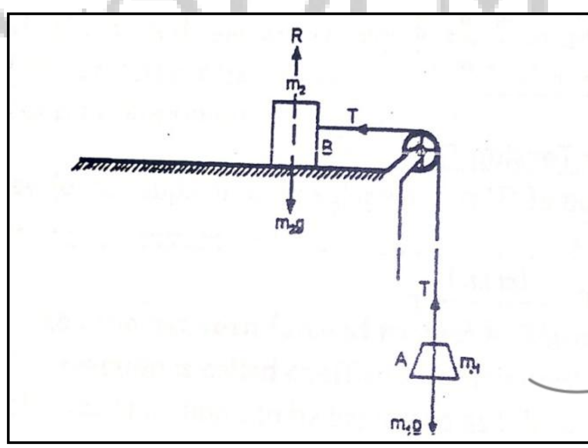
ii) HORIZONTAL MOTION OF BODY 'B"

Since body 'A' moves on horizontal direction,

Net force = Tension

$F_2 = T$

$m_2a = T \longrightarrow (2)$



For Acceleration:

While in vertical direction, $R = M_2g$

Where 'R' is the reaction of surface Solution of equations: Adding eq (1) and eq(2)

$m_1a = m_1g - T$

$+ \quad m_2a = T$

$m_1a + m_2a = m_1g$

$a(m_1+m_2) = m_1g$

$$a = \frac{m_1 g}{(m_1 + m_2)}$$

For Tension:

putting the value of 'a' in eq (2),

$m_2 a = T$

$$T = \frac{m_1 m_2 g}{(m1 + m2)}$$

MOMENTUM:

DEFINITION: The product of mass and velocity of a body is called its momentum.'

EXPLANATION: Let 'm' be the mass of a body, moving with velocity 'V' then it is a momentum 'P' is

$$P = mv$$

Momentum is a vector quantity and its direction is parallel to the direction of the velocity.

LAW OF CONSERVATION OF MOMENTUM

STATEMENT:

The law of conservation of momentum states that:

"When some bodies constituting an isolated system act upon one another, the total momentum of the system remains constant."

OR

"The total momentum of an isolated system of interacting bodies remains constant."

OR

"Total momentum of an isolated system before collision is always equal to total momentum after collision."

EXPLANATION:

Consider an isolated system of the interacting bodies 'A' and 'B' of masses 'm_1' and 'm_2' colliding with velocities U_1 and U_2 after colliding they move with velocities V_1 and V_2

Therefore,

Total momentum of the system before collision = $m_1u_1 + m_2u_2$ and total momentum of the bodies collide with each other they come in contact for a time interval 's'. During the interval the average force exerted by the body 'A' on body 'B' is F.

According to the third law of motion, the body 'B' will also exert a force (-F) on the body 'A'.

The average force acting on the body 'B' is equal to the rate of change of its momentum.

$$\vec{F}_{\text{A on B}} = \underline{m_2v_2 - m_2u_2}$$

$$t$$

Similarly the average force acting on body 'A' is,

$$\vec{F}_{\text{B on A}} = \frac{m_1v_1 - m_1u_1}{t}$$

Since,

$$\vec{F}_{\text{B on A}} = \vec{F}_{\text{A on B}}$$

$$\frac{m_2v_2 - m_2u_2}{t} = \frac{m_1v_1 - m_1u_1}{t}$$

$$\boxed{m_1u_1 + m_2u_2 = m_2v_1 + m_2v_2}$$

This proves that,
The total momentum of the system remains constant.

ELASTIC COLLISION:

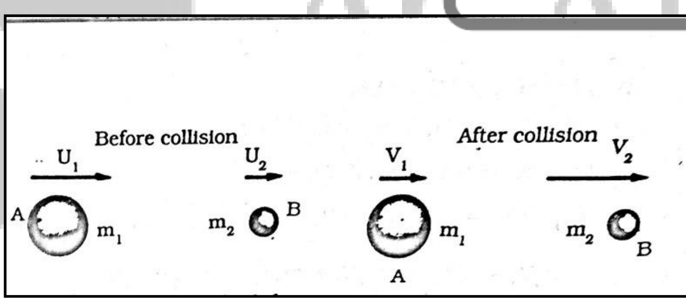
It is the collision in which momentum as well as kinetic energy of the system before and after collision is conserved.

INELASTIC COLLISION:

It is the collision in which momentum of the system remains conserved but kinetic energy before and after collision changes.

ELASTIC COLLISION IN ONE DIMENSION:

Consider two unequal, non rotating spheres of masses 'm_1' and 'm_2' moving with initial velocities 'u_1' and 'u_2'. If $u_1 > u_2$, the body 'A' will collide with body 'B' and both moves with velocities 'V_1' and 'V_2' in the line and direction as shown. According to the law of conservation of momentum,



Initial momentum of the system = Final momentum of the system.

$$m_1u_1 + m_2 u_2 = m_1v_1 + m_2v_2 \quad \text{----------------------(1)}$$

or $\quad m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$

or $\quad m_1(u_1 - v_1) = m_2(v_2 - u_2) \longrightarrow (2)$

And for elastic collision,

Total K.E of the system = Total K.E of the system
Before collision after collision

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$m_1u_1^2 + m_2u_2^2 = m_1v_1^2 + m_2v_2^2$$

$$m_1u_1^2 - m_2u_2^2 = m_1v_1^2 - m_2v_2^2$$

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

$$m_1(u_1 + v_1)(u_1 - v_1) = m_2(v_2 + u_2)(v_2 - u_2) \longrightarrow (3)$$

Dividing eq (2) by eq (1),

$$\frac{m_1(u_1 + v_1)(u_1 - v_1)}{m_1(u_1 - v_1)} = \frac{m_2(v_2 + u_2)(v_2 - u_2)}{m_2(v_2 - u_2)}$$

$$u_1 + v_1 = v_2 + u_2 \longrightarrow (4)$$

FOR V_1:

From eq (3),
$$v_2 = u_1 + v_1 - u_2$$

putting the value of 'V_2' in eq (1),

$$m_1u_1 + m_2u_2 = m_2v_1 + m_2(u_1 + v_1 - u_2)$$

$$m_1u_1 + m_2u_2 = m_2v_1 + m_2u_1 + m_2v_1 - m_2u_2$$

$$m_1u_1 + m_2u_2 - m_2u_1 + m_2u_2 = m_2v_1 + m_2v_1$$

$$(m_1 - m_2)u_1 + 2m_2u_2 = (m_2 + m_2)v_1$$

$$\boxed{\frac{(m_1 - m_2)}{(m_1 + m_2)}u_1 + \frac{2m_2u_2}{(m_1 + m_2)} = V_1} \longrightarrow (A)$$

FOR V_2:

From eq (3),

$v_1 = v_2 + u_2 - u_1$

putting the value of 'V_1' in eq (1),

$m_1u_1 + m_2u_2 = m_1(v_2 + u_2 - u_1) + m_2v_2$

$m_1u_1 + m_2u_2 = m_1v_2 + m_1u_2 - m_1u_1 + m_2v_2$

$m_1u_1 + m_1u_2 - m_1u_2 + m_1u_1 = m_1v_2 + m_2v_2$

$2m_1u_1 + (m_1 - m_1)u_2 = (m_1 + m_2)v_2$

$$v_2 = \frac{2m_1u_1}{(m_1 + m_2)} + \frac{(m_1 - m_2)u_2}{(m_1 + m_2)} \longrightarrow (B)$$

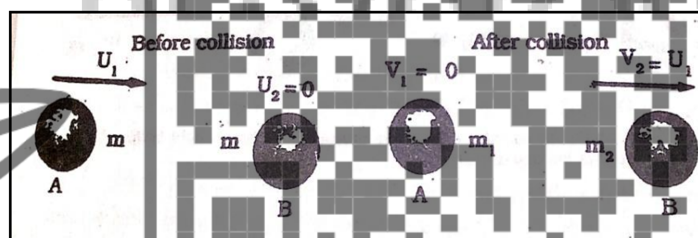
SPECIAL CASES OF ELASTIC COLLISIONS

Case I: if $m_1 = m_2$,

Let $m_1 = m_2 = m$

Then from equ (A)

$$\frac{(m_1 - m_2)u_1}{(m_1 + m_2)} + \frac{2m_2u_2}{(m_1 + m_2)} = V_1$$



Thus,

$$v_1 = \frac{(m - m)u_1}{(m + m)} + \frac{2mu_2}{(m + m)}$$

$$v_1 = 0 + \frac{2mu_2}{2m}$$

$$\mathbf{v_1 = u_2}$$

And from eq (B),

$$v_2 = \frac{2m_1u_1}{(m_1 + m_2)} - \frac{(m_1 - m_2)u_2}{(m_1 + m_2)}$$

Thus,

$$v_2 = \frac{2mu_1}{(m+m)} - \frac{(m-m)u_2}{(m+m)}$$

$$v_2 = \frac{2mu_1}{2m}$$

$$\boxed{v_2 = u_1}$$

This shown that the bodies interchange their velocities after the collision.

Case II: if $m_1 = m_2$, and $u_2 = 0$

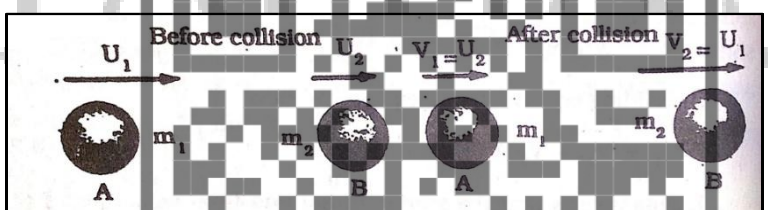
let $m_1 = m_2 = m$, $u_2 = 0$

Then,

$$\frac{(m_1 - m_2)\,u_1}{(m_1 + m_2)} + \frac{2m_2u_2}{(m_1 + m_2)} = V_1$$

$$\frac{(m - m)\,u_1}{(m + m)} + \frac{2m(0)}{(m + m)} = V_1$$

$$\boxed{v_1 = 0}$$



And,

$$V_2 = \frac{2mu_1}{(m+m)} + \frac{(m-m)}{(m+m)} \times 0$$

$$\boxed{v_2 = u_1}$$

This means that body 'A' will stop after collision and 'B' will move with the initial velocity of A.

Case III: if $m_1 << m_2$, and $u_2 = 0$ (Massive body at rest)

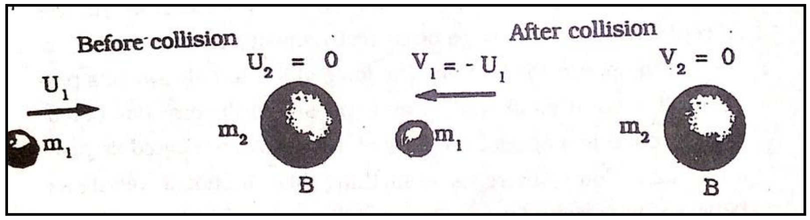
Now, m_1 can be neglected

Since,

$$v_1 = \frac{(m_1 - m_2)\,u_1}{(m_1 + m_2)} + \frac{2m_2u_2}{(m_1 + m_2)}$$

$$v_1 = \frac{(0 - m_2)}{(0 + m_2)} u_1 + \frac{2m_2(0)}{(0 + m_2)}$$

$$\boxed{v_1 = -u_1}$$



and

$$v_2 = \frac{2m_1u_1}{(m_1 + m_2)} + \frac{(m_1 - m_2)u_2}{(m_1 + m_2)}$$

$$v_2 = \frac{2(0)\ u_1}{(0 + m_2)} + \frac{(0 - m_2)(0)}{(0 + m_2)}$$

thus,

$$\boxed{v_2 = 0}$$

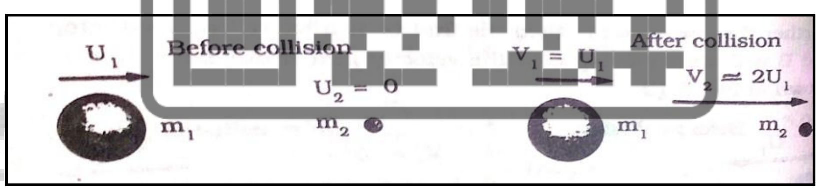
This means that body lighter body "A" will comes back with its initial velocity and the massive body "B" will remain at rest.

Case IV: if $m_1 >> m_2$ and $u_2 = 0$

then m_2 can be neglected,
Thus

$$v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)} u_1 + \frac{2m_2u_2}{(m_1 + m_2)}$$

$$v_1 = \frac{(m_1 - 0)}{(m_1 + 0)} u_1 + \frac{2(0)(0)}{(m_1 + 0)}$$



$$\boxed{v_1 = u_1}$$

And,

$$v_2 = \frac{2m_1u_1}{(m_1 + m_2)} + \frac{(m_1 - m_2)u_2}{(m_1 + m_2)}$$

$$v_2 = \frac{2m_1u_1}{(m_1 + 0)} + \frac{(m_1 - 0)(0)}{(m_1 + 0)}$$

$$\boxed{V_2 = 2u_1}$$

This means that body 'A' will continues its motion with the same velocity and 'B' will move with double the velocity of body A.

FORCE OF FRICTION

Definition:

When a body slides over another body, it gets resistance in its motion. It means a force is set up between the surfaces in contact which tends to oppose the motion. This force between the two surfaces in contact is called force of friction and this is because of roughness of the material surfaces in contact. The force of friction always acts parallel to the surfaces in contact and opposite to the direction of motion.

Explanation:

Suppose a rectangular block is placed on a horizontal surface. Forces acting on the block are its weight 'mg' downward, and reaction of the horizontal surface 'R' upward.If the block is at rest "mg" is balanced by "R". Now small force F is applied on block parallel to surfaces. If the block is still at rest then some force F is set up between surfaces which are opposing the motion. This force F is equal to applied force F and known as static friction. Now on increasing the applied force gradually , opposing force between surfaces will also go on increasing upto the value, when the block is just about to move. This maximum value of force of friction is called limiting friction.

If external force applied on block is R further increased, equilibrium will be lost and it will start moving. The force of friction f between surfaces in this situation is called F sliding friction or kinetic friction. If one body rolls over the surface, then force of friction is called rolling friction. Sliding friction is always less than limiting friction. Whereas rolling friction is less than sliding friction. The angl e .between resultant force of limiting friction and normal reaction "R" is called angle of friction.

Coefficient of Friction:

It is observed experimentally that limiting friction between surfaces is directly proportional to normal reaction "R" i.e.

$$F \propto R$$

$$=> F = \mu R$$

or

$$\boxed{\mu = F/R}$$

Where, μ is the constant of proportionately called co-efficient of friction.

Merits & Demerits of Friction:

Friction plays very important role in our daily life. It is advantageous as well as disadvantageous depending upon desired results. Friction between rubber and ground is desirable, without this it is impossible to walk, to drive vehicles. Friction between moving parts of different machines or engines is undesirable, because it affects the efficiency. For reducing it different lubricants are used. Another astonishing feature of frictional force is that it is a self adjusting force i.e. 'it varies with external force and it depends upon the nature of the two surfaces in contact.

Fluid Friction & Viscosity:

A thick layer of liquid consists of large number of microscopic layers of molecules. When liquid flows each of its layers slides over the other, experiences force which opposes their motion. This internal friction between layers of same liquid, which makes it to flow slowly or resists in flow is called Viscosity: This property is found in all fluids. It is found that opposing tangential force between last stationary layer and any upper layer of given liquid is directly proportional to area of contact, velocity of layer and inversely proportional to distance of layer from stationary layer.

$$F \propto \frac{Av}{d}$$

$$F = \eta \frac{Av}{d}$$

Where η the constant of proportionality is called co-efficient of viscosity of given liquid.

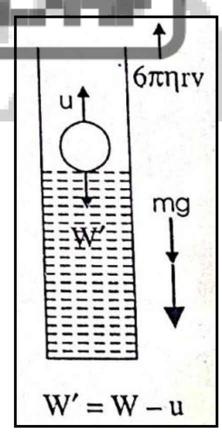
Fluid Friction or Viscous Drag:

When bodies are allowed to move through liquid or gases, they experience force which opposes their motion. This opposing force offered by liquid or gases is called viscous drag or fluid friction.

Stoke studied the effect of viscous drag on small spheres falling through liquids. He found that sphere of radius "r" falling through liquids of viscosity η with velocity v experience retarding force "F" given by,

F= $6\,\pi\,\eta\,r\,v$

This is called Stoke's law. This equation shows that retarding force on sphere depends upon velocity "v". Forces acting on the falling sphere in liquid are mg downward and retarding force $6\,\pi\,\eta\,r\,v$ upward.Net force with which spheres falls in liquid is mg - $6\,\pi\,\eta\,r\,v$

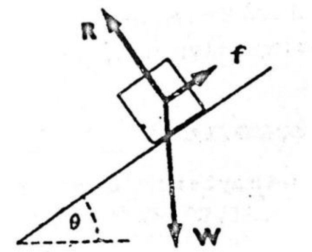


Retarding $6\,\pi\,\eta\,r\,v$ increases with increase in velocity. After falling through some distance, velocity of sphere attains such a value that force ($6\,\pi\,\eta\,r\,v$ becomes equal to mg).Under this condition sphere starts falling down with constant velocity. This constant velocity of sphere in a given liquid at which mg becomes equal to retarding force is called terminal velocity denoted by Vt.

INCLINED PLANE

Definition:

Such a surface which makes an angle θ with horizontal such that, $0° < \theta < 90°$ is called an inclined plane.



Explanation:

Since heavy load may be lifted more easily by pulling it along an inclined plane than by lifting it vertically. Therefore, it is treated as simple machine. Let us consider a block of mass 'm' placed on an inclined surface which makes an angle θ with the horizontal. Here there are three force acting the block i.e.

(i) Frictional force between block and the inclined plane ("f")

(ii) Reaction of the surface on the block (R)

(iii) Weight of the body (W)

The weight W of the block can be resolved into two components and for this purpose.We take x-axis parallel to the inclined plane and y-axis perpendicular .Now the components of weight perpendicular and parallel to the surface are

$$\cos\theta = \frac{W_{\perp}}{W}$$

or

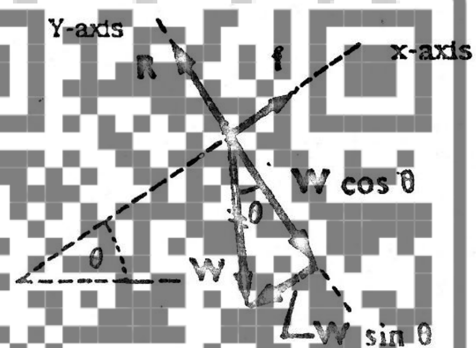
$$\boxed{W_{\perp} = W\cos\theta}$$

and

$$\sin\theta = \frac{W_{\parallel}}{W}$$

or

$$\boxed{W_{\parallel} = W\sin\theta}$$



It is clear from fig. that block will remain at rest if it fulfils conditions of equilibrium i.e. opposite force should be equal to each other i.e

$$f = W_{\parallel}$$

or

$$\boxed{f = W\sin\theta}$$

and

$$R = W_{\perp}$$

or

$$\boxed{R = W\cos\theta}$$

Equation no (i) and (ii) are conditions under which block remains at rest.Now, if force of friction between block and inclined plane becomes small then block can slide down with an acceleration a under the net force,

$$F = W_{\parallel} - f$$

or

$$ma = W\sin\theta - f$$

or

$$ma = mg\sin\theta - f$$

or

$$a = \frac{mg\sin\theta - f}{m}$$

or

$$\boxed{a = g\sin\theta - \frac{f}{m}} \quad \text{---------(i)}$$

This equation represents the acceleration of the body sliding down on an inclined plane having friction "f".

<u>If Friction is Absent:</u>

If $f = 0$, then eq(i) becomes

$$\boxed{a = g\sin\theta} \quad \text{----------(ii)}$$

<u>Special Cases:</u>

(i) If $\theta = 0°$

then eq (ii) => $a = g\sin 0°$

$$a = g(0)$$

$$\boxed{a = 0}$$

(ii) If $\theta = 90°$

then Eq (ii) => $a = g\sin 90°$

$$a = g(1)$$

$$\boxed{a = g}$$

i.e. Free Fall Motion.

M.C.Qs.

1. The area between a velocity time graph and the time axis is equal to the:

(a) Velocity (b) Distance

(c) Displacement (d) Acceleration

2. Swimming is possible on account of:

(a) First law of motion

(b) Second law of motion

(c) Third law of motion

(d) Newton's law of gravitation

3. Inertia of a body is measured in terms of
(a) its weight (b) its applied force
(c) its reaction (d) its mass

4. During free fall, of air friction is negligible then acceleration of bodies of different masses is:
(a) The same for all the masses
(b) Different for different masses
(c) Different for different vertical positions.
(d) Both A & B

5. The frictional resistance between its various layers of fluids is called:
(a) Viscous drag (b) Viscosity
(c) Friction (d) Up thrust

6. If two bodies of equal mass collide elastically then:
(a) their velocities are added to each other
(b) their velocities are subtracted
(c) their velocities do not changed
(d) they exchange their velocities

7. To produce same acceleration in the bodies of masses 10 kg and 20 kg the force applied on the second body should be:
(a) Halved (b) Equal to that on first body
(c) Doubled (d) Three times

8. A truck covers a distance of 360km in 5 hrs. its speed will be:
(a) 180 km/h (b) 360 km/h
(c) 72 km/h (d)36km/h

9. How long does it take by a car going at 30 m/s to stop if it decelerates at 7m/s2:
(a) 4s (b) 5s
(c) 6s (d) 7s

10. Stoke's law holds for:
(a) bodies of all shapes
(b) Motion through free space
(c) horizontal motion of particles
(d) motion through a viscous medium

11. The coefficient of frictional force between two surfaces in contact does NOT depends upon.
(a) The normal force passing one against the other (b) The area of surfaces
(c) Whether the surfaces are stationary or in relative motion
(d) whether a lubricant is used or not.

12. If the time interval is very small ($\Delta t \to 0$), the rate of change of velocity of a body is called:
(a) Average acceleration
(b) acceleration
(c) instantaneous acceleration
(d) constant acceleration

13. If velocity of a body is decreasing, the direction of acceleration is:
(a) in the direction of velocity
(b)opposite to the direction of velocity
(c) perpendicular to the direction of velocity
(d) 60^o to the direction of velocity

14. Kinetic friction is always:
(a) greater than static friction
(b) Equal to static friction
(c) less than static friction
(d) zero

15. If 'F' be the limiting friction and 'R' the normal reaction. Then co-efficient of static friction 'μ' is:
(a) μ= F/R (b) μ= R/F
(c) μ=FR (d) μ= 1/FR

16. A body falls freely .The distance covered by it in 2 sec is:
(a) 9.8 m (b) 19.6 m
(c)39.2 m (d) 100 m

17. The acceleration of body moving down a

frictionless plane inclines at 30^0 will be:

(a) 4.9 m/s^2 (b) 9.8 m/s^2
(c) 98 m/s^2 (d) 10 m/s^2

18. If the rate of change momentum with respect to time is zero then.

(a) The momentum is a function of time
(b) The momentum is not conserved
(c) The momentum is constant
(d) Some force acts

19. Which of the following changes when a particle is moving with uniform velocity?

(a) Speed (b) Velocity
(c) Acceleration (d) Position vector

20. If linear momentum of a particle in doubled, its kinetic energy will.

(a) be double (b) be halved
(c) be quadrupled (d) Remains unchanged

21. A collision in which momentum conserved but K.E is not conserved is called:

(a) Elastic collision (b) In elastic collision
(c) Both A & B (d) either A or B

22. A passenger in a moving bus is thrown forward when the bus is suddenly stopped. This is explained

(a) by Newton's first law
(b) by Newton's second law
(c) by Newton's third law
(d) by the principle of conservation of momentum

23. A gun of mass 1000 kg fires a projectile of mass 1 kg with a horizontal velocity of 100 m/s. The velocity of recoil of the gun in the horizontal direction is

(a) 5 m/s (b) 0.1 m/s
(c) 15 m/s (d) 20 m/s

24. A block of wood is placed on a surface. A force is applied parallel to the surface to move the body. The frictional force developed acts

(a) normal to the surface upwards
(b) normal to the surface downwards
(c) along the direction of the applied force
(d) opposite to the direction of the applied force

25. The inherent property, with which a body resists any change in its state of motion is known as

(a) Force (b) Momentum
(c) Inertia (d) Acceleration

26. 1 Newton = ————

(a) 1 Newton = 1 kg x 1 m/s2
(b) 1 Newton = 1 kg
(c) 1 Newton = 1 kg x 1 m/s
(d) 1 Newton = m/s2

27. When the body is stationary:

(a) There is no force acting on it
(b) The force acting on it are not in contact each other
(c) The forces acting on it are balanced with it
(d) The body is in vacuum

PAST PAPER M.C.Qs.

2022

6.An object is falling through a fluid with terminal velocity, its velocity is

* zero *decreased * increased *** constant**

13. A truck covers a distance of 360km in 5 hrs. its speed will be:

* 180 km/h *360 km/h ***72 km/h** *36km/h

27.A ball is dropped from a height of 100m, its acceleration at half of height will be:

*** 9.8 m/s^2** * 4.9 m/s^2 *10 m/s^2 * 5 m/s^2

2021

(ii) To produce same acceleration in the bodies of masses 10 kg and 20 kg the force applied on the second body should be:

*Halved *Equal to that on first body ***Doubled** *Three times

(iii) A truck covers a distance of 360km in 5 hrs. its speed will be:

* 180 km/h *360 km/h ***72 km/h** *36km/h

(iv) How long does it take by a car going at30m/s to stop if it decelerates at $7m/s^2$:

***4s** *5s *6s *7s

(v) The force acting on a body of 1 kg mass falling freely will be:

*5 N *19.6 N ***9.8 N** * Zero N

(xxvi) The acceleration of a body moving down a frictionless plane inclined at 30° will be:

***4.9m/sec^2** *9.8m/sec^2 *98m/sec^3 *10m/sec^2

38 .If 'F' be the limiting friction and 'R' the normal reaction. Then co-efficient of static friction`μ' is:

*** $\mu = \frac{F}{R}$** *$\mu = \frac{R}{F}$ *$\mu = FR$ *$\mu = \frac{1}{FR}$

2019

4.If 'F' be the limiting friction and 'R' the normal reaction. Then co-efficient of static friction`μ' is:

*** $\mu = \frac{F}{R}$** *$\mu = \frac{R}{F}$ *$\mu = FR$ *$\mu = \frac{1}{FR}$

16. If the time interval is very small ($\Delta t \to 0$), the rate of change of velocity of a body is called:

*Average acceleration *acceleration

***instantaneous acceleration** *constant acceleration

2018

13. A bullet is fired horizontally with 20 m/s, the absence of air friction, its horizontal velocity component after 2 s will be:

*40m/s ***20m/s** * 10 m/s * 5 m/s

16.An object is falling through a viscous fluid with terminal velocity,its velocity:

*is increasing *is decreasing *** remains same** * becomes zero

2017

1. A bus of weight 30000 N is moving with uniform velocity of 14 m/s, its acceleration is:

*14 m/s² ***zero** *7 m/s² *9.8 m/s²

17. Stokes law is applicable to the:

*bodies resting on the surface of liquid

***moving bodies through the viscous medium**

*moving bodies through the non viscous medium

*moving bodies through vacuum

2016

2. If velocity of a body is decreasing, the direction of acceleration is:

*in the direction of velocity ***opposite to the direction of velocity**

*perpendicular to the direction of velocity *60^o to the direction of velocity

17. Kinetic friction is always:

*greater than static friction *Equal to static friction

***less than static friction** *zero

2015

4. The rate of change of linear momentum is:

*acceleration * torque ***force** * velocity

5. If 'F' be the limiting friction and 'R' the normal reaction. Then co-efficient of static friction 'μ' is:

***μ= F/R** *μ= R/F *μ=FR *μ= 1/FR

2013

2. If the average and instantaneous velocities of a body are the same, the body will move with:

*Variable velocity ***Uniform velocity**

*Uniform acceleration *Variable acceleration

6. A body falls freely .The distance covered by it in 2 sec is:

*9.8 m ***19.6 m** *39.2 m *100 m

2012

5. The property of fluids due to which they resists their flow is called:

*co efficient of friction * static friction

*** viscosity** * terminal velocity

17. A helicopter weighing 3920 N is moving up with a constant speed of 4 m/s. The force on the helicopter is:

*4720 N *** 3920N** * 3924 N * 3916N

2011

5. A one kilogram stone, falling freely from a height of 10 meter, strikes the ground with a velocity of:

***14 m/s** *10 m/s *98 m/s *19.6 m/s

14. The acceleration of body moving down a frictionless plane inclines at 30^0 will be:

***4.9 m/s²** * 9.8 m/s²

*98 m/s² *10 m/s²

2010

15. Stokes's law holds good for:
  * the bodies of all shapes
  * motion through non viscous medium
  * motion through vacuum
  * **motion through viscous medium**

16. How many meters will a 20 kg ball, starting from rest, falls freely in one second?
  * 19.6 m
  * **9.8 m**
  * 4 m
  * 4.9 m

TEXTBOOK NUMERICALS

Q.1: In an electron gun of a television set, an electron with an initial speed of 10^3 m/s enters region where it is electrically accelerated. It emerges out of this region after 1 micro second with speed of 4×10^5 m/s. What is the maximum length of the electron gun? Calculate the acceleration.

Data:

Initial Speed $= v_i = 10^3 m/s$

Time $= t = 1\ \mu S = 1 \times 10^{-6} s$

Final Speed $= v_f = 4 \times 10^5 m/s$

Length of Gun $= S = ?$

Acceleration $= a = ?$

Solution:

Using First Equation of Motion

$$\mathrm{v_f} = \mathrm{v_i} + \mathrm{at}$$

$$4 \times 10^5 = 10^3 + \mathrm{a}(1 \times 10^{-6})$$

$$\mathrm{a} = \frac{4\times10^5 - 10^3}{1\times10^{-6}}$$

$$\boxed{\mathrm{a} = 3.99 \times 10^{11} m/s^2}$$

Now,

$$2aS = V_f^2 - V_i^2$$

$$2(3.99 \times 10^{11})S = (4 \times 10^5)^2 - (1 \times 10^3)^2$$

$$S = \frac{(4\times10^5)^2 - (1\times10^3)^2}{2(3.99\times10^{11})}$$

$$\boxed{S = 0.2\ m}$$

Result: The maximum length of the electron gun is 0.2 m and the acceleration is $3.99 \times 10^{11} m/s^2$

Q.2: A car is waiting at a traffic signal and when it turns green, the car stats ahead with a constant acceleration of 2 m/s². At the same time a bus traveling with a constant speed of 10 m/s overtakes and passes the car. (a) How far beyond its starting point will the car overtake the bus? (b) How fast will the car be moving?

Data:

Intial Speed of car $= v_i = 0$

Acceleration of car $= a = 2\ m/s^2$

Speed of Bus $= v = 10\ m/s$

Distance $= S = ?$

Final speed of car $= v_f = ?$

Solution:

For Car:

$$S = v_i t + \frac{1}{2}at^2$$

$$S = 0 \times t + \frac{1}{2}(2)t^2$$

$$S = t^2 \text{ --- (i)}$$

For Bus:

$$S = v \times t$$

$$S = 10\ t \text{ ----- (ii)}$$

Comparing eq (i) and eq (ii)

$$t^2 = 10\ t$$

$$\boxed{t = 10\ sec}$$

Putting in eq (ii)

$$\boxed{S = 10 \times 10 = 100\ m}$$

Final Velocity of Car:

$$v_f = v_i + at$$

$$v_f = 0 + 2 \times 10$$

$$\boxed{v_f = 20\ m/s}$$

Result: The bus overtakes at 100 m distance from starting point and car was moving with 20 m/s.

Q.3: A helicopter ascending at a rate of 12 m/s. At a height of 80m above the ground, a package is dropped.How long does the package take to reach the ground?

Data:

Intial Speed $= v_i = 12\ m/s$

Initial Height $= H = 80\ m$

Total time $= t = ?$

Solution:

During upward Motion

For t_1:

$$\mathrm{v_f = v_i + at}$$

$$0 = 12 + (-9.8)\mathrm{t_1}$$

$$\boxed{\mathrm{t_1} = \frac{12}{9.8} = 1.22\ \mathrm{sec}}$$

For S_1:

$$\mathrm{S = v_i t + \frac{1}{2}at^2}$$

$$\mathrm{S_1} = 12 \times 1.22 + \frac{1}{2}(-9.8)(1.22)^2$$

$$\mathrm{S_1} = 14.64 - 7.29$$

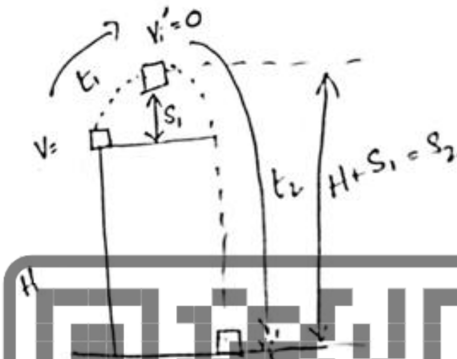
$$\boxed{\mathrm{S_1} = 7.35\ \mathrm{m}}$$

During Downward Motion

For t_2:

$$\mathrm{S = v_i t + \frac{1}{2}at^2}$$

Or



$$\mathrm{S_1 + H} = 0 \times \mathrm{t_2} + \frac{1}{2}(9.8)\mathrm{t_2}^2$$

$$7.35 + 80 = (4.9)\mathrm{t_2}^2$$

$$87.35 = 4.9\mathrm{t_2^2}$$

$$\mathrm{t_2}^2 = 17.82$$

Taking square root

$$\boxed{\mathrm{t_2} = 4.22\ \mathrm{sec}}$$

Now, Total time is given by

$$\boxed{\mathrm{t = t_1 + t_2} = 1.22 + 4.22 = 5.44\ \mathrm{sec}}$$

Result: The total time taken will be 5.44 sec.

Q.4: A boy throws a ball upward from the top of a cliff with a speed of 14.7 m/s. On the way down it just misses the thrower and fall the ground 49 metres below. Find (a) How long the ball rises? (b) How high it goes? (c) How long it is in air and (d) with what velocity it strikes the ground.

Data:

Intial Speed $= v_i = 14.7\ m/s$

Height $= H = 49\ m$

(a) Time for Upward flight $= t_1 = ?$

(b) Height gained by ball $= h_2 = ?$

(c) Total Time $= t = ?$

(d) Final Speed $= v_f = ?$

Solution:

During upward Motion

For t_1:

$$\mathrm{v_f = v_i + at}$$

$$0 = 14.7 + (-9.8)\mathrm{t_1}$$

$$\boxed{\mathrm{t_1} = \frac{14.7}{9.8} = 1.5\ \mathrm{sec}}$$

For S_1:

$$\mathrm{S = v_i t + \frac{1}{2}at^2}$$

$$\mathrm{S_1} = 14.7 \times 1.5 + \frac{1}{2}(-9.8)(1.5)^2$$

$$\mathrm{S_1} = 22.05 - 11.025$$

$\boxed{S_1 = 11.025 \text{ m}}$

During Downward Motion

For t_2:

$$S = v_i t + \frac{1}{2}at^2$$

Or $$S_1 + H = 0 \times t_2 + \frac{1}{2}(9.8){t_2}^2$$

$$11.025 + 49 = (4.9){t_2}^2$$

$$60 = 4.9t_2^2$$

$${t_2}^2 = 12.24$$

Taking square root

$$\boxed{t_2 = 3.5 \text{ sec}}$$

Now, Total time is given by

$$\boxed{t = t_1 + t_2 = 1.5 + 3.5 = 5 \text{ sec}}$$

For Final Velocity:

$$v_f = v_i + at$$

$$v_f = 0 + (9.8)t_2$$

$$v_f = 9.8 \times 3.5$$

$$\boxed{v_f = 34.3 \text{ m/s}}$$

Result: The ball rises 1.5 sec and it goes 11.025 m. It is in air for 5 sec and with 34.3 m/s velocity it strikes the ground

Q.5: A helicopter weighs 3920 Newton. Calculate the force on it if it is ascending up at a rate of 2 m/s². What will be force on helicopter if it is moving up with the constant speed of 4 m/s?

Data:

Weight of Helicopter $= W = 3920N$

(a) Acceleration $= a = 2\ m/s^2$

Force on Helicopter $= F = ?$

(b) Speed of Helicopter $= v = 4\ m/s$

Force on Helicopter $= F = ?$

Solution:

$$m = \frac{W}{g} = \frac{3920}{9.8} = 400\ kg$$

(a) $Force\ applied\ by\ engine = weight + acceleration\ producing\ force$

$$F = W + ma$$

$$F = 3920 + 400 \times 2$$

$$\boxed{F = 4720\ N}$$

(b)

$Force\ applied\ by\ engine = weight(\ acceleration = 0)$

$$F = W$$

$$\boxed{F = 3920\ N}$$

Result: The force on helicopter when accelerated is 4720 N and when velocity is constant it is 3920 N.

Q.6: A bullet having a mass of 0.005 kg is moving with a speed of 100 m/s. It penetrates into a bag of sand and is brought to rest after moving 25cm into the bag. Find the deceleration force on the bullet. Also calculate the time in which it is brought to rest.

Data:

Mass of Bullet $= m = 0.005\ kg$

Initial Speed of Bullet $= v_i = 100\ m/s$

Final Speed of Bullet $= v_f = 0$

Distance covered $= S = 25\ cm = 0.25\ m$

Decelerating Force $= F = ?$

Time $= t = ?$

Solution:

As we know that

$$F = ma \text{ --- (i)}$$

For Acceleration:

$$2aS = V_f^2 - V_i^2$$

$$2a(0.25) = (0)^2 - (100)^2$$

$$a(0.5) = -10000$$

$$\boxed{a = -20000\ m/s^2}$$

Putting values in eq (i)

$$F = ma$$

$$F = 0.005 \times (-20000)$$

$$\boxed{F = -100\ N}$$

For Time:

$$v_f = v_i + at$$

$$0 = 100 + (-20000) \times t$$

$$t = \frac{100}{20000}$$

$$\boxed{t = 0.005\ sec}$$

Result: The deceleration force on the bullet is 100 Nand the time in which it is brought to rest is 0.005 sec.

Q.7: A car weighing 9800 N is moving with a speed of 40 km/h. On the application of the brakes it comes to rest after traveling a distance of 50 metres. Calculate the average retarding force.

Data:

Weight of Car $= W = 9800\ N$

Initial Speed of Car $= v_i = 40\frac{km}{h} = \frac{40\times1000}{3600} = 11.11\ m/s$

Final Speed of Car $= v_f = 0$

Distance covered $= S = 50\ m$

Retarding Force $= F = ?$

Solution:

$$m = \frac{W}{g} = \frac{9800}{9.8} = 1000\ kg$$

As we know that

$F = ma$ --- (i)

For Acceleration:

$$2aS = V_f^2 - V_i^2$$

$$2a(50) = (0)^2 - (11.11)^2$$

$$a(100) = -123.43$$

$$\boxed{a = -1.234\ m/s^2}$$

Putting values in eq (i)

$$F = ma$$

$$F = 1000 \times (-1.234)$$

$$\boxed{F = -1234\ N}$$

Result: The retarding force on the car is 1234 N.

Q.8: An electron in a vacuum tube starting from rest is uniformly accelerated by an electric field so that it has a speed 6x10⁶ m/s after covering a distance of 1.8cm. Find the force acting on the electron. Take the mass of electron as 9.1x10⁻³¹ kg.

Data:

Mass of Electron $= m = 9.1\text{x}10^{-31}$ kg

Initial Speed of Bullet $= v_i = 0$

Final Speed of Bullet $= v_f = 6\times10^6 m/s$

Distance covered $= S = 1.8\ cm = 0.018\ m$

Force $= F = ?$

Solution:

As we know that

$F = ma$ --- (i)

For Acceleration:

$$2aS = V_f^2 - V_i^2$$

$$2a(0.018) = (6\times10^6)^2 - (0)^2$$

$$a(0.036) = 3.6\times10^{13}$$

$$\boxed{a = 1\times10^{15}\ m/s^2}$$

Putting values in eq (i)

$$F = ma$$

$$F = 9.1\text{x}10^{-31} \times (1\times10^{15})$$

$$\boxed{F = 9.1\text{x}10^{-16}\ N}$$

Result: The force acting on the electron is $9.1\text{x}10^{-16}\ N$.

Q.9: Two bodies A & B are attached to the ends of a string which passes over a pulley, so that the two bodies hang vertically. If the mass of the body A is 4.8 kg. Find the mass of body B which moves down with an acceleration of 0.2 m/s² . The value of g can be taken as 9.8 m/s² .

Data:

Mass of Body A (Lighter body) $= m_2 = 4.8$ kg

Mass of Body B (Havier body) $= m_1 = ?$

Acceleration $= a = 0.2\ m/s^2$

Acceleration due to gravity $= g = 9.8\ m/s^2$

Solution:

As we know that

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$$

$$0.2 = \left(\frac{m_1 - 4.8}{m_1 + 4.8}\right) 9.8$$

$$0.2(m_1 + 4.8) = (m_1 - 4.8)9.8$$

$$0.2m_1 + 0.96 = 9.8\text{m}_1 - 47.04$$

$$0.2m_1 - 9.8\text{m}_1 = -47.04 - 0.96$$

$$-9.6\text{m}_{1=} - 48$$

$\boxed{m_1 = 5\text{ kg}}$

Result: The mass of body B which moves down is 5 kg.

Q.10: Two bodies of masses 10.2 kg & 4.5 kg are attached to the ends of a string which passes over a pulley in such a way that the body of mass 10.2 kg lies on a smooth surface and the other body hangs vertically. Find the acceleration of the bodies and tension of the string and also the force, which the surface exerts, on the body of mass 10.2 kg.

Data:

Mass of Body A $= m_2 = 10.2$ kg

Mass of Body B $= m_1 = 4.5$ kg

Acceleration $= a = ?$

Tension $= T = ?$

Force on $m_2 = R = ?$

Solution:

(i) $a = \left(\frac{m_1}{m_1+m_2}\right) g$

$a = \left(\frac{4.5}{4.5+10.2}\right) 9.8$

$\boxed{a = 3\ m/s^2}$

(ii) $T = \left(\frac{m_1 m_2}{m_1+m_2}\right) g$

$T = \left(\frac{4.5\times10.2}{4.5+10.2}\right) 9.8$

$\boxed{T = 30.6\ N}$

(iii) $R = w_2 = m_2 g$

$R = 10.2 \times 9.8$

$\boxed{R = 99.96\ N}$

Result: The acceleration of the bodies is 3 m/s² and tension of the string is 30.6 N.The force, which the surface exerts, on the body of mass 10.2 kg is 99.96 N

Q.11: A 100 grams bullet is fired from a 10 kg gun with a speed of 1000 m/s. What is the speed of recoil of the gun?

Data:

Mass of Bullet $= m_1 = 100\text{ g} = 0.1$ kg

Mass of Gun $= m_2 = 10$ kg

Initial Speed of Bullet $= u_1 = 0$

Initial Speed Gun $= u_2 = 0$

Final Speed of Bullet $= v_1 = 1000\ m/s$

Final Speed of Gun $= v_2 = ?$

Solution:

According to law of Conservation of Momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$(0.1)(0) + (10)(0) = (0.1)(1000) + (10)v_2$

$0 + 0 = 100 + (10)v_2$

$\boxed{v_2 = -\frac{100}{10} = -10\ m/s}$

Result: The speed of recoil of gun is 10 m/s.

Q.12: A 50 grams bullet is fired into a 10 kg block that is suspended by a long cord so that it can swing as a pendulum. If the block is displaced so that its centre of gravity rises by 10cm, what was the speed of the bullet?

Data:

Mass of Bullet $= m_1 = 50\text{ g} = 0.05$ kg

Mass of Block $= m_2 = 10$ kg

Height covered by block $= h = 10\ cm = 0.1\ m$

Initial Speed of Bullet $= u_1 = ?$

Solution:

According to law of Conservation of Momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$0.05 \text{ x } u_1 + 10 \text{ x } 0 = 0.05 \text{ x v} + 10 \text{ x v}$

$0.05 \text{ x } u_1 = 10.05 \text{ x v}$

$u_1 = \frac{10.05}{0.05} \text{ x v}$ ------ (i)

According to law of Conservation of Energy

Loss in K.E. = Gain in P.E

$\frac{1}{2}mv^2 = mgh$

$v^2 = 2gh \qquad => v = \sqrt{2gh}$

$v = \sqrt{2 \text{ x } 9.8 \text{ x } 0.1} = \sqrt{1.96}$

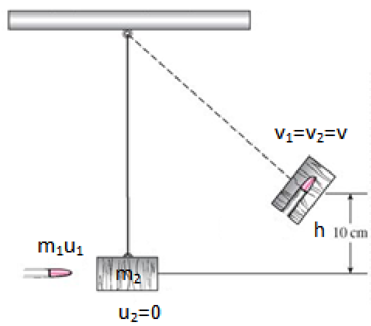
$\boxed{v = 1.4\ m/s}$

Putting in eq (i)

$u_1 = \frac{10.05}{0.05} \text{ x } 1.4$

$\boxed{u_1 = 281.4\text{ m/s}}$

Result: The Initial speed of bullet is 281.4 m/s.



Q.13: A machine gun fires 10 bullets per second into a target. Each bullet weighs 20 gm and had a speed of 1500 m/s. Find the force necessary to hold the gun in position.

Data:

No. of Bullet $= n = 10$

Time $= t = 1\ sec$

Mass of Bullet $= m = 20$ g $= 0.02$ kg

Initial Speed of Bullet $= v_i = 1500\ m/s$

Final Speed of Bullet $= v_f = 0$

Force $= F = ?$

Solution:

According to the definition of Force

$$F = \frac{\Delta P}{\Delta t}$$

Force due to one bullet is

$$f = \frac{m v_f - m v_i}{t}$$

$$f = \frac{0.02\times 0 - 0.02\times 1500}{1}$$

$$\boxed{f = -30\ N}$$

Now Force due to "n" Bullets

$$F = f \times n$$

$$F = -30 \times 10$$

$$\boxed{F = -300\ N}$$

Result: The force necessary to hold the gun in position is 300N

Q.14: A cyclist is going up a slope of 30° with a speed of 3.5 m/s. If he stops pedaling, how much distance will he move before coming to rest? (Assume the friction to be negligible).

Data:

Slope $= \theta = 30°$

Initial Speed of cyclist $= v_i = 3.5\ m/s$

Final Speed of cyclist $= v_f = 0$

Distance covered $= S = ?$

Solution:

Using 3rd equation of motion

$$2aS = V_f^2 - V_i^2 \text{ --- (i)}$$

When friction is absent

$$a = g\ Sin\theta$$

$$a = -9.8 \times\ Sin30°$$

$$\boxed{a = -4.9\ m/s^2}$$

Putting in eq (i)

$$2(-4.9)S = (0)^2 - (3.5)^2$$

$$-9.8 \times S = -12.25$$

$$\boxed{S = 1.25\ m}$$

Result: He will move1.25 m before coming to rest.

Q.15: The engine of a motorcar moving up 45° slope with a speed of 63 km/h stops working suddenly. How far will the car move before coming to rest? (Assume the friction to be negligible).

Data:

Slope $= \theta = 45°$

Initial Speed of Motor car $= v_i = 63\frac{km}{h} = \frac{63\times 1000}{3600} = 17.5\ m/s$

Final Speed of Motor Car $= v_f = 0$

Distance covered $= S = ?$

Solution:

Using 3rd equation of motion

$$2aS = V_f^2 - V_i^2 \text{ --- (i)}$$

When friction is absent

$$a = g\ Sin\theta$$

$$a = -9.8 \times Sin45°$$

$$\boxed{a = -6.92\ m/s^2}$$

Putting in eq (i)

$$2(-6.92)S = (0)^2 - (17.5)^2$$

$$-13.84 \times S = -306.25$$

$$\boxed{S = 22.1\ m}$$

Result: The car will move 22.1 m before coming to rest.

Q.16: In question no. 15, find the distance that the car moves, if it weighs 19,600N and the frictional force is 2000 N.

Data:

Slope $= \theta = 45°$

Initial Speed of Motor car $= v_i = 63\frac{km}{h} = \frac{63\times1000}{3600} = 17.5\ m/s$

Final Speed of Motor Car $= v_f = 0$

Weight of Motor car $= W = 19600\ N$

Friction $= f = 2000N$

Distance covered $= S = ?$

Solution:

$$m = \frac{W}{g} = \frac{19600}{9.8} = 2000\ kg$$

Using 3rd equation of motion

$$2aS = V_f^2 - V_i^2 \text{ --- (i)}$$

When friction is Present

$$a = g\ Sin\theta - \frac{f}{m}$$

$$a = -9.8 \times Sin45° - \frac{2000}{2000}$$

$$\boxed{a = -7.92\ m/s^2}$$

Putting in eq (i)

$$2(-7.92)S = (0)^2 - (17.5)^2$$

$$-15.84 \times S = -306.25$$

$$\boxed{S = 19.3\ m}$$

Result: The car will move 19.3 m before coming to rest.

2022

iv) A cyclist is going up a slope of 30° with a speed of 3.5 m/s. if he stops pedalling, how much distance will he move before coming to rest? (Assume, friction in negligible).

Data:

Slope $= \theta = 30°$

Initial Speed of cyclist $= v_i = 3.5\ m/s$

Final Speed of cyclist $= v_f = 0$

Distance covered $= S = ?$

Solution:

Using 3rd equation of motion

$$2aS = V_f^2 - V_i^2 \text{ --- (i)}$$

When friction is absent

$$a = g\ Sin\theta$$

$$a = -9.8 \times Sin30°$$

$$\boxed{a = -4.9\ m/s^2}$$

Putting in eq (i)

$$2(-4.9)S = (0)^2 - (3.5)^2$$

$$-9.8 \times S = -12.25$$

$$\boxed{S = 1.25\ m}$$

Result: He will move1.25 m before coming to rest.

2019

2(i) A body starts from rest and moves with constant acceleration of 10 m/s^2.How much distance will it travel in the 4th second of its motion?

Data:

Initial velocity= v_i = 0 m/s

Acceleration= a = 10 m/s^2

Distance during 4th sec = $S_4 - S_3$ =?

Solution:

(i) When t= 3 sec

$$S = v_i t + \frac{1}{2}at^2$$

$$S_3 = 0 + \frac{1}{2}(10)(3)^2$$

$$\boxed{S_3 = 5\ \times 9 = 45\ m}$$

(ii) When t= 4 sec

$$S = v_i t + \frac{1}{2}at^2$$

$$S_4 = 0 + \frac{1}{2}(10)(4)^2$$

$$\boxed{S_4 = 5\ \times 16 = 80\ m}$$

$$\therefore S_4 - S_3 = 80 - 45 = 35\ m$$

Result: Distance travelled during 4th second is equal to 35 m

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Q.2 (xv) Textbook Numerical 12

2018

Q. 2(v) A brick of mass 2 kg is dropped from a height of 5m above the ground. What is its velocity at a height of 3m above the ground?

Data:

Mass = m = 2kg

Initial Height = h_1=5m

Final Height = h_2=3m

Initial Velocity = v_i = 0 m/s

Final Velocity = v_f = ?

Solution:

$$h = h_1 - h_2 = 5 - 3 = 2\ m$$

According to Third equation of motion

$$2as = v_f^2 - v_i^2$$

$$2(9.8)(2) = v_f^2 - (0)^2$$

$$39.2 = v_f^2$$

Taking square root on both sides

$$\sqrt{39.2} = \sqrt{v_f^2}$$

Or

$$\boxed{v_f = 6.26\ m/s}$$

Result: The velocity of block will be 6.26 m/s.

Q.2(xiii) A balloon tied up with a wooden piece is moving upward with velocity of 15 m/s.At a height of 300 m above the ground, the wooden piece is detached from the balloon. How much time will it take to reach the ground?

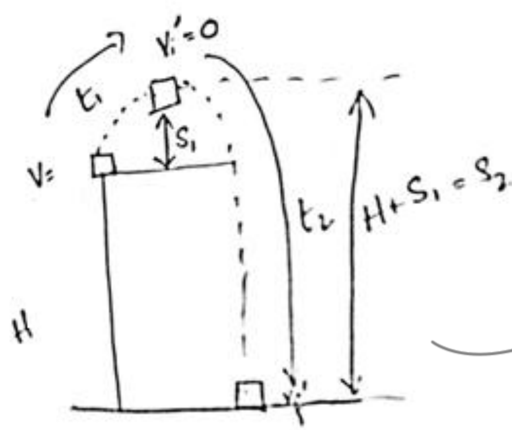
Data:

Initial Velocity = V_i=15 m/s

Final Velocity = V_f = 0 m/s

Height = H = 300 m

Total time = t = ?



Solution:

During upward Motion

For t_1:

$$v_f = v_i + at$$
$$0 = 15 + (-9.8)t_1$$
$$\boxed{t_1 = \frac{15}{9.8} = 1.5\ \text{sec}}$$

For S_1:

$$S = v_i t + \frac{1}{2}at^2$$
$$S_1 = 15 \times 1.5 + \frac{1}{2}(-9.8)(1.5)^2$$
$$S_1 = 22.5 - 11.025$$
$$\boxed{S_1 = 11.475\ \text{m}}$$

During Downward Motion

For t_2:

$$S = v_i t + \frac{1}{2}at^2$$

Or $$S_1 + H = 0 \times t_2 + \frac{1}{2}(9.8){t_2}^2$$
$$11.475 + 300 = (4.9){t_2}^2$$
$$311.475 = 4.9t_2^2$$
$${t_2}^2 = 63.56$$

Taking square root

$$\boxed{t_2 = 7.9\ \text{sec}}$$

Now, Total time is given by

$$\boxed{t = t_1 + t_2 = 1.5 + 7.9 = 9.4\ \text{sec}}$$

Result: The total time taken will be 9.4 sec.

2017

No Numerical

2016

Q.2 (xii) A car starts from rest and moves with a constant acceleration. During the 5th second of its motion, it covers a distance of 36 m. Calculate: (a) Acceleration of the car (b) Distance covered by the car during this time.

Data:

Initial velocity= $v_i = 0$

Acceleration= $a = ?$

Distance during 5th sec = $S_5 - S_4 = 36$ m

Total distance in 5 sec = $S_5 = ?$

Solution:

When t= 4 sec

$$S = v_i t + \frac{1}{2}at^2$$
$$S_4 = 0 + \frac{1}{2}(a)(4)^2$$
$$\boxed{S_4 = 8a} \text{ ---- (i)}$$

When t= 5 sec

$$S = v_i t + \frac{1}{2}at^2$$
$$S_5 = 0 + \frac{1}{2}(a)(5)^2$$
$$\boxed{S_5 = 12.5a} \text{ ---- (ii)}$$
$$\therefore S_5 - S_4 = 12.5a - 8a$$
$$36 = 4.5a$$
$$\boxed{a = 8\ m/s^2}$$

Putting value in eq (ii)

$$\boxed{S_5 = 12.5a = 12.5 \times 8 = 112.5\ m}$$

Result: The acceleration of body is 8 m/s² and Distance travelled in 5 seconds is equal to 100 m.

2015

Q.2 xiii) Textbook Numerical 11

2014

Q.2 (iv) A car starts from rest and moves with a constant acceleration. During the 4th second of its motion, it covers a distance of 24 meters. Calculate the acceleration and the total distance covered by the car during this time.

Data:

Initial velocity= $v_i = 0$

Acceleration= a =?

Distance during 4th sec = $S_4 - S_3$ =24 m

Total distance in 4 sec = S_4 = ?

Solution:

When t= 4 sec

$$S = v_i t + \frac{1}{2}at^2$$

$$S_4 = 0 + \frac{1}{2}(a)(4)^2$$

$$\boxed{S_4 = 8a} \text{ ---- (i)}$$

When t= 3 sec

$$S = v_i t + \frac{1}{2}at^2$$

$$S_3 = 0 + \frac{1}{2}(a)(3)^2$$

$$\boxed{S_3 = 4.5a} \text{ ---- (ii)}$$

$$\therefore S_4 - S_3 = 8a - 4.5a$$

$$24 = 3.5a$$

$$\boxed{a = 6.85\ m/s^2}$$

Putting value in eq (i)

$$\boxed{S_4 = 8a = 8 \times 6.85 = 54.8\ m}$$

Result: The acceleration of body is 6.85 m/s^2 and Distance travelled in 4 seconds is equal to 54.8 m.

2013

Q.2 (vi) A stone if dropped from the peak of a hill. It covers a distance of 30 meters in the last second of its motion. Find the height of the peak.

Data:

Initial velocity= v_i = 0 m/s

Acceleration= g = 9.8 m/s^2

Distance during last sec = $S_t - S_{(t-1)}$ = 30 m

Height of peak = S_t = ?

Solution:

(i) When t= t sec

$$S = v_i t + \frac{1}{2}at^2$$

$$S_t = 0 + \frac{1}{2}(9.8)(t)^2$$

$$\boxed{S_t = 4.9t^2} \text{ ---- (i)}$$

(ii) When t= (t-1) sec

$$S = v_i t + \frac{1}{2}at^2$$

$$S_{t-1} = 0 + \frac{1}{2}(9.8)(t-1)^2$$

$$S_{t-1} = 4.9(t^2 - 2t + 1)$$

$$\boxed{S_{t-1} = 4.9t^2 - 9.8t + 4.9}$$

$$\therefore S_t - S_{t-1} = 4.9t^2 - (4.9t^2 - 9.8t + 4.9)$$

$$30 = 4.9t^2 - 4.9t^2 + 9.8t - 4.9$$

$$30 = 9.8t - 4.9$$

$$t = \frac{34.9}{9.8} = 3.56\ s$$

Putting in eq (i)

$$S_t = 4.9(3.56)^2$$

$$\boxed{S_t = 62.1\ m}$$

Result: The height of the peak is 62.1 m.

Q.2 (ix) A car is waiting at a traffic signal. As it turns green, the car starts ahead with a constant acceleration of 3 m/s^2.At the same time, a bus travelling with a constant speed of 20 m/s overtakes and passes the car.

(a) How far beyond the starting point will the car overtake the bus?

(b) What will be the velocity of the car at that time?

Data:

Initial Velocity of car = v_i = 0

Acceleration of car = a = 3 m/s^2

Speed of Bus = v = 20m/s

Distance from signal = S = ?

Final Velocity of car = v_f = ?

Solution:

For Car:

$$S = v_i t + \frac{1}{2}at^2$$

$S = 0 \times t + \frac{1}{2}(3)t^2$

$S = 1.5t^2$ --- (i)

For Bus:

$S = v \times t$

$S = 20\ t$ ----- (ii)

Comparing eq (i) and eq (ii)

$1.5t^2 = 20\ t$

$\boxed{t = 13.33\ sec}$

Putting in eq (ii)

$\boxed{S = 20 \times 13.33 = 266.6\ m}$

Final Velocity of Car:

$v_f = v_i + at$

$v_f = 0 + 3 \times 13.33$

$\boxed{v_f = 39.99\ m/s}$

Result: The bus overtakes at 266.6 m distance from starting point and car was moving with 39.9 m/s.

2012

No Numerical

2011

Q.2(vii) **Textbook Numerical 12**

2010

Q.2 (ix) **Textbook Numerical 5**

THEORY NOTES

PROJECTILE

DEFINITION:

An object falling freely in a gravitational field, having been projected with a velocity ‘v’ and at an angle of elevation ‘θ’ with the horizontal is called projectile.

PROJECTILE MOTION:

When an object is projected with a velocity ‘v’ it will move in a semi-parabolic path called ‘trajectory’. Three assumptions are made, when considering a projectile motion.

ASSUMPTION:

i) The acceleration due to gravity, ‘g’, is constant over the range of motion and is directed downward.

ii) The effect of air resistance is negligible.

iii) The rotation of earth does not affect the motion.

At any instant project motion can be described in two parts.

a) Horizontal Motion: b) Vertical Motion:

When an object is projected with an initial velocity vo and with on angle ‘θ’ with the horizontal in travels in horizontal directions as well as in vertical direction.

The velocity can be resolved, at any instant, in two components. $V_x = V_{ox} = V_o Cos\theta$

a) Horizontal Velocity:

it remains unchanged throughout the motion because there is no acceleration in the motion.

b) Vertical Velocity:

It continuously changes because of the force of attraction of the earth.

$$V_y = V_{oy} = V_o Sin\theta$$

Thus,

The net velocity $= Vo = \sqrt{V_{ox}^2 + V_{oy}^2}$

FOR HORIZONTAL MOTION:

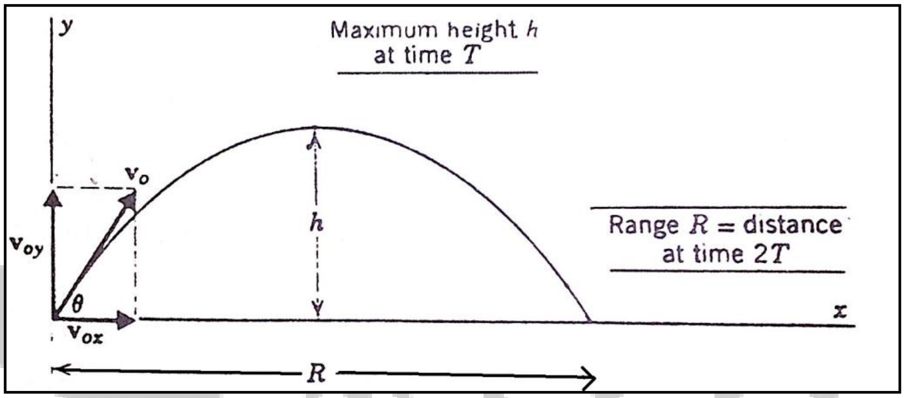
Acceleration $= ax = 0$

Velocity $= V_x = V_{ox}$

Displacement$= X = V_{ox}.x\ t$

FOR VERTICAL MOTION:

Acceleration $= a_y = -g$
Velocity $= V_y = V_{oy} - gt$
Displacement $= y = V_{oy}t - \frac{1}{2}gt^2$



DERIVATIONS:

A) **TIME TO REACH MAXIMUM HEIGHT:**

Initial Velocity $= V_{oy} = V_o\sin\theta$
Final Velocity $= V_y = 0$
Time $= T$
Acceleration $= -g$

We know that

$$V_f = V_i + at$$
$$V_y = V_{oy} + (-g)T$$
$$0 = V_o\sin\theta - gT$$
$$gT = V_o\sin\theta$$

$$T = \frac{V_o\sin\theta}{g}$$

B) **TIME OF FLIGHT:**

Total time of flight $= T`$
$T` = 2T$

$$T` = \frac{2V_o\sin\theta}{g}$$

C) **MAXIMUM HEIGHT:**

Distance = Height $= y = h_{max}$
Initial Velocity $= V_{oy} = V_o\sin\theta$
Acceleration $= a = -g$

$$\text{Time} = T = \frac{V_o \sin\theta}{g}$$

We know that,

$$S = Vit + 1/2\ at^2$$

$$Y = V_{oy}T + 1/2\ (-g)\ T^2$$

$$h_{max} = V_o \sin\theta \times \frac{V_o \sin\theta}{g} - 1/2 g \left(\frac{V_o \sin\theta}{g}\right)^2$$

$$h_{max} = \frac{V_o^2 \sin^2\theta}{g} - \frac{V_o^2 \sin^2\theta}{2\ g}$$

$$h_{max} = \frac{2Vo^2 \sin^2\theta - Vo^2 \sin^2\theta}{2g}$$

$$\boxed{h_{max} = \frac{Vo^2 \sin^2\theta}{2g}}$$

D) RANGE:

The maximum horizontal distance traveled by a projectile is called ' rang ".

Distance = X = R

Velocity = $V_{ox} = V_o \cos\theta$

Time = T'

Since,

$$S = V \times t$$

$$X = V_o \times T$$

$$R = V_o \cos\theta \times \frac{2\ V_o \sin\theta}{g}$$

$$R = \frac{V_o^2\ 2\cos\theta\ \sin\theta}{g}$$

$$\boxed{R = \frac{V_o^2 \sin 2\theta}{g}} \qquad \left(\sin 2\theta = 2\sin\theta \cos\theta \right)$$

E) MAXIMUM RANGE:

Horizontal Range is given as,

$$R = \frac{V_o^2 sin2\theta}{g}$$

Above expression shows that, for constant velocity of projection (V0) and gravitational acceleration (g), horizontal range depends on the factor sin2θ and it will be maximum at the maximum value of sin2θ. The maximum value of sin is 1.

$$\text{Sin } 2\theta = 1$$

or

$$2\theta = \text{Sin}^{-1}(1)$$

or

$$2\theta = 90^0$$

or

$$\theta = 45^0$$

It shows that, "when a projectile is projected with 45^0, its horizontal range will be maximum."

F) <u>PROJECTILE TRAJECTORY:</u>

The path followed by a projectile is called its trajectory.

Knowing the displacement along horizontal and vertical direction, position of projectile can be determined.

A) Horizontal displacement – X

Since,

$$S = vt$$

$$X = V_o x t$$

$$X = V_o \text{Cos}\theta . t$$

$$t = \frac{X}{V_o \text{Cos}\theta} \text{ ---------------(1)}$$

B) Vertical displacement – Y

Since,

$$S = Vit + 1/2\, at^2$$

$$y = V_o y \times t + 1/2\,(-g)t^2$$

$$y = V_o \text{Sin}\theta \frac{x}{V_o \text{Cos}\theta} - 1/2 g . \frac{x^2}{V_o^2 \text{Cos}^2\theta}$$

$$y = x \frac{\text{Sin}\theta}{\text{Cos}\theta} - \frac{gx^2}{2V_o^2 \text{Cos}^2\theta}$$

$$\boxed{y = x \text{ Tan}\theta - \frac{gX^2}{2V_o^2 \text{Cos}^2\theta}}$$

This expression is known as Equation of Trajectory.

ANGULAR MOTION OR CIRCULAR MOTION:

When a body is moving along a circular path, it is called as circular Motion or angular motion, in this type of motion, the change in position, of a moving body is measured by the angle subtended by it at the center of its circular path. The universe is full of a large number of objects such as the planets revolve around the sun and the moon moves around are earth in nearly circular orbits.

ANGULAR DISPLACEMENT:

The angle, through which a body moves while moving along a circular path, is called as angular displacement. It is the angle subtended at the center during angular motion.

Angular displacement is measured in degree and radian.

(I) DEGREE: When a rotating object completes one revolution is subtends and angle of 360 degrees at the center of its circular path and thus its angular displacement is 360^{o}.

II) RADIAN: It is the angle subtended at the center of a circle by an arc equal in length to its radius.

RELATION BETWEEN RADIAN AND DEGREE:

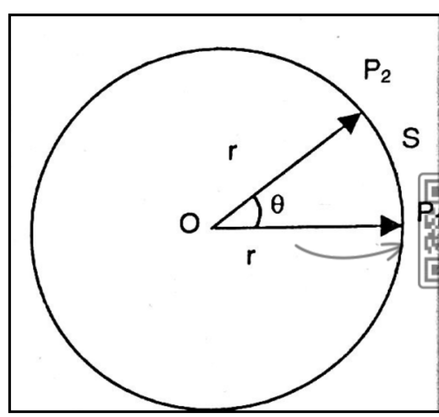
Consider a circle of radius "r" with o as center Arc AB is taken in length equal to r. The circumference of the circle is equal in length to$2\pi r$. As the arc of length r subtends an angle of 1 radian at the center, so the whole circumference will subtend an angle of $2\pi r$ radius at the center. But the whole circumference subtends an angle of 360^{o} degree at the center, therefore,

2π radian = 360 degrees

π radian = 180 degrees

1 radian = $\frac{180}{\pi}$

1 radian = 57.3^{o}



RELATION BETWEEN LINEAR AND ANGULAR DISPLACEMENT:

It is clear from the figure that the arc length is directly proportional to the angle subtended at the center.
Mathematically we can write as,

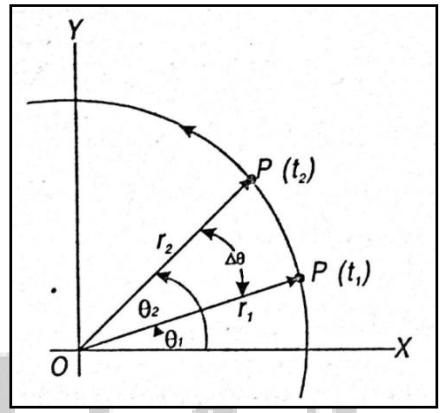
$$S \propto \theta$$

or

$$\boxed{S = r\theta}$$

Where, r is the radius of circle and angle is measured in Radian.

ANGULAR VELOCITY:



In circular motion of a particle P the rate of change of angular displacement is called angular velocity and it is denoted by “(i)”. If ‘AB’ is the change in angular displacement during time interval Δt due to the motion of particle P along a circular path then average angular velocity will be

$$\omega_{av} = \frac{\Delta\theta}{\Delta t}$$

$$\omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$

If the interval Δt is very small i.e., Δt → 0 then angular velocity during such a short time interval will be called instantaneous angular velocity. i.e.

$$\omega_{ins} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

UNIT OF ANGULAR VELOCITY:

The angular velocity may be measured in deg / sec, rad /sec or rev / sec. But radian does not appear in the final answer so the unit of angular velocity is sec^{-1}, which is also the unit of frequency. So we may call ‘ω’, as angular frequency.

The angular velocity is also measured in R.P.M.

i.e 1 revolution = 2π rad

1 rpm = 2π 60 rad/sec

1 rpm = 0.105 rad/sec

DIRECTION OF ANGULAR VELOCITY:

The angular velocity is always directed along axis of rotation of the circle. If curl of fingers of right hand points the direction of rotation then thumb will point out the direction of omega “ω”.

In the case of counter clock wise rotation the ω will be directed out of the page. In case of Clock wise rotation ω will be directed into the page.

RELATION BETWEEN LINEAR AND ANGULAR VELOCITIES:

We know that

$$S = r\theta$$

But in case of small angular displacement due to circular motion of particle P along circle of radius r, we may write this equation

$$\Delta S = r\Delta\theta$$

Divide both sides by Δt

$$\frac{\Delta S}{\Delta t} = r\frac{\Delta\theta}{\Delta t}$$

Taking limits on both sides

$$\lim_{\Delta t\to 0}\frac{\Delta S}{\Delta t} = r\lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t} \text{ ------(i)}$$

As we know that

$$v = \lim_{\Delta t\to 0}\frac{\Delta S}{\Delta t}$$

And

$$\omega = \lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t}$$

Therefore eq(i) , becomes

$$\boxed{V = r\omega}$$

or

$$\boxed{\vec{V} = \vec{r}\times\vec{\omega}}$$

ANGULAR ACCELARATION:

The rate of change of angular velocity is called angular acceleration and it is denoted by '$\propto$'. If $\Delta\omega$ is the change in angular velocity during time interval Δt due to the motion of a point along circular path then average angular acceleration will be,

$$\propto_{av} = \frac{\Delta\omega}{\Delta t}$$

$$\propto_{av} = \frac{\omega_2-\omega_1}{t_2-t_1}$$

Now if the time interval for this change is very small then angular acceleration on during such a short time interval is called instantaneous angular acceleration.i.e.

$$\propto_{ins} = \lim_{\Delta t\to 0}\frac{\Delta\omega}{\Delta t}$$

UNIT OF ANGULAR ACCELARATION:

The angular acceleration may be measured in rad/sec^2, deg / sec^2 or rev / sec^2. But S. I Unit is rad/sec^2

DIRECTION OF ANGULAR ACCELERATION:

The direction of angular acceleration may either be parallel or anti parallel to the direction of $\vec{\omega}$ which is along the axis of rotation. If $\vec{\omega}$ increases then it will be parallel to the direction of $\vec{\omega}$. Similarly if it decreases then it will be in opposite direction to that of $\vec{\omega}$ along the axis of rotation.

RELATION BETWEEN LINEAR AND ANGULAR ACCELERATION:

We have the relation between linear and angular velocities as $V = r\omega$. Suppose an object rotating about a fixed axis, changes it angular velocity by $\Delta\omega$ in a time change Δt then the change in tangential velocity.

$$V_t = r\omega$$

Divide both sides by Δt

$$\frac{\Delta V_t}{\Delta t} = r\frac{\Delta\omega}{\Delta t}$$

Taking limits on both sides

$$\lim_{\Delta t\to 0}\frac{\Delta V_t}{\Delta t} = r\lim_{\Delta t\to 0}\frac{\Delta\omega}{\Delta t} \text{ ------(i)}$$

As we know that

$$a_t = \lim_{\Delta t\to 0}\frac{\Delta V_t}{\Delta t}$$

And

$$\propto = \lim_{\Delta t\to 0}\frac{\Delta\omega}{\Delta t}$$

Therefore eq(i) , becomes

$$\boxed{a_t = r\propto}$$

or

$$\boxed{\vec{a_t} = \vec{r}\times\vec{\propto}}$$

CENTRIPETAL ACCELARATION

DEFINITION:

The acceleration in the motion of a body only due to the rate of change in direction of velocity is called centripetal acceleration as it is always directed towards centre of the circle or centre of curvature of the track.

FORMULA:

$$a_c = \frac{v^2}{r}$$

DERIVATION:

Let us consider a particle of mass m moving with uniform speed 'v' along a circular path of radius 'r'. Suppose its linear velocity vector at P_1 is $\vec{v_1}$ at time t_1 whereas velocity vector at P_2 is $\vec{v_2}$ at time t_2 , as shown in the figure.

In case of uniform motion

$$|v_1| = |v_2|$$

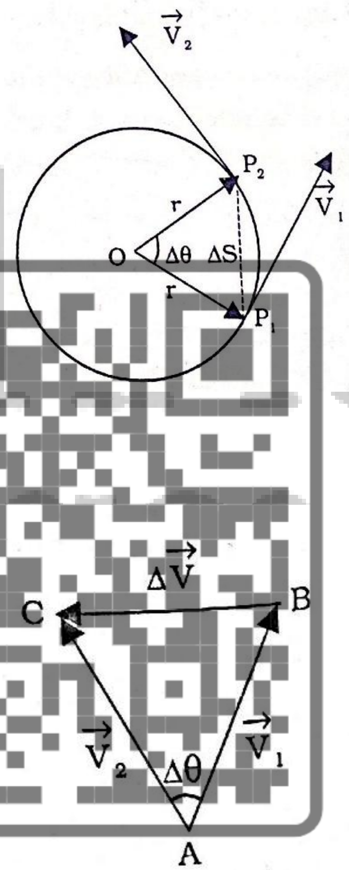
but from the figure

$$\vec{V_1} + \Delta\vec{V} = \vec{V_2}$$

$\Delta\vec{V} = \vec{V_2} - \vec{V_1}$

This change in velocity is only because of the change in its direction. So its rate of change will be called centripetal acceleration. As we know that the angle between perpendiculars to the two lines is same as the angle between these two lines, therefore, the angle between V_1 and V_2 is $\Delta\theta$ as the angle between radial lines is $\Delta\theta$.

It is clear from fig (i) and (ii) that the isosceles triangle ΔP_1OP_2 and ΔBACare congruent. Then according to geometry,



$$\frac{\Delta S}{r} = \frac{\Delta V}{V}$$

$$\Delta V = \frac{V\Delta S}{r}$$

Divide both sides by Δt

$$\frac{\Delta V}{\Delta t} = \frac{V\Delta S}{r\Delta t}$$

Taking limit both sides as $\Delta t \to 0$

$$\lim_{\Delta t\to 0}\frac{\Delta V}{\Delta t} = \frac{V}{r}\lim_{\Delta t\to 0}\frac{\Delta S}{\Delta t} \text{ ------------(i)}$$

As we know that

$$V = \lim_{\Delta t\to 0}\frac{\Delta S}{\Delta t}$$

and

$$a_c = \lim_{\Delta t\to 0}\frac{\Delta V}{\Delta t}$$

Then eq (i) becomes

$$\boxed{a_c = \frac{v^2}{r} \text{------(ii)}}$$

As we know that

$$V = r\omega$$

putting in eq(ii)

$$a_c = \frac{(r\omega)^2}{r}$$

$$\boxed{a_c = r\omega^2}$$

CENTRIPETAL FORCE:

DEFINITION:

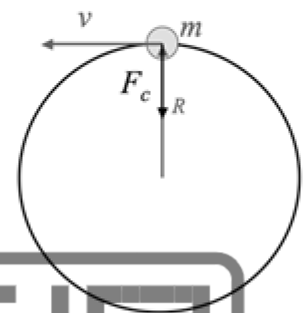
"The force that causes an object to move along a curve (or a curved path) is called centripetal force."

MATHEMATICAL EXPRESSION:

We know that the magnitude of centripetal acceleration of a body in a uniform circular motions is directly proportional to the square of velocity and inversely proportional to the radius of the path. Newton's Second Law of Motion:

$$F = ma$$

$$\Rightarrow F_c = mv^2/r$$



Where,
F_c = Centripetal Force
m = Mass of object
v = Velocity of object
r = Radius of the curved path

1. In the absence of air friction projectile has maximum range when fired at an angle.
(a) 30° with the horizontal
(b) 45° with the vertical
(c) 30° with the vertical
(d) 60° with the horizontal

2. A horizontal range attained by a projectile can be found by the formula:
(a) $\frac{v_o sin\theta}{g}$ (b) $\frac{2v_o sin\theta}{g}$
(c) $\frac{{v_o}^2 sin\theta}{g}$ (d) $\frac{{v_o}^2 sin\, 2\theta}{g}$

3. During the projectile motion, the horizontal component of velocity:
(a) Change with time

(b) Becomes zero
(c) Does not change but remains constant.
(d) Increases with time

4. The maximum height of a projectile is directly proportional to.
(a) The initial velocity (b) Launch angle
(c) Square of the initial velocity (d) None of these

5. A body is moving in a circle at a constant speed which of the following statements about the body is true?
(a) There is no acceleration.

(b) There is no force acting on it
(c) There is force acting at a tangent to the circle
(d) There is force acting towards the centre of

the circle

6. The acceleration in uniform circular motion.

(a) Varies inversely with the velocity of the particle. (b) Varies inversely with the radius of the orbit.

(c) Varies directly with the square of the velocity.

(d) both (b) and (c)

7. If a body is rotating in a circle with variable linear speed, it must have:

(a) Only centripetal acceleration.

(b) Only tangential acceleration

(c) Both centripetal and tangent acceleration

(d) None of these

8. The direction of angular velocity can be find out by

(a) Left hand rule (b) Angular displacement

(c) Direction of movement (d) Right hand rule

9.If a particle moves in a circle describing equal angles in equal intervals, then

(a) Angular velocity change and linear velocity constant.

(b) Angular velocity constant and linear velocity constant

(c) Angular velocity constant and linear velocity changes.

(d) None of these

10. The rate of change of angular displacement with time is called:

(a) Angular acceleration. (b) Linear velocity

(c) Angular velocity (d) None of these

11. The centripetal acceleration produced in a rotating body is commonly due to the change in __________ of the velocity:

(a) Magnitude (b) Direction

(c) Value (d) None of these

12. An object is launched in an arbitrary direction in space with a certain initial velocity and of moves freely under gravity. Its path will be a.

(a) Straight line (b) circle

(c) parabola (d) hyperbola

13. The velocity component with which a projectile covers certain vertical distance is minimum at the moment of:

(a) Projection (b) Hitting the ground

(c) Highest point (d) None of these

14. A body, moving along the circumference of a circle, completes two revolutions. If the radius of circle is R, the ratio of displacement to the covered path will be :

(a) zero (b) πR

(c) $2\pi R$ (d) $4\pi R$

15. The angle between centripetal acceleration and tangential acceleration in circular motion is:

(a) 180^o (b) 0^o

(c) 90^o (d) 45^o

16. A projectile has its speed maximum at the moment of:

(a) Projection (b) Hitting the ground

(c) Both of these (d) None of these

17. The horizontal range of a projectile depend upon:

(a) The angle of projection

(b) The velocity of projection

(c) Both of these

(d) None of these

18. If a projectile is projected at an angle of 30^o , it hits certain target. It will have the same range if it is projected at an angle of :

(a) 45^o (b) 55^o (c) 90^o (d) 60

o

19. The linear and angular velocity of a particle, moving about the centre of a circle of radius r, are related by :

(a) v = ω / r (b) v = r x ω
(c) ω = v x r (d) ω = r x v

20. A ball is thrown at 40 m/s with the angle of projection of 30^0 with the horizontal, the vertical velocity, of the projectile after 1 sec:

(a) 20 m/s (b) 15 m/s
(c) 10 m/s (d) Zero

21. A car moving at a constant speed of 20 ms-1 on a circular path of radius 100m what is the acceleration?

(a) 0.4 m/s^2 (b) 6 m/s^2
(c) 4.0 m/s^2 (d) 33 m/s^2

22. The missile is fired at 20 m/s at 60^o with respect to the horizontal, the horizontal and vertical component of the velocity at the maximum height is respectively:

(a) 10 m/s, 10 m/s (b) 10 m/s, 5 m/s
(c) 10 m/s, 0 (d) 0, 10 m/s

23. The cyclist cycling around a circular racing track skids because:

(a) the centripetal force upon him is less than the limiting friction
(b) the centripetal force upon him is greater than the limiting friction
(c) the centripetal force upon him is equal to the limiting friction
(d) none of them

24. When the angular velocity of a disk increases, angular acceleration and angular velocity are:

(a) parallel (b) non parallel
(c) perpendicular (d) none of these

25. A 100 kg body is rotating in circular path of radius 200m, at 50 m/sec. find the centripetal force acting on the body:

(a) 225 N (b) 1250 N
(c) 525 N (d) 500 N

26. If a body covers 5 rotations in 2 seconds, around a path of radius 2m the linear velocity of body is:

(a) π m/s (b) 10 π m/s
(c) 5 π m/s (d) 20 π m/s

27. The angular speed of an hour's hand of a watch in radian / minute is:

(a) π/6 (b) π/30
(c) π/180 (d) π/360

28. An angle subtended at its centre by an arc whose length is double to that of its radius is:

(a) 2° (b) 57.3°
(c) 80° (d) 114.6°

29. The unit of angular velocity is:

(a) radian/cm (b) metre/sec
(c) radian/sec (d) radian/sect

PAST PAPER M.C.Qs.

2022

7. The angle between centripetal and tangential acceleration in circular motion is:

*180°　　　* zero　　　*** 90°**　　　*45°

23.The acceleration of projectile at the top of trajectory is:

*** g**　　　*1/2 g　　　* zero　　　* 2g

2021

(vi) A car is travelling at a constant speed of 20 m/s round a curve of radius 100m. Its acceleration is:

*2m/s^2　　　* 3m/s^2　　　*** 4m/s^2**　　　*Zero m/s^2

(xviii) During the projectile motion the acceleration of the projectile along the horizontal direction will:

*Decrease　　　* Increase　　　***be zero**　　　* remain constant

(xx) One radian is equal to:

*0.017^0　　　*** 57.3^0**　　　* 35.7^0　　　*0.117^0

2019

7.An angle subtended at its centre by an arc whose length is double to that of its radius is:

*2°　　　*57.3°　　　*80°　　　***114.6°**

13. A projectile is thrown upward with a certain velocity. Its time of flight will be minimum,if it is launched at an angle of:

***30°**　　　*45°　　　*60°　　　*75°

2018

7. A body, moving along the circumference of a circle, completes two revolutions. If the radius of circle is R, the ratio of displacement to the covered path will be :

*** zero**　　　* πR　　　* 2 πR　　　* 4πR

2017

2. The angular speed of the minute hand of a clock is:

*$\frac{\pi}{30}$　　　*$\frac{\pi}{60}$　　　*$\frac{\pi}{1800}$　　　*$\frac{\pi}{3600}$

3. A projectile is fired at an angle θ with the horizontal will be minimum at:

***the highest point**　　　*the point of projection

*all points of its path　　　*the point of landing on ground

2016

11.The angle between centripetal acceleration and tangential acceleration in circular motion is:

*180^0　　　*0^0　　　***90^0**　　　*45^0

13. One radian is equal to:

*1^0　　　*75.3^0　　　***57.3^0**　　　*0.017^0

2015

7. The unit of angular velocity is:

*radian/cm　　　* metre/sec　　　*** radian/sec**　　　* radian/sect

13. The angle between centripetal and tangential acceleration in circular motion is:

*180°   * zero   **<u>* 90°</u>**   *45°

<u>2014</u>

9. An angle subtended at its centre by an arc whose length is double to that of its radius is:

*84.3°   *57.3°   **<u>*114.6°</u>**   *168.6°

<u>2012</u>

3. If the axis of rotation of a rotating body passes through the body itself, then its motion is called:

*linear motion   * orbital motion
<u>* spin motion</u>   * S.H.M

<u>2011</u>

6. When the angular velocity of a disk increases, angular acceleration and angular velocity are:

<u>*parallel</u>   *non parallel   *perpendicular   *none of these

<u>2010</u>

7. The cyclist cycling around a circular racing track skids because:

*the centripetal force upon him is less than the limiting friction
<u>* the centripetal force upon him is greater than the limiting friction</u>
* the centripetal force upon him is equal to the limiting friction
*none of them

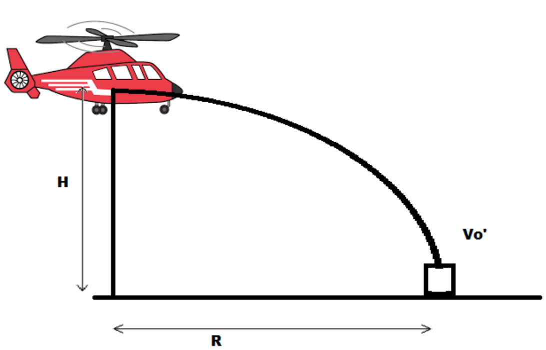
17. The horizontal range of a projectile depends upon:

* the angle of projection   * 'g' at the place
* the velocity of projection   **<u>* all of them</u>**

TEXTBOOK NUMERICALS

Q.1: A rescue helicopter drops a package of emergency ration to a stranded party on the ground. If the helicopter is traveling horizontally at 40 m/s at a height of 100 m above the ground, (a) where does the package strike the ground relative to the point at which it was released? (b) What are the horizontal and vertical component of the velocity of the package just before it hits the ground?

<u>Data:</u>



Horizontal velocity component $= v_{0x} = 40 m/s$

Height of helicopter $= H = 100\ m$

(a) Range of projectile $= R = ?$

(b) X-component of Final Velocity $= v_{ox}{}' = ?$

Y-component of Final Velocity $= v_{oy}{}' = ?$

<u>Solution:</u>

(a) As we know that

$$s = v \times t$$

so,

$$R = v_{ox} \times T \text{ ---(i)}$$

For time (T) :

Using Second equation of motion

$$s = v_i t + \frac{1}{2}at^2$$

$$H = v_{oy} \times T + \frac{1}{2}(9.8) \times T^2$$

$$-100 = 0 \times T + \frac{1}{2}(-9.8) \times T^2$$

$$100 = 4.9 \times T^2$$

$$T^2 = 20.4$$

Or $\boxed{T = 4.5\ sec}$

Putting value in eq(i)

$$R = 40 \times 4.5$$

$$\boxed{R = 180\ m}$$

Now,

(b) Final Velocities:

As we know that

$$\boxed{v'_{ox} = v_{ox} = 40\ m/s}$$

and

$$v_f = vi + at$$

$$v_{oy}{}' = v_{oy} + gt$$

$$v'_{oy} = 0 - 9.8 \times 4.5$$

$$\boxed{v'_{oy} = -44.1\ m/s}$$

Result: The package strike the ground at 180 m relative to the point at which it was released and x component of final velocity is 40 m/s and y component is 44.1 m/s (downwards).

Q.2: A long-jumper leaves the ground at an angle of 20° to the horizontal and at a speed of 11m/s (a) How far does he jump? What is the maximum height reached? Assume the motion of the long jumper is that of projectile.

Data:

Launch angle $= \theta = 20°$

Initial Speed $= v_o = 11\ m/s$

(a) Range of long jumper $= R = ?$

(b) Maximum Height attained $= H_{max} = ?$

Solution:

(a) Range of projectile is given by

$$R = \frac{v_o^2 Sin\,2\theta}{g}$$

$$R = \frac{(11)^2 \times \sin\,2(20)}{9.8}$$

$$R = \frac{121\ \times \sin\,(40)}{9.8}$$

$$\boxed{R = 7.93\ m}$$

(b) The Maximum height attained by projectile is given by

$$H_{max} = \frac{v_o^2\, Sin^2\theta}{2g}$$

$$H_{max} = \frac{(11)^2 \times (Sin\ 20)^2}{2 \times 9.8}$$

$$H_{max} = \frac{121 \times 0.116}{19.6}$$

$$\boxed{H_{max} = 0.72\ m}$$

Result: He jumped 7.93 m far and maximum height reached is 0.72 m.

Q.3: A stone is thrown upward from the top of a building at an angle of 30 ° to the horizontal and with a initial speed of 20 m/s. If the height of building is 45 m. (a) Calculate the total time the stone in flight (b) What is the speed of stone just before it strikes the ground? (c) Where does the stone strike the ground?

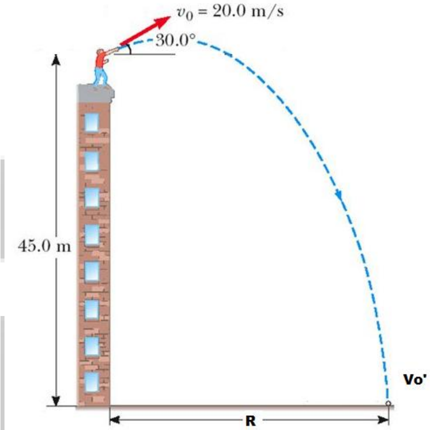
Data:

Launch angle $= \theta = 30°$

Initial Speed $= v_o = 20\ m/s$

Height of Building $= H_1 = 45\ m$

(a) Time of flight $= T' = ?$
(b) Final speed $= v_o' = ?$
(c) Range of stone $= R = ?$



Solution:

(a) First we calculate time to reach maximum height

$$T_1 = \frac{v_o \sin\theta}{g}$$

$$T_1 = \frac{20 \times \sin 30}{9.8}$$

$$\boxed{T_1 = 1.02\ s}$$

Now, Height reached during this time

$$H_2 = \frac{v_o^2 \sin^2\theta}{2g}$$

$$H_2 = \frac{(20)^2 \times (\sin 30)^2}{2 \times 9.8}$$

$$H_2 = \frac{400 \times 0.25}{19.6}$$

$$\boxed{H_2 = 5.1\ m}$$

Now, total height is given by

$$\boxed{H = H_1 + H_2 = 45 + 5.1 = 50.1\ m}$$

Now, using this height to find time for downward flight

$$s = v_i t + \frac{1}{2}at^2$$

$$H = v_i T_2 + \frac{1}{2}a{T_2}^2$$

$$50.1 = 0 \times T_2 + \frac{1}{2}(9.8)(T_2^2)$$

$$50.1 = 4.9(T_2^2)$$

$$T_2^2 = 10.22$$

Taking Square root O.B.S

$$\boxed{T_2 = 3.19\ s}$$

Now, Total Time is given by

$$\boxed{T = T_1 + T_2 = 1.02 + 3.19 = 4.21\ s}$$

(b) **Final Velocities:**

As we know that

$$v_{ox}' = v_{ox} = v_o \cos\theta$$

$$v_{ox}' = 20 \times \cos 30$$

$$\boxed{v_{ox}' = 17.3\ m/s}$$

and

$$v_f = vi + at$$

$$v_{oy}' = v_{oy} + gT_2$$

$$v_{oy}' = 0 + 9.8 \times 3.19$$

$$\boxed{v_{oy}' = 31.26\ m/s}$$

Net Final velocity:

$$v_o' = \sqrt{(v_{ox}')^2 + (v_{0y}')^2}$$

$$v_o' = \sqrt{(17.3)^2 + (31.26)^2}$$

$$\boxed{v_o' = 35.7\ m/s}$$

(c) **Range of stone:**

$$R = v_{ox} \times T$$

$$R = 17.3 \times 4.21$$

$$\boxed{R = 73\ m}$$

Result: The total time the stone in flight is 4.21 s
(b) The speed of stone just before it strikes the ground is 35.7 m/s and the stone strikes the ground 73 m away.

Q.4: A ball is thrown in horizontal direction from a height of 10 m with a velocity of 21 m/s (a) How far will it hit the ground from its initial position on the ground? and with what velocity?

Data:

Initial Speed $= v_{ox} = 21\ m/s$
Height $= H = 10\ m$
(a) Range of stone $= R = ?$
(b) Final speed $= v_o' = ?$

Solution:

(a) The Range of ball is given by

$$R = v_{ox} \times T \text{ ---- (i)}$$

For time (T) :

Using Second equation of motion

$$s = v_i t + \frac{1}{2}at^2$$

$$H = v_{oy} \times T + \frac{1}{2}(9.8) \times T^2$$

$$-10 = 0 \times T + \frac{1}{2}(-9.8) \times T^2$$

$$10 = 4.9 \times T^2$$

$$T^2 = 2.04$$

Or $\boxed{T = 1.42\ sec}$

Putting value in eq(i)

$$R = 21 \times 1.42$$

$$\boxed{R = 30\ m}$$

(b) **(b) Final Velocities:**

As we know that

$$\boxed{v_{ox}' = v_{ox} = 21\ m/s}$$

and

$$v_f = vi + at$$

$$v_{oy}' = v_{oy} + gt$$

$$v_{oy}' = 0 - 9.8 \times 1.42$$

$$\boxed{v_{oy}' = -13.9\ m/s}$$

Net Final velocity:

$$v_o' = \sqrt{(v_{ox}')^2 + \left(v_{0y}'\right)^2}$$

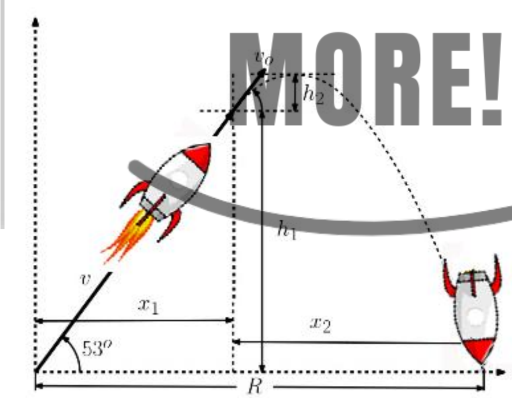
$$v_o' = \sqrt{(21)^2 + (13.9)^2}$$

$$\boxed{v_o' = 25.2\ m/s}$$

Result: It will hit the ground 30m from its initial position on the ground and with velocity 25.2 m/s.

Q.5: A rocket is launched at an angle of 53° to the horizontal with an initial speed of 100 m/s. It moves along its initial line of motion with an acceleration of 30 m/s² for 3s. At this time the engine falls and the rocket proceeds to move as a free body. Find (a) the maximum altitude reached by the rocket (b) its total time of flight, and (c) its horizontal range.

Data:



Launch angle $= \theta = 53°$

Initial Speed $= v_i = 100\ m$

Acceleration $= a = 30\ m/s^2$

Time $= t = 3\ s$

(a) Maximum altitude reached by the rocket $= H = ?$

(b) Time of flight $= T' = ?$

(c) Horizontal range $= R = ?$

Solution:

During Constant acceleration phase:

$$S = v_i t + \frac{1}{2} a t^2$$

$$S = 100 \times 3 + \frac{1}{2}(30) \times (3)^2$$

$$S = 300 + 135$$

$$\boxed{S = 435\ m}$$

i) Horizontal Distance(x_1):

$$x_1 = Scos\theta$$

$$x_1 = 435 \times cos53°$$

$$\boxed{x_1 = 261.7\ m}$$

ii) Vertical Distance(h_1):

$$h_1 = S \times Sin\theta$$

$$h_1 = 435 \times Sin\ 53°$$

$$\boxed{h_1 = 347.4\ m}$$

Now at the time of failure of engine

$$v_f = v_i + at$$

$$v_f = 100 + 30 \times 3$$

$$\boxed{v_f = 190\ m/s}$$

Now, the rocket becomes a projectile and its initial velocity is 190 m/s

$$v_0 = v_f = 190 \frac{m}{s}$$

During Projectile Motion phase:

Maximum height reached is given by

$$h_2 = \frac{v_o^2 \sin^2\theta}{2g}$$

$$h_2 = \frac{(190)^2 \times (\sin 53)^2}{2 \times 9.8}$$

$$h_2 = \frac{36100 \times 0.63}{19.6}$$

$$\boxed{h_2 = 1178.77\ m}$$

Time to reach maximum height

$$t_1 = \frac{v_o \sin\theta}{g}$$

$$t_1 = \frac{190 \times \sin 53}{9.8}$$

$$\boxed{t_1 = 15.4\ s}$$

Now, Total height is given by

$$\boxed{H = h_1 + h_2 = 347.4 + 1178.77 = 1526.17\ m}$$

Now, Time for Downward Motion:

$$S = v_i t + \frac{1}{2}at^2$$

$$H = 0 \times t_2 + \frac{1}{2}(9.8) \times (t_2)^2$$

$$1526.17 = 4.9\ t_2^2$$

$$t_2^2 = 311.46$$

Taking Square root O.B.S

$$\boxed{t_2 = 17.6\ s}$$

$$Total\ Time\ of\ Flight\ is$$

$$T' = t + t_1 + t_2 = 3 + 15.4 + 17.6$$

$$\boxed{T' = 36\ s}$$

To calculate x_2 we have to find total time of projectile phase.

$$T = t_1 + t_2 = 15.4 + 17.6 = 33\ s$$

Now, range is given by

$$R = v_{ox} \times T$$

or

$$x_2 = v_o cos\theta \times T$$

$$\boxed{x_2 = 190 \times cos53 \times 33 = 3773.3\ m}$$

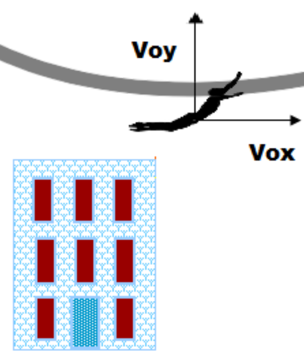
Now, Total Range is

$$\boxed{R = x_1 + x_2 = 261.7 + 3773.3 = 4035\ m}$$

Result: The maximum altitude reached by the rocket is 1526.17 m (b) its total time of flight is 36 s and (c) its horizontal range is 4035 m.

Q.6: A diver leaps from a tower with an initial horizontal velocity component of 7 m/s and upward velocity component of 3 m/s. find the component of her position and velocity after 1 second.

Data:



Initial Horizontal velocity $= v_{ox} = 7\ m/s$

Initial Vertical velocity $= v_{oy} = 3\ m/s$

Time $= t = 1\ s$

Final Horizontal velocity $= v'_{ox} = ?$

x component of position $= x = ?$

Final Vertical velocity $= v'_{oy} = ?$

y component of position $= y = ?$

Solution:

Since the x component of velocity remains constant

$$\boxed{v'_{ox} = v_{ox} = 7\ m/s}$$

Now her position

$$x = v_{ox} \times t$$

$$x = 7 \times 1$$

$$\boxed{x = 7\ m}$$

The y component of velocity is given by

$$v_f = v_i + at$$

or $$v'_{oy} = v_{oy} + (-g)t$$

$$v'_{oy} = 3 - (9.8 \times 1)$$

$$v'_{oy} = 3 - (9.8)$$

$$\boxed{v'_{oy} = -6.8\ m/s}$$

Now her position, using second equation of motion

$$s = v_i t + \frac{1}{2}at^2$$

$$y = 3 \times 1 + \frac{1}{2}(-9.8)(1)^2$$

$$\boxed{y = -1.9\ m}$$

Result:

(i) The final x component of velocity will be 7 m/s and y component will be 6.8 m/s(downward)

(ii) Her displacement in x axis is 7 mand in y axis is 1.9 m(downward)

Q.7: A boy standing 10m from a building can just barely reach the roof 12m above him when he throws a ball at the optimum angle with respect to the ground. Find the initial velocity component of the ball.

Data:

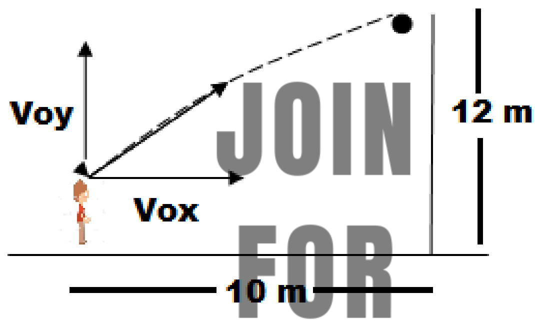
$H_{max} = 12\ m$

$R = 10\ m$

$V_{0x} = ?$

$V_{0y} = ?$

Solution:



As we know that

$$H_{max} = \frac{v_{0y}^2}{2g}$$

$$v_{0y}^2 = H_{max} \times 2g$$

$$v_{0y}^2 = 10 \times 2 \times 9.8$$

$$v_{0y} = \sqrt{196}$$

$$\boxed{v_{0y} = 14\ m/s}$$

Now,

$$s = v \times t$$

Or $R = v_{ox} \times T$ ------ (i)

The time to reach maximum height is given by

$$T = \frac{v_{oy}}{g} = \frac{14}{9.8} = 1.42\ sec$$

Putting values in eq (i)

$$12 = v_{ox} \times 1.42$$

Or $\boxed{v_{ox} = 8.4\ m/s}$

Result: The horizontal component of velocity is $8.4\ m/s$ and the vertical component of velocity is $14\ m/s$

Q.8: A marter shell is fired at a ground level target 500m distance with an initial velocity of 90 m/s. What is its launch angle?

Data:

Range = R = 500 m

Initial Velocity = V_0 = 90 m/s

Launch Angle = θ = ?

Solution:

$$R = \frac{v_0^2\ Sin\ 2\theta}{g}$$

$$500 = \frac{(90)^2\ Sin\ 2\theta}{9.8}$$

$$Sin\ 2\theta = \frac{500 \times 9.8}{8100}$$

$$Sin\ 2\theta = 0.604$$

$$2\theta = Sin^{-1}(0.604)$$

$$2\theta = 37.1$$

Or $\boxed{\theta = 18.5^0}$

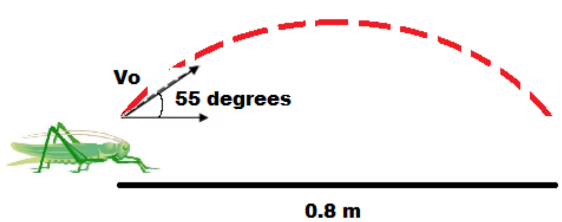
The larger angle is given by

$$\boxed{\theta' = 90 - \theta = 90 - 18.5 = 71.4^0}$$

Result: The two possible values of launch angles are 18.5^0 and 71.4^0.

Q.9: What is the take off speed of a locust if its launch angle is 55° and its range is 0.8m?

Data:



Range = R = 0.8 m

Initial Velocity = V_0 = ?

Launch Angle = $\theta = 55^0$

Solution:

The Range of Projectile is given by

$$R = \frac{v_0^2\ Sin\ 2\theta}{g}$$

$$0.8 = \frac{v_0^2 \; Sin\; 2(55)}{9.8}$$
$$7.84 = v_0^2 \times \sin 110°$$
$$7.84 = v_0^2 \times 0.9396$$
$$v_0^2 = 8.34$$
Taking Square root on both sides

$$\boxed{v_o = 2.88\; m/s}$$

Result: The take off speed of the locust is 2.88 m/s.

Q.10: A car is traveling on a flat circular track of radius 200m at 20 m/s and has a centripetal acceleration a_c = 4.5 m/s² (a) If the mass of the car is 1000 kg, what frictional force is required to provide the acceleration? (b) if the coefficient of static frictions μ_s is 0.8, what is the maximum speed at which the car can circle the track?

Data:

Radius of Track = R = 200 m

Centripetal Acceleration = $a_c = 4.5\; m/s^2$

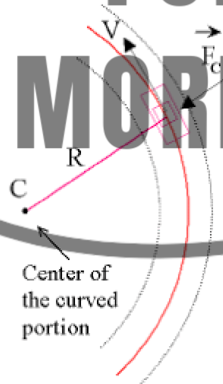
mass of Car = m = 1000 kg

(a) Frictional Force = f = ?

(b) Co-efficient of friction = $\mu_2 = 0.8$

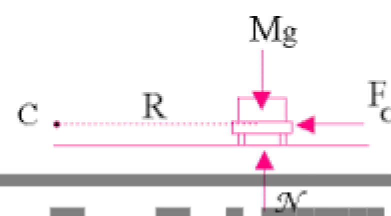
Maximum speed of car = v_{max} =?

Solution:



(a) As we know that Frictional force is equal to the centripetal force

$$f = F_c = ma_c$$
$$f = 1000 \times 4.5$$
$$\boxed{f = 4500\; N}$$



(b) According to the definition of co efficient of friction

$$\mu_s = \frac{F}{R} \text{ ---(i)}$$

In this Case

$$F = F_c = \frac{m v_{max}^2}{r}$$

and

$$R = W = mg$$

Putting values in eq(i)

$$\mu_s = \frac{\frac{m v_{max}^2}{r}}{mg}$$
$$\mu_s = \frac{v_{max}^2}{rg}$$
$$v_{max}^2 = \mu_s r g$$
$$v_{max}^2 = 0.8 \times 200 \times 9.8$$
$$v_{max}^2 = 1568$$

Taking Square root O.B.S.

$$\boxed{v_{max} = 39.5\; m/s}$$

Result: The frictional force required to provide the acceleration is 4500 N and the maximum speed at which the car can circle the track is 39.5 m/s.

Q.11: The turntable of a record player rotates initially at a rate of 33 rev/min and takes 20s to come to rest (a) What is the angular acceleration of the turntable, assuming the acceleration is constant? (b) How many rotation does the turntable make before coming to rest? (c) If the radius of the turntable is 0.14m, what is the initial linear speed of a bug riding on the rim?(d) What is the magnitude of the tangential acceleration

of the bug at time t = 0?

Data:

Initial angluar velocity $= \omega_i = 33\frac{rev}{min} = \frac{33\times 2\pi}{60} = 3.45\ rad/s$

Time $= t = 20\ s$

Final angluar velocity $= \omega_f = 0$

(a) Angular Acceleration $= \alpha = ?$

(b) No. of rotations= $\theta = ?$

(c) Radius of turtable $= r = 0.14\ m$

Initial Tangential velocity $= v_i = ?$

(d) Tangential Acceleration $= a = ?$

Solution:

(a) Using First Equation of Motion

$$\omega_f = \omega_i + \alpha t$$
$$0 = 3.45 + \alpha(20)$$
$$-3.45 = \alpha(20)$$
$$\boxed{\alpha = -0.172\ rad/s^2}$$

(b) Using Second Equation of Motion

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$
$$\theta = 3.45\times 20 + \frac{1}{2}(-0.172)\times(20)^2$$
$$\theta = 69 - 34.4$$
$$\theta = 34.6\ radians$$

In Rotations:

$$\theta = \frac{34.6}{2\pi} = \frac{34.6}{6.28}$$
$$\boxed{\theta = 5.5\ rotations}$$

(c) As we know that

$$v_i = r\omega_i$$
$$v_i = 0.14\times 3.45$$
$$\boxed{v_i = 0.483\ m/s}$$

(d) As we know that

$$a = r\alpha$$
$$a = 0.14\times(-0.172)$$
$$\boxed{a = -0.024\ m/s^2}$$

Result: (a) The angular acceleration of the turntable is $-0.172\ rad/s^2$ (b) 5.5 rotations the turntable makes before coming to rest (c) The initial linear speed of a bug riding on the rim is 0.783 m/s (d) The tangential acceleration of the bug is $-0.024\ m/s^2$

Q.12: Tarzan swings on a vine of length 4m in a vertical circle under the influence of gravity. When the vine makes an angle of θ = 20° with the vertical, Tarzan has a speed of 5 m/s. Find (a) his centripetal acceleration at this instant, (b) his tangential acceleration, and (c) the resultant acceleration.

Data:

Length of vine $= r = 4\ m$

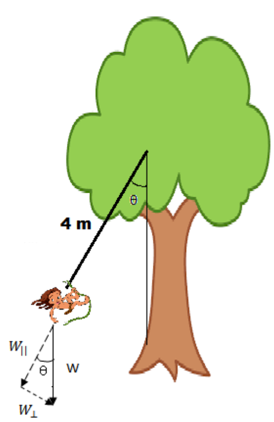
Angle with the vertical $= \theta = 20^0$

Speed of Tarzan $= v = 5\ m/s$

Centripetal Acceleration $= a_c = ?$

Tangential Acceleration $= a = ?$

Resultant Acceleration $= a_r = ?$



Solution:

The centripetal acceleration is given by

$$a_c = \frac{v^2}{r}$$
$$a_c = \frac{(5)^2}{4}$$
$$\boxed{a_c = 6.25\ m/s^2}$$

The net force on Tarzan

$$F = W_\perp$$
$$F = W sin\theta$$
$$ma = mg sin\theta$$
$$a = g\times sin\theta = 9.8\times\sin 20°$$
$$\boxed{a = 3.35\ m/s^2}$$

Now,

$$a_r = \sqrt{a^2 + a_c^2}$$
$$a_r = \sqrt{(3.35)^2 + (6.25)^2}$$
$$\boxed{a_r = 7.09\ m/s^2}$$

Result: The centripetal acceleration is 3.2 m/s^2 , tangential acceleration is 4.9 m/s^2 and resultant acceleration is 7.09 m/s^2

PAST PAPER NUMERICALS

2022

vi) What is the ratio of maximum range and maximum height of a projectile for an angle at which range is maximum.

Solution:

The maximum range is given by

$$R_{max} = \frac{v_0^2}{g} \text{ -----(i)}$$

Maximum height is given by

$$H_{max} = \frac{v_0^2 \sin^2\theta}{2g}$$

For maximum range $\theta = 45°$

$$H_{max} = \frac{v_0^2 (\sin 45)^2}{2g}$$

$$H_{max} = \frac{v_0^2}{4g} \text{ ------(ii)}$$

Dividing eq(i) by eq(ii)

$$\frac{R_{max}}{H_{max}} = \frac{v_0^2}{g} \div \frac{v_0^2}{4g}$$

$$\frac{R_{max}}{H_{max}} = \frac{v_0^2}{g} \times \frac{4g}{v_0^2}$$

$$\boxed{\frac{R_{max}}{H_{max}} = 4}$$

The ratio of maximum range and maximum height of a projectile for an angle at which range is maximum is 4:1

2019

No Numerical

2018

2(ii) Calculate the angle of projection for which the maximum height of projectile is equal to 1/3 of its horizontal range.

Data:

Angle of projection= $\theta = ?$

$H_{max} = \frac{1}{3}R$

Solution:

According to the given condition

$$H_{max} = \frac{1}{3}R$$

$$\frac{v_0^2 Sin^2\theta}{2g} = \frac{1}{3}\left(\frac{v_0^2 Sin2\theta}{g}\right)$$

$$\frac{Sin^2\theta}{2} = \left(\frac{Sin 2\theta}{3}\right)$$

$$\because Sin2\theta = 2\, sin\theta\, Cos\theta$$

$$\therefore \frac{Sin^2\theta}{2} = \left(\frac{2\, sin\theta \;\; Cos\theta}{3}\right)$$

$$\frac{sin\theta}{Cos\theta} = \left(\frac{4}{3}\right)$$

$$\tan\theta = \left(\frac{4}{3}\right)$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right)$$

$$\boxed{\theta = 53.1^0}$$

Result: The angle of projection for which the maximum height of projectile is equal to 1/3 of its horizontal range is 53.1^0.

2017

Q.2 (viii) **Textbook Numerical 7**

2016

Q.2 (iii) Tarzan swings on a vine, of length 5m, in a vertical circle, under the influence of gravity. When the vine makes an angle of 30° with the Vertical, Tarzan has a speed of 4 m/s. Find:
(a) Centripetal acceleration at this instant (b) His tangential acceleration

Data:

$Length\ of\ vine = r = 5m$

$Angle\ with\ the\ vertical = \theta = 30^0$

$Speed\ of\ Tarzan = v = 4\ m/s$

$Centripetal\ Acceleration = a_c = ?$

$Tangential\ Acceleration = a = ?$

Solution:

The centripetal acceleration is given by

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{(4)^2}{5}$$

$$\boxed{a_c = 3.2\ m/s^2}$$

The net force on Tarzan

$$F = W_{\perp}$$

$$F = W sin\theta$$

$$ma = mg sin\theta$$

$$a = g \times sin\theta = 9.8 \times \sin 30$$

$$\boxed{a = 4.9\ m/s^2}$$

Result: The centripetal acceleration is 3.2 m/s^2 and tangential acceleration is 4.9 m/s^2.

2015

Q.2 vii) A boy whose mass is 100 kg when resting on the ground at the equator if the radius of earth 'Re' is 6.4 x 10⁶ m. Calculate the centripetal acceleration and centripetal force.

Data:

Mass of boy = m = 100 kg

Radius of Earth = Re = 6.4 x 10^6 m

Centripetal Acceleration = a_c = ?

Centripetal Force = F_c = ?

Solution:

The centripetal Acceleration is given by

$$a_c = R\omega^2$$

And

$$\omega = \frac{2\pi}{T}$$

So,

$$a_c = R\left(\frac{2\pi}{T}\right)^2 = \frac{6.4 \text{ x } 10^6 \times 4 \times 3.14^2}{(86400)^2}$$

$$\boxed{a_c = 0.034\ m/s^2}$$

Now, Centripetal force is given by

$$\boxed{F_c = ma_c = 100 \times 0.034 = 3.4\ N}$$

Result: the centripetal acceleration is 0.034 m/s^2 and centripetal force is 3.4 N.

2014

Q.2 (ix) A mortar shell is field at a ground level target of 400 m distance with an initial velocity 85m/sec. Calculate the maximum time to hit the target.

Data:

Range = R = 400 m

Initial Velocity = V_0 = 85 m/s

Max. Time to Hit = T = ?

Solution:

First we will find the angle of projection

$$R = \frac{v_0^2\ Sin\ 2\theta}{g}$$

$$400 = \frac{(85)^2\ Sin\ 2\theta}{9.8}$$

$$Sin\ 2\theta = \frac{400 \times 9.8}{7225}$$

$$Sin\ 2\theta = 0.542$$

$$2\theta = Sin^{-1}(0.542)$$

$$2\theta = 32.8^0$$

Or $$\boxed{\theta = 16.4^0}$$

The larger angle is given by

$$\theta' = 90 - \theta = 90 - 16.4 = 73.5^0$$

For maximum time we use greater angle and find time of flight

$$T = 2\frac{(85)Sin\ (73.5)}{9.8}$$

$$T = 16.6\ sec$$

Result: The maximum time to hit the target is 16.6 sec.

Q.2 (xii) Calculate the centripetal acceleration and centripetal force on a man whose mass is 80 kg when resting on the ground at the equator if the radius of earth is 6.4 x 10^6 metres.

Data:

Mass of man= m =80 kg

Radius of Earth = Re = 6.4 x 10^6 m

Centripetal Acceleration = a_c =?

Centripetal Force = F_c =?

Solution:

The centripetal Acceleration is given by

$$a_c = R\omega^2$$

And

$$\omega = \frac{2\pi}{T}$$

So,

$$a_c = R\left(\frac{2\pi}{T}\right)^2 = \frac{6.4 \text{ x } 10^6 \times 4 \times 3.14^2}{(86400)^2}$$

$$a_c = 0.034\ m/s^2$$

Now, Centripetal force is given by

$$F_c = ma_c = 80 \times 0.034 = 2.72\ N$$

Result: the centripetal acceleration is 0.034 m/s^2 and centripetal force is 2.72 N.

2013

Q.2 (viii) Textbook Numerical 12

2012

Q.2 (vi) A diver leaps from a tower with an initial horizontal velocity component of 7m/sec and upward velocity component of 5m/sec. Find the components of his velocity along x and y axis after 1.5 sec.

Data:

Initial Horizontal velocity = $v_{ox} = 7\ m/s$

Initial Vertical velocity = $v_{oy} = 5\ m/s$

Time = $t = 1.5\ s$

Final Horizontal velocity = v'_{ox} =?

Final Vertical velocity = v'_{oy} =?

Solution:

Since the x component of velocity remains constant

$$v'_{ox} = v_{ox} = 7\ m/s$$

The y component of velocity is given by

$$v_f = v_i + at$$

or

$$v'_{oy} = v_{oy} + (-g)t$$

$$v'_{oy} = 5 - (9.8 \times 1.5)$$

$$v'_{oy} = 5 - (14.7)$$

$$v'_{oy} = -9.7\ m/s$$

Result: The final x component will be 7 m/s and y component will be 9.7 m/s(downward)

Q.2 (viii) Same as 2018 Q.2(ii)

2011

Q.2 (ix) Same as 2014 Q.2(xii)

2010

No Numerical

THEORY NOTES

TORQUE

DEFINITION:

The turning effect of force is called torque Mathematically it is defined as the product of force and the force arm that is the perpendicular distance between the point of application of force and the fixed point or fulcrum about which body rotates.

EXPLANATION:

From above definition,
Torque = Force x Momemt arm

$$\vec{\tau} = \vec{r} \times \vec{F}$$

rFSinθ

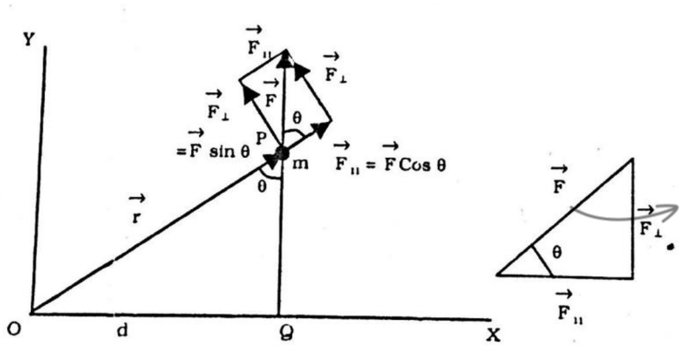
Where 'θ' is the angle between 'F' and 'r'

MAGNITUDE:

Let us consider a particle of mass 'm' which is acted upon by a force F . Let r be the position vector of the particle which is also the position vector of the point of application of the force. The force P can be resolved into its rectangular components i.e.

(i) $F_{\parallel}$, i.e. parallel to the vector r

(ii) $F_{\perp}$, i.e. perpendicular to the vector r as shown in fig.



It is clear from fig. that $F_{\parallel}$ is the pulling component and $F_{\perp}$ is the rotating compo- nent, i.e. $F_{\parallel}$ only is responsible to rotate the body about point O, therefore the magnitude of torque produced b the force F about point O will be

$$\tau = (r)(F_{\perp})$$
$$\tau = (r)(FSin\theta)$$

Where θ is the smaller angle between the positive directions of $\vec{r}$ and $\vec{F}$.

As Torque is the Vector quantity then

$$\vec{\tau} = (r)(FSin\theta)\hat{n}$$

$$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$

Where $\hat{n}$ is the unit vector perpendicular to $\vec{r}$ and $\vec{F}$.

IN RECTANGULAR COMPONENT FORM:

Let $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

and $\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}$

then

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

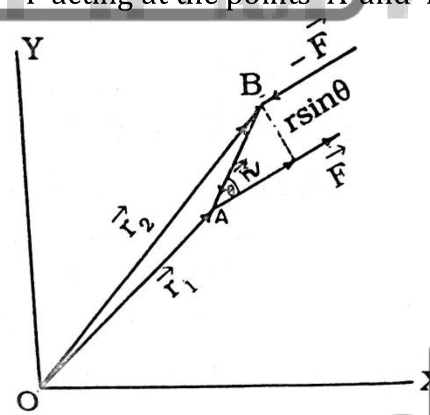
COUPLE:

DEFINITION:

Two force which are equal in magnitude but opposite in direction and not acting along with the same direction constitute a couple"

EXPLANATION:

Let the forces constituting the couple are represented by F and –F acting at the points 'A' and 'B'. The moment of force F about 'O' is



$$\vec{\tau}_1 = \vec{r_1} \times \vec{F}$$

And, The moment of force – F about the same point is

$$\vec{\tau}_2 = \vec{r_2} \times -\vec{F}$$

The total moment of the two forces is given by,

$$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2$$
$$\vec{\tau} = \vec{r_1} \times \vec{F} + \vec{r_2} \times -\vec{F}$$
$$\vec{\tau} = \vec{r_1} \times \vec{F} - \vec{r_2} \times \vec{F}$$

Taking common

$$\vec{\tau} = (\vec{r_1} - \vec{r_2}) \times \vec{F}$$

A/c to the figure , $\vec{r} = \vec{r_1} - \vec{r_2}$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

The Magnitude of $\vec{\tau}$ is given by

$$\tau = (r)(FSin\theta)$$

or $$\tau = Fd$$

Where d $= rSin\theta$ is the perpendicular distance between the line of action of forces.

Centre of Mass

DEFINITION:

The centre of mass of a body, or a system of particles, is a point on the body that moves in the same way that a single particle would move under the influence of the same external forces. The whole mass of the body is supposed to be concentrated at this point. This point is called Centre of Mass.

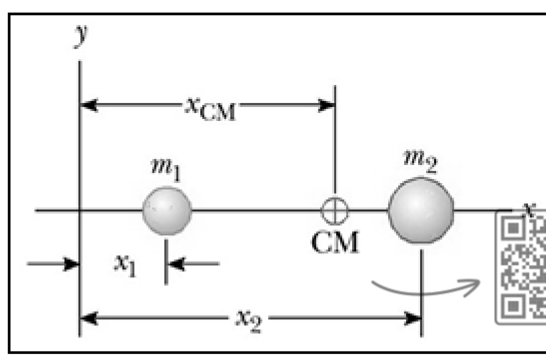
EXPLANATION

During translational motion each point of a body moves in the same manner i.e., different particles of the body do not change their position w.r.t each other. Each point on the body undergoes the same displacement as any other point as time goes on. So the motion of one particle represents the motion of the whole body. But in rotating or vibrating bodies different particles move in different manners except one point called centre of mass. The centre of mass of a body or a system of particle is a point which represents the movement of the entire system. It moves in the same way that a single particle would move under the influence of same external forces.

CENTRE OF MAS AND CENTRE OF GRAVITY

In a completely uniform gravitational field, the centre of mass and centre of gravity of an extended body coincides. But if gravitational field is not uniform, these points are different.

DETERMINATION OF CENTRE OF MASS



Consider a system of two particles having masses m_1, m_2 having position coordinates $x_1$and x_2.Now the centre of mass of this system is the arithmetic mean between positions of the masses.

Mathematically we can write as,

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

and for "n" number of particles

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \cdots + m_n x_n}{m_1 + m_{2+\cdots+m_n}}$$

Or

$$\boxed{x_{cm} = \frac{\sum m_i x_i}{\sum m_i}}$$

Similarly for y co-ordinate of centre of mass,

$$\boxed{y_{cm} = \frac{\sum m_i y_i}{\sum m_i}}$$

and for z co-ordinate of centre of mass,

$$\boxed{z_{cm} = \frac{\sum m_i z_i}{\sum m_i}}$$

EQUILIBRIUM:

A body is said to be equilibrium if it is at rest or is moving with uniform velocity.

1) **STATIC EQUILIBRIUM:**

A body at rest is said to be in static equilibrium

2) **DYNAMIC EQUILIBRIUM:**

A body in uniform motion along a straight line is said to be in dynamic equilibrium.

In both the cases the bodies do not posses any acceleration neither linear nor angular.

CONDITIONS OF EQUILIBRIUM:

There are two conditions of equilibrium.

A) FIRST CONDITION OF EQUILIBRIUM: (TRANSLATION EQUILIBRIUM):

STATEMENT:

A body will be in equilibrium if the resultant of all the forces on it is equal to zero."

EXPLANATION:

Let $\vec{F_1}$, $\vec{F_2}$,…… $\vec{F_n}$ be the external forces acting on a body. Thus, according to the first condition.

$$\vec{F_1} + \vec{F_2} + \ldots + \vec{F_n} = 0$$

or

$$\boxed{\sum_{i=1}^{n} \vec{F_i} = 0}$$

A FORCES ACTING ALONG X-AXIS

Let $\overrightarrow{F_{ix}}, \overrightarrow{F_{2x}}, \ldots\ldots\ldots \overrightarrow{F_{nx}}$ be forces acting along x-axis

Thus, from first condition,

$$\overrightarrow{F_{1x}} + \overrightarrow{F_{2x}} + \ldots\ldots + \overrightarrow{F_{nx}} = 0$$

Or $(F_{1x} + F_{2x} + \ldots.. F_{nX})\,\hat{\imath} = 0$

or

$$\boxed{\sum_{i=1}^{n} \overrightarrow{F_{ix}} = 0}$$

B FORCES ACTING Y-AXIS:

Let $\overrightarrow{F_{iy}}, \overrightarrow{F_{2y}}, \ldots\ldots\ldots \overrightarrow{F_{ny}}$ be forces acting along x-axis

Thus, from first condition,

$$\overrightarrow{F_{1y}} + \overrightarrow{F_{2y}} + \ldots\ldots + \overrightarrow{F_{ny}} = 0$$

Or $(F_{1y} + F_{2y} + \ldots.. F_{ny})\,\hat{\jmath} = 0$

$$\boxed{\sum_{i=1}^{n} \overrightarrow{F_{iy}} = 0}$$

I.e. For body to be in equilibrium, the sum of x-components of all the forces and the sum of y-components of all the forces must be equal to zero separately. Therefore First condition of equilibrium can be written as

$$\boxed{\sum_{i=1}^{n} \overrightarrow{F_{ix}} = 0\ , \sum_{i=1}^{n} \overrightarrow{F_{iy}} = 0 \ and\ over\ all \sum_{i=1}^{n} \vec{F_i} = 0}$$

B) SECOND CONDITION OF EQUILIBRIUM:(ROTATIONAL EQUILIBRIUM)

STATEMENT:

"If the vector sum of the torques acting on a body is zero, the body is said to in rotational equilibrium."

EXPLANATION:

Let τ_1, τ_2, ……, τ_n are the torques on the body then

$\tau_1 + \tau_2 + \ldots\ldots + \tau_n = 0$

Or $\boxed{\sum_{i=1}^{n} \vec{\tau} = 0}$

This is the required condition.

For this condition it is necessary that" Sum of clockwise torque is equal to sum of anticlockwise torque" Where Anticlockwise Torque is taken Positive and Clockwise Torque is taken Negative.

ANGULAR MOMENTUM:

DEFINITION: Angular momentum of an object moving in a circle is the cross product of linear momentum and position vector from the origin."

EXPLANATION:

A body having rotatory motion possesses angular velocity and angular momentum.

Consider a particle of mass 'm' let r be its position vector and P be the linear momentum with respect to origin.

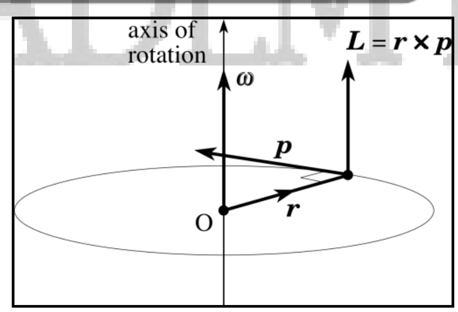
From Definition,

Angular momentum = Position Vector X Linear Momentum

$\vec{l} = \vec{r} \times \vec{P}$

$\vec{l} = \vec{r} \times \vec{mv}$ [P = mv]

$\vec{l} = m\vec{r} \times \vec{v}$



where, $\vec{v}$ be the velocity of the particle.

The direction of angular momentum is normal to the plane formed by $\vec{r}$ and $\vec{P}$ as given by right and rule.

$l = rp\sin\theta$

or $l = mvr\sin\theta$

Where 'θ' is the angle between $\vec{r}$ and $\vec{P}$

When $\theta = 90^o$ [Sin90 = 1]

$$l = rp = mvr$$

In Cartesian Coordinate system when,

$$\vec{l} = \vec{r} \times \vec{P} = (x\hat{\imath} + y\hat{\jmath} + z\hat{k}) \times (P_x\hat{\imath} + P_y\hat{\jmath} + P_z\hat{k})$$

$$\vec{l} = \vec{r} \times \vec{P} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x & y & z \\ P_x & P_y & P_z \end{vmatrix}$$

DIMENSION AND UNIT:

The dimension of angular momentum is

$[l] = [r]\,[p] = [r]\,[m]\,[v] = L.M.L/T = L^2MT^{-1}$

In S.I system its units is NmS = J.S.

LAW OF CONSERVATION OF ANGULAR MOMENTUM

STATEMENT:

The angular momentum of a particle is conserved (constant) if the torque acting on it is zero."

EXPLANATION:

If F is the force acting on a particle of mass 'm' moving with velocity $\vec{V}$ and $\vec{P}$ is the linear momentum, then,

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$= \vec{r} \times \frac{\vec{P_f} - \vec{P_i}}{\Delta t} \quad \because \text{Force is the rate of change of momentum}$$

$$= \frac{\vec{r} \times \vec{P_f} - \vec{r} \times \vec{P_i}}{\Delta t}$$

$$= \frac{\vec{L_f} - \vec{L_i}}{\Delta t} \qquad \because \vec{L} = \vec{r} \times \vec{P}$$

$$\vec{\tau} = \frac{\Delta \vec{L}}{\Delta t}$$

This equation states that the torque acting on a particle is the time rate of change of its angular momentum.

If net torque acting on the particle is zero, Then,

$$\frac{\Delta \vec{L}}{\Delta t} = 0$$

or $\quad \Delta\vec{L} = 0 \quad => \quad \boxed{\vec{L} = Constant}$

thus, angular momentum of a particle is conserved, i.e. law of conservation of angular momentum.

M.C.Qs.

1. Torque is also known as:
(a) Angular speed (b) Angular momentum
(c) Moment of inertia (d) Moment of force

2. The rate of change of angular momentum is called:
(a) Force (b) Torque
(c) Momentum (d) Equilibrium

3. A force of 8 N is applied to the spanner perpendicularly at a distance of 0.12 m from the centre of nut ,the moment of force acting on the nut is:
(a) 0.96 Nm (b) 1.5 Nm
(c) 2.1 Nm (d)3 Nm

4. Torque is zero, if angle θ between force and momentum arm is:
(a) 0° (b) 60° (c) 90° (d) 180^0

5. The motion of the body can describe by the motion of it's:
(a) Center of gravity (b) Origin
(c) Center of mass (d) None of these

6. The ratio of SI unit of angular momentum to linear momentum is:
(a) J.s (b) N/J
(c) J.N (d) J/N

7. The two forces constitute couple are:
(a) Equal in magnitude
(b) Opposite in direction
(c) Not acting along the same line
(d) All of these

8. The angular momentum of a particle changes from 0 to 720 is in 4 sec, the magnitude of torque acting will be:
(a) 1440J (b) 360J
(c) 180J (d) 4.5J

9. The point at which whole weight of the body is concentrated is called.
(a) Centre of mass (b) Centre of gravity
(c) Origin (d) Centre of action

10. The magnitude of couple depends upon:
(a) The distance of F from origin
(b) The distance of –F from origin
(c) Distance between F and –F
(d) None of these

11. If linear momentum of body is doubled and parallel to the axis of rotation then angular momentum will be:
(a) Doubled (b) Halved
(c) Quadrupled (d) Zero

12. The centre of mass coincides with centre of gravity of body, if it is placed:
(a) In a non-uniform gravitation field.
(b) In a uniform gravitation field

(c) At the centre of earth
(d) At the poles

13. The magnitude of the angular momentum is given by:
(a) L = rp cos θ (b) L = rp/sin θ
(c) L = rp sin θ (d) both A & B

14. The angular momentum of tyre of car of mass 10kg and radius 0.5m and the car moving with velocity of 10m/s is:
(a) 50 (b) 25 (c) 100 (d) Zero

15. If the net torque acting on a body is zero then the ___ of the body is conserved:
(a) Force (b) Liner momentum
(c) Torque (d) Angular momentum

16. According to law of conservation of angular momentum.
(a) $\frac{dl}{dt} = 0$ (b) $\frac{dl}{dt} = constant$
(c) $\frac{dl}{dt} = \frac{df}{dx}$ (d) Both a and b

17. The product of moment of inertia and angular acceleration is:
(a) Angular momentum (b) Torque
(c) Couple (d) None of these

18. If the force of F = 2i + 4j – 3k acting on a body pivoted at 2m from the axis of rotation along x-axis. Then the magnitude of rotational analogue of the force is:
(a) 12N-m (b) 10N-m
(c) 8N-m (d) 6N-m

19. A body will be in translation equilibrium if the vector sum of external forces acting on a body is :
(a) Maximum (b) Minimum
(c) Square (d) Zero

20. If the axis of rotation passes through the body itself the corresponding rotator motion is called the:
(a) Spin –motion (b) Orbital motion
(c) Vibratory motion (d) To and fro motion

21. For which of the following does the centre of mass lie outside the body?
(a) Pen (b) Dice
(c) Rectangular tile (d) Bangle

22. Two bodies of masses 5kg and 15kg are located in the cartesian plane at (1,0) and (0,1). What is the location of their centre of mass?
(a) 1/4, 1/4 (b) 3/4, 3/4
(c) 3/4, 1/4 (d) 1/4, 3/4

23. A dancer on ice starts spinning faster, when she folds her arms. This is due to
(a) decrease in friction at the skates
(b) increase in angular momentum
(c) decrease in angular momentum
(d) constant angular momentum and decrease in moment of inertia

24. When a torque acting on a system is increased, then which one of the following quantities will increase
(a) linear momentum (b) Angular momentum
(c) force (d) Displacement

25. The resultant force acting in the couple is ______
(a) Zero (b) Infinite
(c) Twice the magnitude of the single force
(d) Half the magnitude of the single forcea

PAST PAPER M.C.Qs.

2022

1. The SI unit of angular momentum is:

***J-s** *J/s *s/J *J-s^2

14. The point which describes the motion of the whole system or body is known as the:

***center of mass** *inertia *centre of gravity *moment of inertia

2021

(vii) The rate of change of angular momentum is called

*Power *** Torque** *Momentum *Force

(viii) A 400N force acting perpendicularly to an object at the distance of 200cm from the axis of rotation, the moment of force generated is:

*100N m *200 Nm * 400Nm *** 800 Nm**

(xl) A body will be in complete equilibrium is:

*1s condition of equilibrium only *2nd condition of equilibrium only

***Both 1" and 2nd condition of equilibrium** *Neither 1st nor 2d condition of equilibrium

(xlii) The SI unit of angular momentum is:

***J-s** *J/s *s/J *J-s^2

2019

9. The point which describes the motion of the whole system or body is known as the:

***center of mass** *inertia

*centre of gravity *moment of inertia

15. Two forces of equal magnitude but opposite direction in direction and not acting along the same straight line form a:

*circle ***couple**

*power *torque

2018

14. The angular momentum of particle moving in circle is conserved if:

***net torque acting on particle is zero** *the acceleration of particle is zero

* net angular displacement of particle is zero * net force acting on particle is zero

17.A force of 8 N is applied to the spanner perpendicularly at a distance of 0.15 m from the centre of nut ,the moment of force acting on the nut is:

***1.2 Nm** *1.5 Nm * 2.1 Nm * 3 Nm

2017

4. Torque is maximum when force:
   - *is parallel to moment arm
   - *as anti parallel to moment arm
   - *makes angle 60 degree with moment arm
   - *** is perpendicular to moment arm**

14. The ratio of SI unit of angular momentum to linear momentum is:
   - *J.s
   - *N/J
   - *J.N
   - ***J/N**

2016

3.The rate of change of angular momentum is also known as:
   - *Linear momentum
   - ***Torque**
   - *Force
   - *Energy

2015

6. When torque acting on a system is zero, this will be constant:
   - *force
   - ***angular momentum**
   - * linear momentum
   - * velocity

2013

7. The centre of mass of a body:
   - *always coincides with the centre of gravity
   - * never coincides with the centre of gravity
   - ***coincides with the centre of gravity only in uniform field**
   - *is lower than the centre of gravity

9. The angular momentum of a particle changes from 0 to 720 is in 4 sec, the magnitude of torque acting will be:
   - *1440J
   - *360J
   - ***180J**
   - *4.5J

16. The sum of torques acting on a body is zero, and then this will be constant:
   - ***angular momentum**
   - *force
   - *linear momentum
   - *pressure

2012

12. The magnitude of couple depends upon:
   - *The distance of F from origin
   - * The distance of –F from origin
   - *** Distance between F and –F**
   - * None of these

2011

12. The rate of change of angular momentum with respect to time is:
   - *force
   - * angular velocity
   - *angular acceleration
   - ***torque**

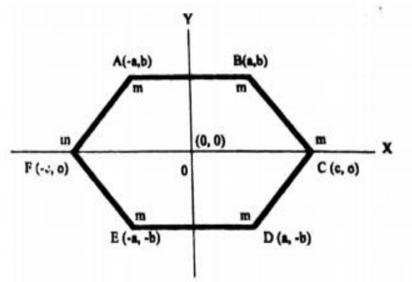
13. Two forces of equal magnitude but opposite in direction and not acting on the same line constitute:
   - ***a couple**
   - * power
   - *a circle
   - *a force

2010

14. Torque is defined as the time rate of change of:
   - *** angular momentum**
   - * linear momentum
   - * angular velocity
   - * angular acceleration

TEXTBOOK NUMERICALS

Q.1: Locate the centre of mass of a system of particles each of mass 'm', arranged to correspond in position to the six corners of a regular (planar) hexagon.



Data:

Mass of each particle $= m$

Centre of Mass $= C(x, y) = ?$

Solution:

The x co-ordinate of centre of mass is given by

$$x = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots + m_nx_n}{m_1 + m_2 + m_3 + \cdots + m_n}$$

$$x = \frac{m(-a) + m(a) + m(c) + m(a) + m(-a) + m(-c)}{m + m + m + m + m + m}$$

$$x = \frac{-ma + ma + mc + ma - ma + mc}{6m}$$

$$x = \frac{0}{6m}$$

$$\boxed{x = 0}$$

The y co-ordinate of centre of mass is given by

$$y = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots + m_ny_n}{m_1 + m_2 + m_3 + \cdots + m_n}$$

$$y = \frac{m(b) + m(b) + m(0) + m(-b) + m(-b) + m(0)}{m + m + m + m + m + m}$$

$$y = \frac{mb + mb - mb - mb}{6m}$$

$$y = \frac{0}{6m}$$

$$\boxed{y = 0}$$

Result: The centre of mass of given hexagon is at origin (0,0).

Q.2: Find the position of centre of mass of five equal-mass particles located at the five corners of a square-based right pyramid with sides of length 'l' and altitude 'h'.

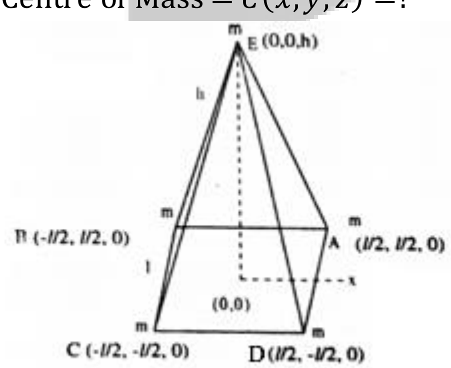
Data:

Mass of each particle $= m$

Length of base $= l$

Height of Pyramid $= h$

Centre of Mass $= C(x, y, z) = ?$



Solution:

The x co-ordinate of centre of mass is given by

$$x = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots + m_nx_n}{m_1 + m_2 + m_3 + \cdots + m_n}$$

$$x = \frac{m(l/2) + m(-l/2) + m(-l/2) + m(l/2) + m(0)}{m + m + m + m + m}$$

$$x = \frac{m(l/2) - m(l/2) - m(l/2) + m(l/2)}{5m}$$

$$x = \frac{0}{5m}$$

$$\boxed{x = 0}$$

The y co-ordinate of centre of mass is given by

$$y = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots + m_ny_n}{m_1 + m_2 + m_3 + \cdots + m_n}$$

$$y = \frac{m(l/2) + m(l/2) + m(-l/2) + m(-l/2) + m(0)}{m + m + m + m + m}$$

$$y = \frac{m(l/2) + m(l/2) - m(l/2) - m(l/2)}{5m}$$

$$y = \frac{0}{5m}$$

$$\boxed{y = 0}$$

The z co-ordinate of centre of mass is given by

$$z = \frac{m_1 z_1 + m_2 z_2 + m_3 z_3 + \cdots + m_n z_n}{m_1 + m_2 + m_3 + \cdots + m_n}$$

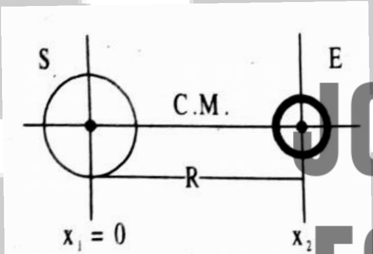
$$z = \frac{m(0) + m(0) + m(0) + m(0) + m(h)}{m + m + m + m + m}$$

$$z = \frac{mh}{5m}$$

$$\boxed{z = h/5}$$

Result: The centre of mass of given Pyramid is at 1/5th of its height (0,0,h/5).

Q.3: The mass of the sun is 329.390 times the earth's mass and the mean distance from the centre of the sun to the centre of the earth is 1.496x10⁸ km. Treating the earth and sun as particles with each mass concentrated at the respective geometric centre, how far from the centre of the sun is the C.M (centre of mass) of the earth-sun system? Compare this distance with the mean radius of the sun (6.9960x10⁵ km)



Data:

Mass of Sun $= M_s = 329.390\ M_e$

Mean distance b/w sun & earth $= R =$ 1.496x10^{8}km

Mean radius of the sun $= R_s =$ 6.9960x10^5 km

Distance of Centre of mass of system$= x =?$

Ration of distance to radius of sun$= \frac{x}{R_s} =?$

Solution:

The x co-ordinate of centre of mass is given by

$$x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$x = \frac{M_s x_1 + M_e x_2}{M_s + M_e}$$

$$x = \frac{329.390\ M_e \times 0 + M_e \times 1.496\text{x}10^8}{329.390\ M_e + M_e}$$

$$x = \frac{M_e \times 1.496\text{x}10^8}{330.390\ M_e}$$

$$\boxed{x = 4.52 \times 10^5 km}$$

Now,

$$\frac{x}{R_s} = \frac{4.52 \times 10^5\ km}{6.9960\text{x}10^5\ \text{km}}$$

$$\boxed{\frac{x}{R_s} = 0.64}$$

Result: The centre of mass of Sun-Earth system is $4.52 \times 10^5 km$ far from the sun and this distance is 0.64 times of radius of sun.

Q.4: A particle of mass 4 kg moves along the x-axis with a velocity v = 15t m/s, where t = 0 is the instant that the particle is at the origin.

(a) At t= 2 s , what is the angular momentum of particle about a point P located on +ve y axis 6 m from the origin?

(b) What torque about P acts on the particle?

Data:

Mass of particle $= m = 4\ kg$

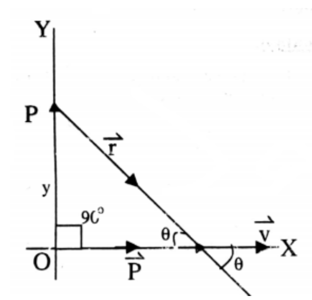
Velocity of particle $= v =$ 15 t m/s

Angular Momentum $= l =?$

Distance of Particle$= r =$ 6 m

Time $= t =$ 2 s

Torque $= \tau =?$



Solution:

The angular momentum of particle is given by

$$l = mvr$$
$$l = m \times 15 \times t \times r$$
$$l = 4 \times 15 \times 2 \times 6$$
$$\boxed{l = 720\ kgm^2/s}$$

Now, Torque is given by

$$\tau = \frac{\Delta l}{\Delta t} = \frac{720 - 0}{2}$$
$$\boxed{\tau = 360\ Nm}$$

Result: The angular momentum of particle is $720\ kgm^2/s$ and torque is $360\ Nm$.

Q.5: A particle of mass 'm' is located at the vector position r and has a linear momentum vector p. The vector r and p are non zero. If the particle moves only in the x, y plane, prove that $L_x = L_y = 0$ and $L_z \neq 0$

Proof:

According to the def. of Angular Momentum

$$\vec{l} = \vec{r} \times \vec{P}$$
$$\vec{l} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x & y & z \\ P_x & P_y & P_z \end{vmatrix}$$
$$l_x\hat{\imath} + l_y\hat{\jmath} + l_z\hat{k} = \hat{\imath}\begin{vmatrix} y & z \\ P_y & P_z \end{vmatrix} - \hat{\jmath}\begin{vmatrix} x & z \\ P_x & P_z \end{vmatrix} + \hat{k}\begin{vmatrix} x & y \\ P_x & P_y \end{vmatrix}$$
$$l_x\hat{\imath} + l_y\hat{\jmath} + l_z\hat{k} = \hat{\imath}(yP_z - zP_y) - j(xP_z - zP_x) + \hat{k}(xP_y - yP_x)$$

As Motion is in the x-y plane therefore z=0 and Pz=0

$$l_x\hat{\imath} + l_y\hat{\jmath} + l_z\hat{k} = \hat{\imath}(y(0) - (0)P_y) - j(x(0) - (0)P_x) + \hat{k}(xP_y - yP_x)$$
$$l_x\hat{\imath} + l_y\hat{\jmath} + l_z\hat{k} = \hat{k}(xP_y - yP_x)$$

By equating the components we get

$$L_x = L_y = 0 \text{ and } L_z = \hat{k}(xP_y - yP_x)$$

Hence it is proved that $L_x = L_y = 0$ and $L_z \neq 0$

Q.6: A light rigid rod 1m in length rotates in the xy-plane about a pivot through the rod's centre. Two particles of mass 2kg and 3kg are connected to its ends. Determined the angular momentum of the syste m about the origin at the instant the speed of each particle is 5m/s.

Data:

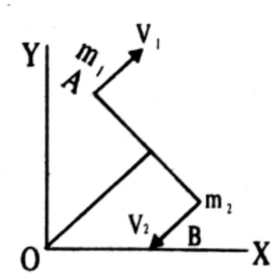
length of rod $= r = 1\ m$

Mass of 1st particle $= m_1 = 2\ kg$

Mass of 2nd particle $= m_2 = 3\ kg$

Angular Momentum $= l = ?$

Velocity of particles $= v_1 = v_2 = 5$ m/s



Solution:

The angular momentum of 1st particle is given by

$$\vec{l}_1 = \vec{r_1} \times \vec{P_1}$$
$$l_1 = r_1 P_1 \sin 90°$$
$$l_1 = \frac{r}{2} m_1 v_1$$
$$l_1 = \frac{1}{2} \times 2 \times 5$$
$$\boxed{l_1 = 5\ kgm^2/s}$$

Now, The angular momentum of 2nd particle is given by

$$\vec{l}_2 = \vec{r_2} \times \vec{P_2}$$
$$l_2 = r_2 P_2 \sin 90°$$
$$l_2 = \frac{r}{2} m_2 v_2$$
$$l_2 = \frac{1}{2} \times 3 \times 5$$
$$\boxed{l_2 = 7.5\ kgm^2/s}$$

Now, total angular momentum is

$$\boxed{l = l_1 + l_2 = 5 + 7.5 = 12.5\ kgm^2/s}$$

Result: The angular momentum of the system is $12.5\ kgm^2/s$.

Q.7: A uniform beam of mass 'M' supports two masses m_1 and m_2. If the knife edge of the support is under the beam's centre of gravity and m_1 is at a distance 'd' from the centre, determine the position of m_2 such that the system is balanced.

Data:

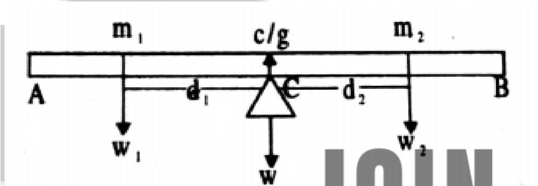
Mass of Beam $= M$

Mass of 1st particle $= m_1$

Mass of 2nd particle $= m_2$

Distance of m_1 from centre $= d_1 = d$

Distance of 2 from centre $= d_2 = D = ?$



Solution:

Using 2nd condition of equilibrium

$$\sum \tau = 0$$

$$W_1 \times d_1 - W_2 \times d_2 + W \times 0 = 0$$

$$W_1 \times d_1 = W_2 \times d_2$$

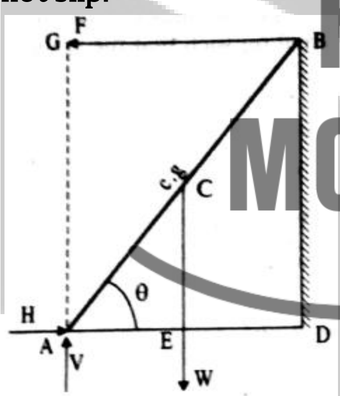
$$m_1 g \times d = m_2 g \times D$$

$$D = \frac{m_1 g \times d}{m_2 g}$$

$$\boxed{D = \frac{m_1}{m_2}\boldsymbol{d}}$$

Result: The position of m_2 such that the system is balanced is $\frac{m_1}{m_2} d$ from the centre.

Q.8: A uniform ladder of length l and weight W = 50 N rests against a smooth vertical wall. If the coefficient of friction between the ladder and the ground is 0.40, find the minimum angle θ_{min} such that the ladder may not slip.



Data:

Length $= \overrightarrow{AB} = \mathrm{L}$

Weight $=$ W $=$ 50N

Co-efficient of friction$= \mu = 0.4$

Minimum angle $= \theta = ?$

Solution:

Using first condition of equilibrium

$\sum F_x = 0$; $\sum F_y = 0$

$H = F$ ----(i)

And

$V = W$----(ii)

Dividing equation (i) by Equation (ii)

$$\frac{H}{V} = \frac{F}{W}$$

Since

$$\mu = \frac{H}{V}$$

Therefore

$$\mu = \frac{F}{W}$$

$$0.4 = \frac{F}{50}$$

$$\boxed{F = 20N}$$

Using 2nd condition of equilibrium

$$\sum \tau = 0$$

$$F \times \overrightarrow{AG} + H \times 0 + v \times 0 - W \times \overrightarrow{AE} = 0$$

$F \times \overrightarrow{BD} = W \times \overrightarrow{AE}$ --- (iii)

From ΔABD

$$sin\theta = \frac{\overrightarrow{BD}}{\overrightarrow{AB}}$$

$$sin\theta = \frac{\overrightarrow{BD}}{L}$$

$$\boxed{\overrightarrow{BD} = L \sin \theta}$$

and From ΔACE

$$Cos\theta = \frac{\overrightarrow{AE}}{\overrightarrow{AC}}$$

$$Cos\ \theta = \frac{\overrightarrow{AE}}{L/2}$$

$$\boxed{\overrightarrow{AE} = \frac{1}{2} L \cos \theta}$$

Putting values of $\overrightarrow{BD}$ and $\overrightarrow{AE}$ in eq (iii)

$$F\ L\ sin\theta = W \frac{L}{2} cos\theta$$

$$\frac{sin\theta}{cos\theta} = \frac{W}{2F}$$

Or

$$Tan\theta = \frac{W}{2F}$$

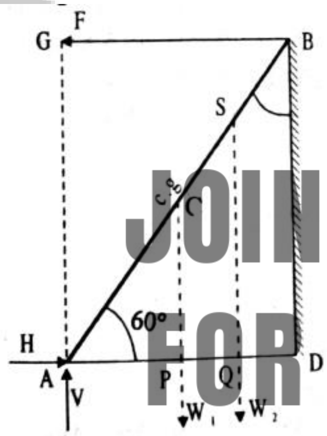
$$Tan\theta = \frac{50}{2 \times 20}$$

$$\theta = Tan^{-1}\frac{5}{4}$$

$$\boxed{\theta = 51.3^0}$$

Result: The minimum angle should be 51.3^0 .

Q.9: A ladder with a uniform density and a mass 'm' rests against a frictionless vertical wall at an angle of 60° . The lower end rests on a flat surface where the coefficient of friction (static) is 0.40. A student with a mass (M = 2m) attempts to climb the ladder. What fraction of the length 'L' of the ladder will the student have reached when the ladder begins to slip?



Data:

Angle = θ= 60^0

Length = $\overrightarrow{AB}$ = L

Mass of Ladder = m

Mass of Student = $M = 2m$

Weight of Ladder = W_1

Weight of Student = W_2

Co-efficient of friction=μ =0.4

Fraction of the ladder covered = $\frac{\overrightarrow{AS}}{\overrightarrow{AB}}$ =?

Solution:

Using first condition of equilibrium

$\sum F_x = 0 ; \sum F_y = 0$

$H = F$ ----(i)

And

$V = W_1 + W_2$

$V = mg + 2mg$

$$\boxed{V = 3\ mg}$$

Dividing equation (i) by Equation (ii)

$$\frac{H}{V} = \frac{F}{3\ mg}$$

Since

$$\mu = \frac{H}{V}$$

Therefore

$$\mu = \frac{F}{3\ mg}$$

$$0.4 = \frac{F}{3\ mg}$$

$$\boxed{F = 1.2\ mg}$$

Using 2nd condition of equilibrium

$$\sum \tau = 0$$

$$F \times \overrightarrow{AG} + H \times 0 + v \times 0 - W_1 \times \overrightarrow{AP} - W_2 \times \overrightarrow{AQ} = 0$$

$$F \times \overrightarrow{BD} = W_1 \times \overrightarrow{AP} + W_2 \times \overrightarrow{AQ} \text{ --- (iii)}$$

From ΔABD

$$sin\theta = \frac{\overrightarrow{BD}}{\overrightarrow{AB}}$$

$$\boxed{\overrightarrow{BD} = \overrightarrow{AB} \sin\theta}$$

and from ΔACP

$$Cos\theta = \frac{\overrightarrow{AP}}{\overrightarrow{AC}}$$

$$Cos\ \theta = \frac{\overrightarrow{AE}}{\overrightarrow{AB}/2}$$

$$\boxed{\overrightarrow{AE} = \frac{1}{2}\overrightarrow{AB}\cos\theta}$$

from ΔASQ

$$Cos\theta = \frac{\overrightarrow{AQ}}{\overrightarrow{AS}}$$

$$\boxed{\overrightarrow{AQ} = \overrightarrow{AS}\cos\theta}$$

Putting values of $\overrightarrow{BD}$ and $\overrightarrow{AP}$ and $\overrightarrow{AQ}$ in eq (iii)

$$F \times \overrightarrow{AB}\sin\theta = W_1 \times \frac{1}{2}\overrightarrow{AB}\cos\theta + W_2 \times \overrightarrow{AS}\cos\theta$$

Putting values

$$1.2\ mg \times \overrightarrow{AB}\sin 60° = mg \times \frac{1}{2}\overrightarrow{AB}\cos 60° +$$

$$2mg \times \overrightarrow{AS}\cos 60°$$

$1.2 \times \overrightarrow{AB} \times 0.866 = \frac{1}{2}\overrightarrow{AB} \times 0.5 + 2 \times \overrightarrow{AS} \times 0.5$

$1.0392 \times \overrightarrow{AB} = \overrightarrow{AB} \times 0.25 + \overrightarrow{AS}$

$1.0392 \times \overrightarrow{AB} - \overrightarrow{AB} \times 0.25 = \overrightarrow{AS}$

$\overrightarrow{AB}(1.0392 - 0.25) = \overrightarrow{AS}$

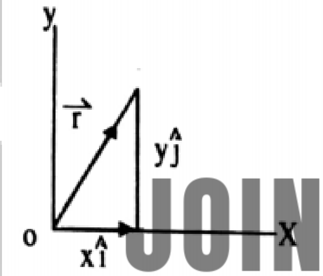
$\overrightarrow{AB}(0.789) = \overrightarrow{AS}$

Or $\frac{\overrightarrow{AS}}{\overrightarrow{AB}} = 0.789$

In Percentange: $\boxed{\frac{\overrightarrow{AS}}{\overrightarrow{AB}}\% = 78.9\ \%}$

Result: The student have reached 78.9 % of length of ladder.

Q.10: A particle of mass 0.3 kg moves in the xy-plane. At the instant its coordinates are (2, 4)m, its velocity is (3i + 4j)m/s. At this instant determine the angular momentum of the particle relative to the origin.



Data:

Mass of particle = m = 0.3 kg

Co-ordinates of position = $\vec{r}$ = (2, 4) =2i + 4j

Velocity of particle = $\vec{v}$ = 3i + 4 j

Angular momentum of particle = $\vec{l}$ =?

Solution:

According to the def. of Angular Momentum

$\vec{l} = m(\vec{r} \times \vec{v})$ ---- (i)

$$\vec{r} \times \vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 4 & 0 \\ 3 & 4 & 0 \end{vmatrix}$$

$$\vec{r} \times \vec{v} = \hat{\imath}\begin{vmatrix} 4 & 0 \\ 4 & 0 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 2 & 0 \\ 3 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & 4 \\ 3 & 4 \end{vmatrix}$$

$\vec{r} \times \vec{v} = \hat{\imath}(0 - 0) - j(0 - 0) + \hat{k}(8 - 12)$

$\boxed{\vec{r} \times \vec{v} = -4\hat{k}}$

Putting in eq (i)

$\vec{l} = 0.3 \times (-4\hat{k})$

Or $\boxed{\vec{l} = -1.2\ \hat{k}}$

Result: The Magnitude of angular momentum is 1.2 j.s and direction is along –ve z axis.

Q.11: A uniform horizontal beam of length 8m and weighing 200N is pivoted at the wall with its far end supported by a cable that makes an angle of 53° with the horizontal. If a person weighing 600N stands 2m from the wall, find the tension and the reaction force at the pivot.

Data:

Length of Beam = $\overrightarrow{AB} = 8m$

Weight of Beam = $W_1 = 200\ N$

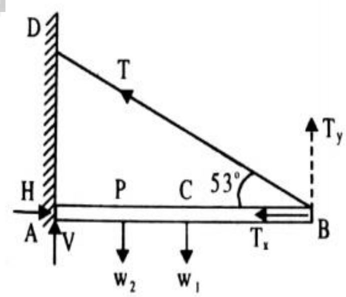
Weight of Person = $W_2 = 600\ N$

Distance of Person from wall = $\overrightarrow{AP} = 2m$

Angle = = $\theta = 53°$

Tension = = T =?

Reaction force at pivot = = R =?



Solution:

Using first condition of equilibrium

$\sum F_x = 0 ; \sum F_y = 0$

$H = T_x$

$H = T\cos\theta$

$H = T\cos 53°$

$H = 0.6\,T$ --- (i)

$V + T_y = W_1 + W_2$

$V + T\sin\theta = 200 + 600$

$V + T\sin 53 = 800$

$V + 0.798\,T = 800$ ---(ii)

Using 2nd condition of equilibrium

$\sum \tau = 0$

Taking Moment about A where Tx is parallel to moment arm so its torque is zero.

$$T_y \times \overrightarrow{AB} + H \times 0 + V \times 0 - W_1 \times \overrightarrow{AC} - W_2 \times \overrightarrow{AP} = 0$$

$$0.798\,T \times 8 - 200 \times \frac{\overrightarrow{AB}}{2} - 600 \times 2 = 0$$

$$6.4\,T - 200 \times 4 - 600 \times 2 = 0$$

$$6.4\,T \times 8 - 2000 = 0$$

$$\boxed{T = \frac{2000}{6.4} = 312.5\,N}$$

For H Putting in eq(i)

$H = 0.6 \times 312.5$

$\boxed{H = 187.5\,N}$

For V Putting in eq(ii)

$V + 0.798 \times 312.5 = 800$

$\boxed{V = 553.4\,N}$

Now,Reaction force at pivot is given by

$R = \sqrt{H^2 + V^2}$

$R = \sqrt{(187.5)^2 + (553.4)^2}$

$\boxed{R = 584.3\,N}$

<u>Result:</u> The tension in the scable is 312.5 Nand the reaction force at the pivot is 584.3 N

PAST PAPER NUMERICALS

<u>2022</u> Q.2 (ix) Textbook Numerical 5

<u>2019</u> Q.2 (ix) Textbook Numerical 8

<u>2018, 2017</u> No Numerical

<u>2016</u>

Q.2 (iv) Textbook Numerical 8

2015 No Numerical

<u>2014</u>

Q.2(i) A particle of mass 500 gm rotates in a circular orbit of radius 25 cm at a constant rate of 1.5 revolutions per second. Find the angular momentum with respect to centre of the orbit .

<u>Data:</u>

Mass of particle =500 gm = 0.5 kg

Radius of circular orbit =25 cm = 0.25 m

Angular velocity = 1.5 rev / sec = 1.5 × 2π = $3\pi\ rad\ /\ sec$

Angular momentum =?

<u>Solution:</u>

As we know that

$L = m\,v\,r$

Or $L = m\,(r\omega)\,r = mr^2\omega$

$L = (0.5)(0.25)^2(3 \times 3.14)$

$\boxed{L = 0.294\,kg\,m^2/s}$

<u>Result:</u> The angular momentum of particle is $0.294\ kg\frac{m^2}{s}$.

<u>2013</u>

No Numerical

2012

Q.2 (xi) A particle of mass 0.5 kg moves along xy-plane. At that instant, the coordinates are (3, 4)m and its velocity is (4i +5j) m/sec. Determine the angular momentum relative to origin at that time.

Data:

Mass of particle = m = 0.5 kg

Co-ordinates of position = $\vec{r}$ = (3, 4) =3i + 4j

Velocity of particle = $\vec{v}$ = 4i + 5 j

Angular momentum of particle = $\vec{l}$ =?

Solution:

According to the def. of Angular Momentum

$$\vec{l} = m(\vec{r} \times \vec{v}) \text{ ---- (i)}$$

$$\vec{r} \times \vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 4 & 0 \\ 4 & 5 & 0 \end{vmatrix}$$

$$\vec{r} \times \vec{v} = \hat{\imath}\begin{vmatrix} 4 & 0 \\ 5 & 0 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 3 & 0 \\ 4 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} 3 & 4 \\ 4 & 5 \end{vmatrix}$$

$$\vec{r} \times \vec{v} = \hat{\imath}(0-0) - \hat{\jmath}(0-0) + \hat{k}(15-16)$$

$$\boxed{\vec{r} \times \vec{v} = -\hat{k}}$$

Putting in eq (i)

$$\vec{l} = 0.5 \times (-\hat{k})$$

Or $$\boxed{\vec{l} = -0.5\,\hat{k}}$$

Result: The Magnitude of angular momentum is 0.5 j.s and direction is along –ve z axis.

2011

Q.2 (x) **Textbook Numerical 8**

2010

Q.2 (xv) **Textbook Numerical 8**

THEORY NOTES

NEWTON'S LAW OF GRAVITATION:

STATEMENTS:

This is law that everybody in this universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers."

EXPLANATION:

Consider tow bodies of masses 'm_1' and 'm_2' with their centers distance 'r' apart.
The magnitude of force of attraction between them is,

$F \propto m_1 . m_2 \longrightarrow (1)$

Also $F \propto \frac{1}{r^2} \longrightarrow (2)$

Combining these factors,

$F \propto m_1 . m_2 / r^2$

Or $F = G\, m_1 . m_2 / r^2 \longrightarrow (3)$

Where 'G' is a constant of proportionality and is called the Universal Gravitational Constant. In S. I system, its value is $6.673\ 10^{-11}$ $N.m^2\ Kg^{-2}$

The gravitational force between two bodies form a pair of action and reaction forces. The body of mass 'm_1' attracts the body of mass 'm_2' by a force $\vec{F}_{12}$, while the body of mass 'm_2' is attracted by the body of mass 'm_1' with a force $\vec{F}_{21}$. These forces are equal and opposite thus,

$\vec{F}_{12} = \vec{F}_{21}$

MASS OF EARTH:

Consider a body of mass 'm' placed on the surface of the earth. Let the mass of the earth is 'M_e' and radius of the earth is 'R_e' .Neglecting the radius of body if compared with that of the earth.

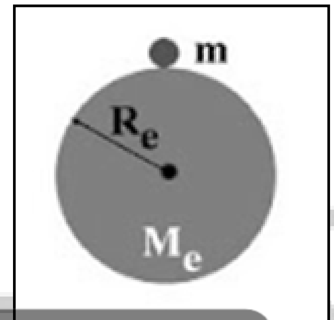
Gravitational force of attraction between earth and body is

$$F = G\, m\, M_e / R_e^2 \quad \text{.......}(1)$$

We know that the force of attraction of the earth on a body is equal to weight the weight of body. i.e

$$F = W \quad \text{......................}(2)$$

Therefore Comparing (1) and (2), we get,



$$W = G\, m\, M_e / R_e^2$$

But $W = mg$

$$mg = G\, m\, M_e / R_e^2$$

or $g = G\, M_e / R_e^2$

or $M_e = g \times R_e^2 / G$

From astronomical data:

$g = 9.8\ m/s^2$

$R_e = 6.38 \times 10^6\ m$

$G = 6.67 \times 10^{-11}\ N\text{-}m^2/kg^2$

Putting these values in the above equation.

$$M_e = 9.8\,(6.38 \times 10^6)^2 / 6.67 \times 10^{-11}$$

or

$$\boxed{M_e = 5.98 \times 10^{24}\ Kg}$$

DENSITY OF EARTH:

Let the average density be 'ρ' of the earth, which is given by,

$$\rho = \frac{M_e}{v_e} \qquad \left(\rho = \frac{Mass}{Volume}\right)$$

Since,

$$V_e = \frac{4}{3}\pi R_e^{\,3}$$

Thus,

$$\rho = \frac{M_e}{\frac{4}{3}\pi R_e^{\,3}}$$

$$\rho = \frac{5.98 \times 10^{24}}{\frac{4}{3} \times 3.14 \times (6.38 \times 10^{6})^{3}}$$

$$\boxed{\rho = 5.5 \text{ x } 10^{3} \text{ Kg / m}^{3}}$$

VARIATION OF 'g' WITH ALTITUDE:

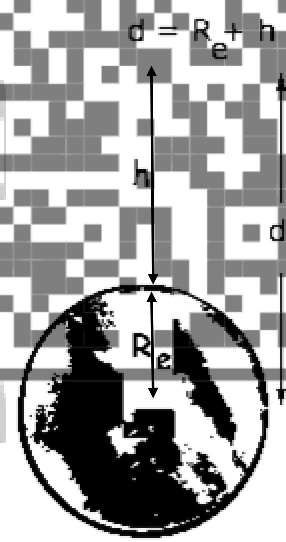
We know that value of "g" at the surface of earth is given by

$$g = \frac{GM_e}{{R_e}^2} \text{-------(i)}$$

If the earth be considered as a sphere of homogenous composition then 'g' any point above its surface will vary inversely as the square of the distance from that point to its centre, which is as below. At a distance (R_e+h),

$$g' = \frac{GM_e}{(R_e+h)^2} \text{---------(ii)}$$

Dividing eq (i) by (ii)



$$\frac{g}{g'} = \frac{\frac{GM_e}{{R_e}^2}}{\frac{GM_e}{(R_e+h)^2}}$$

$$\frac{g}{g'} = \frac{(R_e+h)^2}{{R_e}^2}$$

$$\frac{g}{g'} = \left(\frac{R_e+h}{R_e}\right)^2$$

$$\frac{g}{g'} = \left(\frac{R_e}{R_e} + \frac{h}{R_e}\right)^2$$

$$\frac{g}{g'} = \left(1 + \frac{h}{R_e}\right)^2$$

or
$$\frac{g'}{g} = \left(1 + \frac{h}{R_e}\right)^{-2}$$

Expanding by binomial expansion, $(1+x)^{-n} = 1 - \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} - \cdots$

$$\left(1 + \frac{h}{R_e}\right)^{-2} = 1 - \frac{2\frac{h}{R_e}}{1} + \frac{2(2-1)\frac{h}{R_e}^{2}}{2 \times 1} - \cdots$$

Neglecting higher powers of $\frac{h}{R_e}$

$$\frac{g'}{g} = 1 - \frac{2h}{R_e}$$

$$g' = g\left(1 - \frac{2h}{R_e}\right)$$

From this equation, we conclude that the greater the value of 'h' the smaller is the value of 'g' or ' value of 'g' decreases with altitude".

For example at h 16000m, g = 9.757 m/sec^2

VARIATION OF 'g' WITH DEPTH:

Let g' be the acceleration due to gravity at a depth 'd' below the surface of earth, that is at a distance (R_e-d) from the center or earth.

Where 'ρ' is the density of the earth which is supposed to be uniform everywhere,

M_e = V x ρ

M_e = $\frac{4}{3}\rho\pi R_e^{\,3}$

The value of "g" at the surface of earth is given by ,

$$g = \frac{GM_e}{R_e^{\,2}}$$

Putting the value of M_e

$$g = \frac{G\frac{4}{3}\rho\pi R_e^{\,3}}{R_e^{\,2}}$$

$$g = G\frac{4}{3}\rho\pi R_e \text{ ----------(i)}$$

And at depth 'd' the value of "g" is given by

$$g' = G\frac{4}{3}\rho\pi(R_e - d) \text{ ----------(ii)}$$

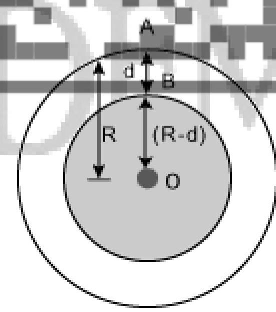
Dividing eq(ii) by eq(i)

$$\frac{g'}{g} = \frac{G\frac{4}{3}\rho\pi(R_e - d)}{G\frac{4}{3}\rho\pi R_e}$$

$$\frac{g'}{g} = \frac{R_e - d}{R_e}$$

$$\frac{g'}{g} = \frac{R_e}{R_e} - \frac{d}{R_e}$$

$$\frac{g'}{g} = 1 - \frac{d}{R_e}$$



or $$g' = g\left(1 - \frac{d}{R_e}\right)$$

That is value of 'g' decreases with depth from surface of the earth.

For example, At d = 4000 Km, g = 8 m/sec^2

WEIGHTLESSNESS

DEFINITION:

"The condition in which the apparent weight of the body becomes zero then it is said to be in state of weightlessness".

EXPLANATION:

To discuss weightlessness in artificial satellites, let us take the example of an elevator having a block of mass (m) suspended by a spring balance attached to the coiling of the elevator. The tension in the thread indicates the weight of the block. Consider following cases.

1. When Elevator is at Rest

$$\boxed{T = m\,g}$$

2. When Elevator is Ascending with an Acceleration 'a'

In this case

$$T > m\,g$$

Therefore, Net force = T - mg

$$m\,a = T - m\,g$$

$$\boxed{T = m\,g + m\,a}$$

In this case of the block appears "heavier".

3. When Elevator is Descending with Acceleration 'a'

In this case

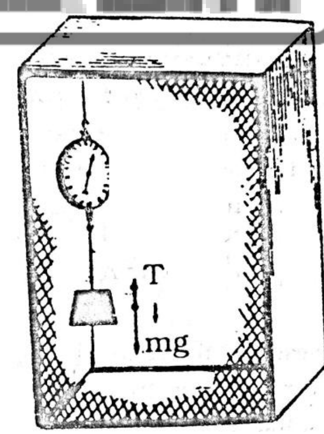
$$m\,g > T$$

Therefore

Net force = m g - T

$$m\,a = m\,g - T$$

$$\boxed{T = m\,g - m\,a} \quad \text{--------(i)}$$



In this case, the body appears lighter

4. When the Elevator is falling freely Under the Action of Gravity

If the cable supporting the elevator breaks, the elevator will fall down with acceleration equal to 'g'

From (i) => $T = m g - m a$

But $a = g$

Therefore

$$T = m g - m g$$

$$\boxed{T = 0}$$

In this case, spring balance will read zero. This is the state of "weightlessness".

In this case gravitation force still acts on the block due to the reason that elevator block, spring balance and string all have same acceleration when they fall freely, the weight of the block appears zero.

<u>WEIGHTLESS NESS IN SATELLITES:</u>

Similarly in case of satellite orbiting around earth, a body experiences gravitational acceleration "g" but also feels an acceleration a_c, that is centripetal acceleration towards the center of earth which equal to the value of "g".So the apparent weight is given by

Eq(i) => $T = m g - m a$

or $T = m g - m a_c$

$T = m g - m g \qquad (a_c = g)$

$$\boxed{T = 0}$$

Therefore the astronaut or body inside satellite experiences weightlessness.

ARTIFICIAL GRAVITY

INTRODUCTION:

All orbiting satellites along with their astronauts and other objects are in a state of free fall and consequently will be in a state of weightlessness. Weightlessness in space craft is highly inconvenient to an astronaut in a number of ways.

For example he cannot pour liquid into a glass, neither he can drink properly. In order to overcome this problem, artificial gravity is produced in the spacecrafts.

EXPLANATION:

In order to produce an artificial gravity in the space craft, the laboratory of space craft is rotated with suitable frequency about its own axis. The rotation is so maintained that the astronaut do not feel weightlessness. The frequency of rotation depends on the length of laboratory of space craft.

Consider a space craft whose laboratory is 'L' meter long consisting of two chambers connected by a tunnel. Let us see how many revolutions per second must the space craft make in order to supply artificial gravity for the astronauts.

Let 'T' be the time for one revolution and 'f' be the frequency of rotation.

When the laboratory revolves, a centripetal accelaration is experienced by the astronauts.

$a_c = \frac{v^2}{r}$

Where a_c is the centripetal acceleration

Since radius of laboratory is R , therefore,

$$a_c = \frac{v^2}{R} \text{ --------(iv)}$$

Now we will determine the linear speed of the laboratory.

As we know that $V = R\omega$

$$a_c = \frac{(R\omega)^2}{R}$$

Now, $\omega = 2\pi f$

so,

$$a_c = \frac{R^2(2\pi f)^2}{R}$$

$$a_c = 4\pi^2 f^2 R$$

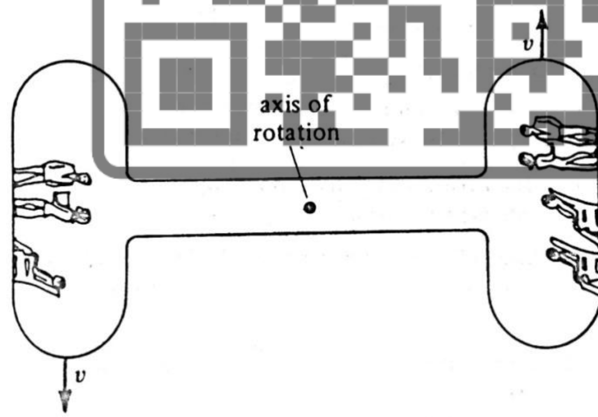
or $$f^2 = \frac{a_c}{4\pi^2 R}$$

Taking square roots on both sides

$$f = \sqrt{\frac{a_c}{4\pi^2 R}}$$

or

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{a_c}{R}}}$$



Let, R=10 m and a_c=g=9.8 m/s^2

now, f becomes => $f = \frac{1}{2(3.14)}\sqrt{\frac{9.8}{10}}$

$$f = 0.158\ rev/sec$$

or $$f = 9.5\ rev/min$$

So, in order to produce artificial gravity on a satellite of length 20m it should be rotated at a rate of 9.5 rev/min.

M.C.Qs.

1. The force of gravitation acts along the.
(a) axis of rotation.
(b) Line joining the interacting bodies.
(c) Line perpendicular to the interacting
(d) None of these

2. According to the law of universal Gravitation:
(a) Everybody in the universes attracts everybody.
(b) The force of attraction is directly proportional to the product of their masses
(c) The force of attraction is inversely proportional to the squire of their distance.
(d) All of the above

3. Force of gravitational attraction of earth on other bodies is given by:
(a) $F = \frac{GM_e m}{r^2}$ (b) $F = \frac{GM_e}{r^2}$
(c) $F = \frac{Gm}{r^2}$ (d) $F = \frac{M_e m}{r^2}$

4. The force of attraction or repulsion between two bodies is:
(a) Inversely proportional to the distance
(b) Directly proportional to the distance
(c) Inversely proportional to the square of the distance (d) None of the above

5. A hole is drilled through the earth along the diameter and a stone is dropped into it. When the stone is at the centre of the earth it has:
(a)Mass (b) Weight
(c) Acceleration (d) Both a and b

6. The acceleration due to gravity on moon is 1/6th of that on earth, what will be the mass of the body on moon, if its mass on earth is m:
(a) m/6 (b) 6 m
(c) m (d) m/3

7. The value of 'g' at the centre of earth is:
(a) Maximum (b) Minimum
(c) Zero (d) None of them

8. The value of g at a certain height above the earth is:
(a) Nearly the same as an the surface of earth
(b) Nearly the same as an the center of earth
(c) Estimated to decreases with altitude
(d) Estimated to depend on the variation of the earth radius

9. If the mass of the earth becomes four times large, the value of g will:
(a) remain unchanged
(b) Becomes four times larger
(c) Be double
(d) sixteen times larger

10. When a lift is moving upward with a uniform velocity, the apparent weight of a body inside the lift will be:
(a) Equal to its actual weight
(b) Less than the actual weight
(c) More than the actual weight
(d) Zero

11. Artificial gravity can be created in the space craft by:
(a) Revolving it around the earth
(b) Spinning it around its own axis
(c) Increasing its velocity
(d) decreasing its velocity

12. The gravitational constant was determined

experimentally by:

(a) Newton (b) Einstein
(c) Cavendish (d) Maxwell

13. If man a goes above the earth's surface to a distance equal to thrice the earth's radius, the value of acceleration due to gravity at that point becomes:

(a) 1/3 g (b) 1/4 g (c) 1/9 g (d) 1/16 g

14. The approximate value of the average density of the earth is:

(a) 5.5×10^3 kg /m³ (b) 6.5×10^3 kg /m³
(c) 7.5×10^3 kg /m³ (d) 8.673×10^3 kg /m³

15. If a planet existed whose mass and radius were both twice that of the earth, then acceleration due to gravity at its surface would be:

(a) 4.9 m/s^2 (b) 19.6 m/s^2
(c) 2.45 m/s^2 (d) same as that of earth

16. When the space ship is at a distance equal to twice of the earth's radius from its centre then the gravitational acceleration is:

(a) 4.9 m/s^2 (b) 19.6 m/s^2
(c) 2.45 m/s^2 (d) 9.8 m/s^2

17. If both masses of two bodies and distance between them are doubled then the gravitational force between them will be:

(a) doubled (b) halved
(c) four times (d) remain same

18. If radius of earth is doubled then the value of gravitational acceleration will be:

(a) doubled (b) halved
(c) four times (d) one fourth

19. Unit of gravitational constant is:

(a) m/s^2 (b) $N\,m^2/kg^2$
(c) N (d) $N\,kg^2/m^2$

20. The value of 'g' for an object falling on any planet is independent of which of the following quantities:

(a) Mass of planet (b) Radius of planet
(c) Mass of falling object (d) all of these

21. If the radius of the Earth was to shrink and their masses were to remains the same, the acceleration due to gravity on the surface of Earth will

(a) Decreases (b) Remains same
(c) Increases (d) None

22. Newton's law does not hold good for particles

(a) moving slowly (b) at rest
(c) moving with high velocity (d) moving with velocity comparable to velocity of light

23. At what depth, the weight of the body becomes three fourth, as compared to the body weight on the surface of earth.

(a) d=R/2 (b) d=R/4
(c) d=2R/3 (d) d=4R/5

24. The distance from the centre of the earth at which the gravitational acceleration becomes one half the value that it has on the earth's surface is calculated as:

(a) 14.1 earth's radius (b) 141.1 earth's radius
(c) 1.41 earth's radius (d) 1.5 earth's radius

25. If distance between two masses is halved, then the force of attraction between them will be

(a) four times (b) doubled
(c) reduced to 1 /2 (d) reduced to 1 /4

PAST PAPER M.C.Qs.

2022

9. The weight of a man is 600 N at the earth, his weight on the moon, where $g_m = g/6$ will be:
*3600N *600N *300N **<u>*100N</u>**

28.If elevator moves upward with the acceleration a , the apparent weight of body of mass m is:
* mg-ma * mg **<u>* mg+ma</u>** * zero

2021

(ix) If the radius of the earth were to shrink by 1% while its mass remains same, the acceleration due to gravity on the earth surface would:
* Decrease **<u>* Increase</u>** *Remain the same *become half

(xxxvi) The value of the gravitational constant (G) was first determined experimentally by:
*Newton **<u>*Cavendish</u>** * Einstein *Maxwell

2019

7. The gravitational constant was determined experimentally by:
*Newton *Einstein **<u>*Cavendish</u>** *Maxwell

2018

1.Artificial gravity in spacecraft can be created by:
*translatory Motion *vibratory Motion **<u>*spin motion</u>** *orbital motion

2017

7. If man a goes above the earth's surface to a distance equal to thrice the earth's radius, the value of acceleration due to gravity at that point becomes:
*1/3 g *1/4 g *1/9 g **<u>*1/16 g</u>**

2016

4. At a distance, equal to twice of the radius of the earth above the surface of the earth, the value of gravitational acceleration will be:
*one half *one fourth *Four times **<u>*one ninth</u>**

2015

8. The value of gravitational constant 'G' was determined experimentally by:
<u>*Cavendish</u> *Newton *Joules *Huygen

2014

1. If a person ascends form the surface of the earth to a distance equal to the radius of the earth, the value of g will be:
* 1/2 g **<u>* 1/4g</u>** *2g *4g

11. Artificial gravity can be created in the spaceship by producing:
*translatory motion *vibratory motion **<u>*spin motion</u>** *orbital motion

2013

8. The weight of a man is 600 N at the earth, his weight on the moon, where $g_m = g/6$ will be:

*3600N   *600N   *300N   ***100N**

2012

8. At a distance equal to the radius of earth above the earth's surface, the value of gravitational acceleration becomes:

*half   *** one forth**   * double   * four times

2011

17. If one moves up from the surface of earth to a distance equal to the radius of the earth the value of acceleration due to gravity will be:

*1/2 g   ***1/4 g**   *2 g   *4 g

2010

1. If we go up from the surface of earth to a distance equal to the radius of the earth the value of acceleration due to gravity will be:

***one fourth**   * one eighth   * one ninth   * double

TEXTBOOK NUMERICALS

Q.1: A 10 kg mass is at a distance of 1 m from a 100 kg mass. Find the gravitational force of attraction when (i) 10 kg mass exerts force on the 100 kg mass (ii) 100 kg mass exerts force on the 10 kg mass

Data:

First Mass $= m_1 = 10\ kg$

Second Mass $= m_2 = 100\ kg$

Distance between masses $= r = 1\ m$

Force of m_1 onto $m_2 = F_{12} = ?$

Force of m_2 onto $m_1 = F_{21} = ?$

Solution:

According to Newton's law of Gravitation

$$F = G\frac{m_1 m_2}{r^2}$$

Then,

Force of m_1 onto m_2:

$$F_{12} = G\frac{m_1 m_2}{r^2}$$

$$F_{12} = 6.67 \times 10^{-11}\frac{10 \times 100}{(1)^2}$$

$$\boxed{F_{12} = 6.67 \times 10^{-8} N}$$

Force of m_2 onto m_1:

$$F_{21} = G\frac{m_2 m_1}{r^2}$$

$$F_{21} = 6.67 \times 10^{-11}\frac{100 \times 10}{(1)^2}$$

$$\boxed{F_{21} = 6.67 \times 10^{-8} N}$$

Result: The force of m_1 onto m_2 and m_2 onto m_1 is equal to $6.67 \times 10^{-8} N$.

Q.2: Compute gravitational acceleration at the surface of the planet Jupiter which has a diameter as 11 times as compared with that of the earth and a mass equal to 318 times that of earth.

Data:

$Diameter\ of\ Jupiter = D_j = 11 \times\ D_E$

$Mass\ of\ Jupiter = M_j = 318 \times\ M_E$

$Value\ of\ g\ on\ Jupiter = g_j = ?$

Solution:

Since

$$Diameter\ of\ Jupiter = D_j = 11 \times\ D_E$$

Dividing by "2" O.B.S.

$$\frac{D_j}{2} = 11 \times \frac{D_E}{2}$$

$$Radius\ of\ Jupiter = R_j = 11 \times\ R_E$$

The acceleration due to gravity is given by

$$g = \frac{GM_E}{{R_E}^2}$$

On the surface of moon

$$g_j = \frac{GM_j}{{R_j}^2}$$

$$g_j = \frac{G(318 \times M_E)}{(11 \times R_E)^2}$$

$$g_j = \frac{(318\ )}{(11\ )^2}\left(\frac{GM_E}{{R_E}^2}\right)$$

$$g_j = \frac{(318\ )}{121}\left(\frac{GM_E}{{R_E}^2}\right)$$

$$g_j = 2.62\ (g)$$

$$g_j = 2.62 \times 9.8$$

$$\boxed{g_j = 25.6\ m/s^2}$$

<u>Result:</u> The value of g on the surface of Jupiter is $25.6\frac{m}{s^2}$.

Q.3: The mass of the planet Jupiter 1.9 x 10^27 kg and that of the sun is 2.0 x 10^30 kg. If the average distance between them is 7.8 x 10^11 m, find the gravitational force of the sun on Jupiter.

<u>Data:</u>

Mass of Jupiter $= m_1 = 1.9 \times 10^{27}\ kg$

Mass of Sun $= m_2 = 2 \times 10^{30}\ kg$

Average Distance $= r = 7.8 \times 10^{11} m$

Force of Sun on jupiter $= F =?$

<u>Solution:</u>

According to Newton's law of Gravitation

$$F = G\frac{m_1 m_2}{r^2}$$

Then,

$$F_{12} = 6.67 \times 10^{-11}\frac{1.9 \times 10^{27} \times 2 \times 10^{30}}{(7.8 \times 10^{11})^2}$$

$$\boxed{F_{12} = 6.24 \times 10^{33}\ N}$$

<u>Result:</u> The force of Sun on Jupiter is equal to $6.24 \times 10^{33}\ N$.

Q.4: The radius of the moon is 27% of the earth's radius and its mass is 1.2% of the earth's mass. Find the acceleration due to gravity on the surface of the moon. How much will a 424N body weight there?

<u>Data:</u>

$$Radius\ of\ moon = R_m = 27\%\ of\ R_E$$

$$= \frac{27}{100}\ R_E = 0.27\ R_E$$

$$Mass\ of\ moon = M_m = 1.2\%\ of\ M_E$$

$$= \frac{1.2}{100}\ M_E = 0.012\ M_E$$

$Value\ of\ g\ on\ moon = g_m = ?$

$Weight\ of\ body = W = 424\ N$

$Weight\ of\ body\ on\ moon = W_m = ?$

<u>Solution:</u>

The acceleration due to gravity is given by

$$g = \frac{GM_E}{{R_E}^2}$$

On the surface of moon

$$g_m = \frac{GM_m}{{R_m}^2}$$

$$g_m = \frac{G(0.012\ M_E)}{(0.27\ R_E)^2}$$

$$g_m = \frac{(0.012\ )}{(0.27\ )^2}\left(\frac{GM_E}{{R_E}^2}\right)$$

$$g_m = \frac{(0.012\ )}{(0.27\ )^2}\left(\frac{GM_E}{{R_E}^2}\right)$$

$$g_m = 0.165\ (g)$$

$$g_m = 0.165 \times 9.8$$

$$\boxed{g_m = 1.62\ m/s^2}$$

The weight of body is given by

$$W = mg \text{----(i)}$$

The weight of body on moon given by

$$W_m = mg_m \text{----(ii)}$$

Dividing eq (ii) by eq (i)

$$\frac{W_m}{W} = \frac{mg_m}{mg}$$

$$W_m = \frac{W \times g_m}{g} = \frac{424 \times 1.62}{9.8}$$

$$\boxed{W_m = 70\ N}$$

<u>Result:</u> The value of g on the surface of moon is

$1.62 \frac{m}{s^2}$ and weight of the body will be 70 N.

Q.5: What is the value of the gravitational acceleration at a distance of (i) earth's radius above the earth's surface? (ii) Twice earth's radius above the earth's surface?

Data:

Value of "g" at height $= g' = ?$

(a) Distance from centre of earth $= R_e' = 2R_e$

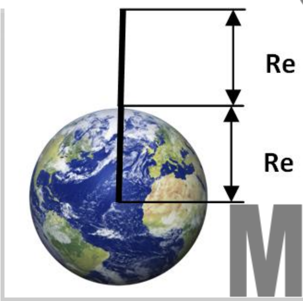
(b) Distance from centre of earth $= R_e' = 3R_e$

Solution:

As we know that

$$g = \frac{GM_e}{R_e^2}$$

(a) When distance from centre of earth = $R_e' = 2R_e$



$$g' = \frac{GM_e}{R_e'^2}$$

$$g' = \frac{GM_e}{(2R_e)^2}$$

or $$g' = \frac{1}{4}\frac{GM_e}{R_e^2}$$

Since

$$g = \frac{GM_e}{R_e^2}$$

Therefore

$$g' = \frac{1}{4} \times g$$

$$g' = \frac{1}{4} \times 9.8$$

$$\boxed{g' = 2.45\ m/s^2}$$

(b) When distance from centre of earth = $R_e' = 3R_e$

$$g' = \frac{GM_e}{R_e'^2}$$

$$g' = \frac{GM_e}{(3R_e)^2}$$

or $$g' = \frac{1}{9}\frac{GM_e}{R_e^2}$$

Since

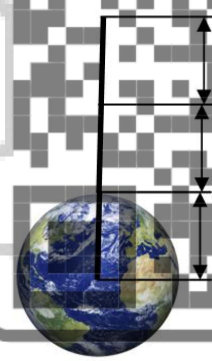
$$g = \frac{GM_e}{R_e^2}$$

Therefore

$$g' = \frac{1}{9} \times g$$

$$g' = \frac{1}{9} \times 9.8$$

$$\boxed{g' = 1.08\ m/s^2}$$



<u>**Result:**</u> (a) When distance from centre of earth = $R_e' = 2R_e$ value of "g" is 2.45 m/s^2 (a) When distance from centre of earth $= R_e' = 3R_e$ value of "g" is 1.08 m/s^2

Q.6: At what distance from the center of the earth does the gravitational acceleration have one half the value that it has on the earth's surface?

Data:

Distance from centre of earth = Re'=?

Value of 'g' at Re' = g' = (1/2) g

Solution:

As we know that

$$g = \frac{GM_e}{R_e^2}$$

$$R_e^2 = \frac{GM_e}{g} \quad \text{---- (i)}$$

At Distance Re' from centre of earth

$$g' = \frac{GM_e}{R_e'^2}$$

$$\frac{1}{2}g = \frac{GM_e}{R_e'^2}$$

$$R_e'^2 = \frac{2GM_e}{g}$$

Or $$R_e'^2 = 2R_e^2$$

Taking Sq.root O.B.S

$$\boxed{R_e'^2 = \sqrt{2}\,R_e = 1.41 \times 6.38 \times 10^6 = 9.022 \times 10^6 m}$$

Result:

At $9.022 \times 10^6\ m$ the value of g becomes one hlaf of the value of g on the surface.

Q.7: Compute the gravitational attraction between two college students of mass 80 & 50 kg respectively, 2m apart from each other. Is this force worth worrying about?

Data:

Mass of 1st student $= m_1 = 80\ kg$

Mass of Second Student $= m_2 = 50\ kg$

Distance b/w students $= r = 2\ m$

Force b/w Students $= F = ?$

Solution:

According to Newton's law of Gravitation

$$F = G\frac{m_1 m_2}{r^2}$$

Then,

$$F_{12} = 6.67 \times 10^{-11}\frac{80 \times 50}{(2)^2}$$

$$\boxed{F_{12} = 6.67 \times 10^{-8} N}$$

Result: The force between students is equal to $6.67 \times 10^{-8} N$ and since this force is negligible therefore this force is not at all worth worying about.

Q.8: Determine the gravitation between the proton and the electron in a hydrogen atom, assuming that the electron describes a circular orbit with a radius of 0.53x10-10 m (mass of proton = 1.67x10-27 kg, mass of electron = 9.1x10-31 kg).

Data:

Mass of Proton $= m_1 = 1.67 \times 10^{-27}\ kg$

Mass of Electron $= m_2 = 9.1 \times 10^{-31}\ kg$

Average Distance $= r = 0.53 \times 10^{-10} m$

Force Between Proton and Electron $= F = ?$

Solution:

According to Newton's law of Gravitation

$$F = G\frac{m_1 m_2}{r^2}$$

then,

$$F_{12} = 6.67 \times 10^{-11}\frac{1.67\times10^{-27}\times9.1\times10^{-31}}{(0.53\times10^{-10})^2}$$

$$\boxed{F_{12} = 3.6 \times 10^{-47} N}$$

Result: The force between Proton and Electron is equal to $3.6 \times 10^{-47}\ N$.

Q.9: A woman with a mass of 45 kg is standing on a scale in an elevator. The elevator accelerates with a constant acceleration of 1.2 m/s² . What is the women's weight as measured by her in the elevator.

Data:

Mass of Woman = m = 45 kg

Apparent Weight of Women = W_{app} = ?

Acceleration of elevator = a = 1.2 m/s^2

Solution:

When elevator is moving upward then its acceleration is given by

$$W_{app} = m(a + g)$$

$$W_{app} = 45(1.2 + 9.8)$$

$$\boxed{W_{app} = 495\ N}$$

Result: The women's weight as measured by her in the elevator is 495 N.

vii) At what height from center of the earth, the value of g becomes 36% of its value on the surface of earth?

Data:

Distance from centre of earth = Re' = ?

Value of 'g' at Re' = g' = $\frac{36}{100}$ g

Solution:

As we know that

$$g = \frac{GM_e}{R_e^2}$$

$$R_e^2 = \frac{GM_e}{g} \quad \text{---- (i)}$$

At Distance Re' from centre of earth

$$g' = \frac{GM_e}{R_e'^2}$$

$$\frac{36}{100}\, g = \frac{GM_e}{R_e'^2}$$

$$R_e'^2 = \frac{100 \times GM_e}{36 \times g}$$

Or $$R_e'^2 = \frac{25}{9}\, R_e^2$$

Taking Sq.root O.B.S

$$R_e'^2 = \frac{5}{3}\, R_e = 1.66 \times 6.38 \times 10^6 = 10.5 \times 10^6 m$$

Result: At $10.5 \times 10^6\ m$ the value of g becomes 36%of the value of g on the surface

2021

Q.2 (vii) Calculate the centripetal force acting on a man whose mass is 64kg when resting on the ground at the equator. (Radius of the earth = 6.4 x10^6 m.)

Data:

Mass of man= m =64 kg

Radius of Earth = Re = 6.4 x 10^6 m

Centripetal Force = F_c =?

Solution:

The centripetal Acceleration is given by

$$a_c = R\omega^2$$

And

$$\omega = \frac{2\pi}{T}$$

So,

$$a_c = R\left(\frac{2\pi}{T}\right)^2 = \frac{6.4 \text{ x } 10^6 \times 4 \times 3.14^2}{(86400)^2}$$

$$a_c = 0.034\ m/s^2$$

Now, Centripetal force is given by

$$F_c = ma_c = 64 \times 0.034 = 2.17\ N$$

Result: the centripetal acceleration is $0.034\ m/s^2$ and centripetal force is $2.17\ N$.

2019

Q.2(vi) How many times in a second does a spaceship of diameter 30 m, needed to be rotated in order to create artificial gravity?

Data:

Frequency= f = ?

Diameter = d = 30 m

Radius = d/2 = 30/2 =15 m

Solution:

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{r}}$$

$$f = \frac{1}{(2 \times 3.14)}\sqrt{\frac{9.8}{15}}$$

$$f = 0.128\ rev/sec$$

Result: The spaceship must be rotated 0.128 rev per second.

2018

Q.2(xi) A man weighing 60 kg is standing on the floor of an elevator. Calculate the force exerted by the man when the elevator is ascending at a rate of 2 m/s^2.

Data:

Mass of person = m = 60 kg

Force exerted by the person = F = ?

Acceleration of elevator = a = 2m/s^2

Solution:

When elevator is moving upward then its acceleration is given by

$$F = m(a+g)$$

$$F = 60(2+9.8)$$

$$\boxed{F = 708\ N}$$

Result: The force exerted by the man when the elevator is ascending is $708\ N$.

2017

Q.2 (iv) The ratio of acceleration due to gravity inside a deep mine to that on the surface of earth is 0.99.Find the depth of the mine, assuming that the density of earth is uniform. Take the radius of earth as $6.38\times10^6 m$.

Data:

Acceleration due to gravity inside a deep mine = g'

Acceleration due to gravity on the surface of earth = g

$$\frac{g'}{g} = 0.99$$

Radius of earth $= R_E = 6.38\times10^6 m$

Solution:

The acceleration due to gravity at a depth is given by

$$g' = g\left(1-\frac{d}{R_E}\right)$$

Or we can write as

$$\frac{g'}{g} = \left(1-\frac{d}{R_E}\right)$$

$$0.99 = \left(1-\frac{d}{6.38\times10^6}\right)$$

$$1-0.99 = \frac{d}{6.38\times10^6}$$

$$0.01 = \frac{d}{6.38\times10^6}$$

$$d = 0.01\times6.38\times10^6$$

$$\boxed{d = 6.38\times10^4 m}$$

Result: The depth of the mine is $6.38\times10^4 m$.

2016

Q.2 (v) Textbook Numerical 4

2015

Q.2 ii) At what distance from centre of earth does the gravitational acceleration have one third of the value that it has on the earth's surface?

Data:

Distance from centre of earth = Re'=?

Value of 'g' at Re' = g' = (1/3) g

Solution:

As we know that

$$g = \frac{GM_e}{R_e^2}$$

$$R_e^2 = \frac{GM_e}{g} \quad \text{---- (i)}$$

At Distance Re' from centre of earth

$$g' = \frac{GM_e}{R_e'^2}$$

$$\frac{1}{3}g = \frac{GM_e}{R_e'^2}$$

$$R_e'^2 = \frac{3GM_e}{g}$$

Or $$R_e'^2 = 3R_e^2$$

Taking Sq.root O.B.S

$$\boxed{R_e'^2 = \sqrt{3}\,R_e = 1.73\times6.38\times10^6 = 11.03\times10^6 m}$$

Result:

At $11.03\times10^6 m$ the value of g becomes one third of the value of g on the surface.

2014

Q.2 (xiii) Textbook Numerical 4

2013

Q.2 (ii) .The mass of the moon is approximately one eightieth (1/80) of the earth's mass and its radius is one fourth (1/4) that of earth. Find the acceleration due to gravity on the surface of the moon.

Data:

$$Radius\ of\ moon = R_m = (\frac{1}{4})\ R_E$$
$$= 0.25\ R_E$$
$$Mass\ of\ moon = M_m = (\frac{1}{80})\ M_E$$
$$= 0.0125\ M_E$$
$$Value\ of\ g\ on\ moon = g_m = ?$$

Solution:

The acceleration due to gravity is given by

$$g = \frac{GM_E}{{R_E}^2}$$

On the surface of moon

$$g_m = \frac{GM_m}{{R_m}^2}$$
$$g_m = \frac{G(0.0125\ M_E)}{(0.25\ R_E)^2}$$
$$g_m = \frac{(0.0125\ )}{(0.25\ )^2}\left(\frac{GM_E}{{R_E}^2}\right)$$
$$g_m = 0.2\ (g)$$
$$g_m = 0.2 \times 9.8$$
$$\boxed{g_m = 1.96\ m/s^2}$$

Result: The value of g on the surface of moon is $1.96\ \frac{m}{s^2}$

2012

Q.2 (x) **Same as 2019 Q.2(vi)**

2011, 2010 **No Numerical**

THEORY NOTES

WORK

PHYSICAL DEFINITION:

"Work is said to be done if a force causes a displacement in a body in the direction of force".

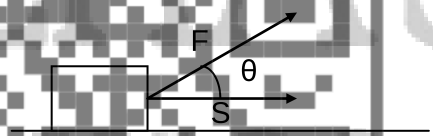
OR

" The work done by a constant force is defined as the product of the component of the force and the displacement in the direction of displacement".

MATHEMATICAL DEFINITION:

" Work is the scalar product or dot product of the force and displacement".

$$W = \vec{F}.\vec{S} = FScos\,\theta = (Fcos\theta)S \text{-------(i)}$$



Where, F= Magnitide of Force

S= Magnitude of Displacement

θ= Angle between $\vec{F}$ *and* $\vec{S}$

or eq(i) can also be written as,

$$W = F(Scos\theta)$$

Where $Scos\theta$ is the component of Displacement in the direction of Force.

DIMENSION AND NATURE:

Work is a scalar quantity and it's dimension is ML^2T^{-2}

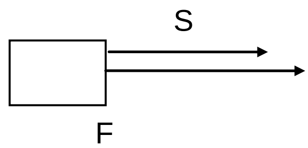
UNITS OF WORK:

- In S.I system: Joule (j)
- In C.G.S. system: Erg
- In F.P.S system: ft X lb

SPECIAL CASES OF WORK :

(i) POSITIVE WORK: if force and displacement are in the same direction , work will be positive or if $\theta > 0$ or $\theta < 90°$

Let $\theta = 0°$



As

Work = FS Cos θ

Work = FS Cos 0°

Work = (F) (S) (1)

Work = FS

(ii) ZERO WORK: if force and displacement are perpendicular to each other, work will be zero. I.e
Since $\theta = 90°$

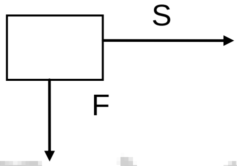
As

Work = FS Cos θ

Work = FS Cos 90°

Work = (F) (S) (0)

Work = 0



iii) NEGATIVE WORK: If force and displacement are in the opposite direction, work will be negative.

Since $\theta = 180°$

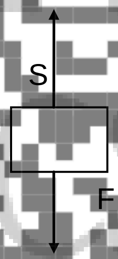
As

Work = FS Cos θ

Work = FS Cos 180°

Work = (F) (S) (-1)

Work = -FS



POWER

DEFINITION:" The rate of work done of a body is called power"

AVERAGE POWER:

Average power of a body doing work is numerically equal to the total work done divided by the time taken to perform the work.

MATHEMATICALLY:

Power = Work done / time

Power = W / Δt

As we know that

$W = \vec{F}.\vec{S}$

Therefore,

$P = \vec{F}.\vec{S} / \Delta t$ -----------(i)

According to the definition of Velocity,

$$v = \vec{S}/\Delta t$$

Therefore,Eq(i)=>

$$\boxed{P = \vec{F}.\vec{v}}$$

or

$$\boxed{P = \vec{F}.\vec{v} = Fv\cos\theta}$$

DIMENSION AND NATURE:

Power is a scalar quantity and its dimension is ML^2T^{-3}

UNITS OF POWER:

1. Watt	[1 watt = 1joule / sec]
2. Kilo watt	[1 Kw = 1000 watt]
3. Mega watt (Mw)	[1Mw = 10^6 watt]
4. Horse Power	[1 Hp = 746]

CONSERVATIVE FIELD

DEFINITION:

A force is said to be conservative if the work done by moving a body along a closed path equals to zero.

FOR EXAMPLE:

1. Gravitational Field 2. Electrostatic Field 3.Magnetic Field

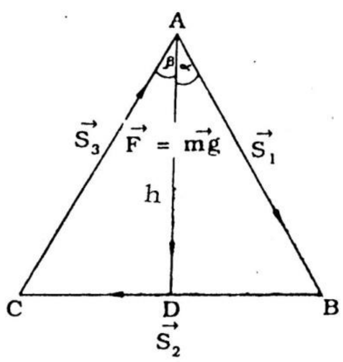
GRAVITATIONAL FIELD IS A CONSERVATIVE FIELD

PROOF:

Suppose a closed path is triangular pat is ABCA in gravitational field as shown in figure

Now we calculate the work in moving a body from A to B,B to C and C to A.

1. WORKDONE FROM A TO B:



$W_{A\to B}$ =F . S_1 =F S_1 Cosα------(i)

In Δ BAD,

Cosα=h/ S_1

or S_1 Cosα=h

Then $W_{A\to B}$ = Fh

Or

$\boxed{W_{A\rightarrow B} = mgh}$ ----------(ii)

2. WORKDONE FROM B TO C:

$W_{B\rightarrow C} = F.S_2 \quad = FS_2 \, Cos90^o$

$W_{B\rightarrow C} = S_2\ (0)$

Or $\boxed{W_{B\rightarrow C} = 0}$ ------------(ii)

3. WORKDONE FROM C TO A:

$W_{C\rightarrow A} = F\ .\ S_3 \quad = F\ S_3\ Cos(180\text{-}\beta)$

$W_{C\rightarrow A} = F\ .\ S_3 \quad = F\ S_3\ Cos(\text{-}\beta)$

$W_{C\rightarrow A} = F\ .\ S_3 \quad = \text{-}F\ S_3\ Cos(\beta)$

In Δ CAD,

$Cos\beta = h\ /\ S_3$

or $S_3\ Cos\beta = h$

Then $W_{C\rightarrow A} = \text{-}\ Fh$

Or

$\boxed{W_{C\rightarrow A} = \text{-}mgh}$

4. TOTAL WORKDONE:

Now the total work done along the path ABCA is

Work = $W_{A\text{->}B} + W_{B\text{->}C} + W_{C\text{->}A}$

W = mgh + 0+ (-mgh)

$\boxed{W = 0}$

Hence it is proved that the gravitational field is conservative field.

ENERGY

DEFINITION:

" The ability of a body to perform work is called Energy". A body cannot perform work if it does not possess energy. A body cannot perform work more than the amount of energy.

DIMENSION AND NATURE:

Energy is a scalar quantity and it's dimension is ML^2T^{-2}

UNITS OF ENERGY:

(i) Joule

(ii) Calorie [1 Calorie =4.2 joule]
(iii) Kilo Watt Hour [1Kwh=3.6 x 10^6 J]

POTENTIAL ENERGY

DEFINITION:

Energy stored by a body due to its position in gravitational field is known as 'Gravitational potential energy'".

FORMULA:

Potential Energy=P.E = mgh

DERIVATION:

Consider a body of mass "m" placed at a height of "h" from the surface earth.

Force = Weight = W

But displacement (S) = h

According o the definition of Work Done

Work done = Fs

Or Work done = Wh [but W = mg]

Work done = mgh

As we know that the energy stored in a body when work is done on it against the gravitational field is known as potential energy .Therefore,

Potential Energy=P.E = mgh

KINETIC ENERGY

DEFINITION:

" Energy posses by a body by virtue of its motion is refused to as ' Kinetic Energy' " .

FORMULA:

K.E = 1 / 2 mv^2

Where m is the mass of body and v is the speed of body.

DERIVATION:

Consider a body of mass "m" starts moving from rest. After a time interval ''t" its speed becomes v. If initial velocity of the body is $V_i = 0$, final velocity $V_f = V$ and the displacement of body is ''S''.
Then
First of all we will find the acceleration of body.
Using 3rd equation of motion

$2aS = Vf^2 - Vi^2$

Putting the above mentioned Values

$2aS = v^2 - 0$

or $a = v^2 / 2S$

Now force is given by

$F = ma$

Putting the value of acceleration

$F = m(v^2 /2S)$

As we know that

Work done = FS

Putting the value of "F"

$\text{Work done} = \left(\frac{mv^2}{2S}\right)(S)$

Work done = $mv^2 / 2$

OR

Work done = $1/2\ mv^2$

Since the ability of doing work by a moving body is called "Kinetic Energy". Therefore,

K.E = Work done

OR **K.E. = $1 / 2\ mv^2$**

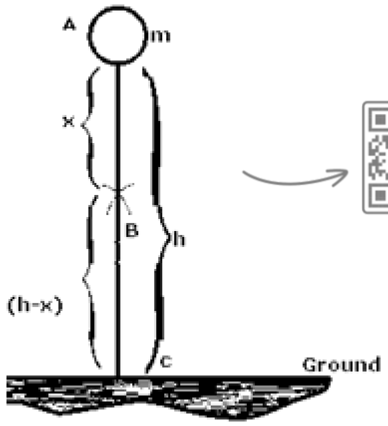
LAW OF CONSERVATION OF ENERGY

STATEMENT:

According to the law of conservation of energy

" Energy can neither be created nor it is destroyed, however energy can be converted from one from energy to any other form of energy".

Explanation:



Consider a body of mass "m" at height h above the ground. Its kinetic energy at that point A is:

1. At Point A:

A/c to the definition of Kinetic Energy

K.E = $1/2(mv^2)$

K.E = $1/2\ m * (0)$ (At point A body is at rest)

K.E = 0 .... (i)

The potential Energy at point A is :

P.E = mgh ....(ii)

So the total energy at point A will be :

T.E = K.E + P.E

E(A) = 0 + mgh

E(A) = mgh -----------(A)

2. At Point B:

Suppose the body is released from this height and falls through a distance x. Its new height will be (h-x). The velocity with which it reaches point B is calculated by using the third equation of motion:

$2gs = V_f^2 - Vi^2$

As we know:

* Vi = 0
* S = x

Therefore,

$2gx = V_f^2 - 0$

$2gx = v^2$

The kinetic energy at point B is:

K.E. = 1/2 mv^2

Substituting the value of v^2:

K.E. = 1/2 x m x 2gx

K.E = mgx

The Potential Energy at point B is:

P.E = mgh

The height of the body is (h-x):

P.E. = mg(h-x)

The total energy at point B is :

E(B) = P.E + K.E.

E(B) = mgx + mg(h-x)

E(B) = mgx + mgh - mgx

E(B) = mgh --------(B)

3. At Point C:

Now the body reaches at point "C" which is just before striking the ground.

The Potential Energy at point C is:

P.E = 0 (h=0)

The velocity with which it reaches point C is calculated by using the third equation of motion:

$2gs = V_f^2 - Vi^2$

As we know:

* Vi = 0
* S = h

Therefore,

$2gh = V_f^2 - 0$

$2gh = v^2$

The kinetic energy at point B is:

K.E. = 1/2 mv^2

Substituting the value of v^2:

$K.E. = 1/2 \times m \times 2gh$

$K.E = mgh$

The total energy at point B is :

$E(C) = P.E + K.E.$

$E(C) = 0 + mgh$

$E(C) = mgh$ ----------(C)

Hence, the total energy at point A ,B and C are same. It means that the total value of energy remains constant. That is, Law of conservation of energy.

INTERCONVERSION OF P.E AND K.E
(WORK ENERGY EQUATION)

DERIVATION:

Let us consider a body of mass "m" is placed at point A at a height h from the surface of earth.At this point the body possesses gravitational potential energy equal to mgh w.r.t point C lying on the ground.

Now consider a point B at a distance x below the point A during downward motion of body.At this stage the height of the body becomes (h-x).

so, potential energy at point B becomes,

$$P.E = mg(h-x)$$

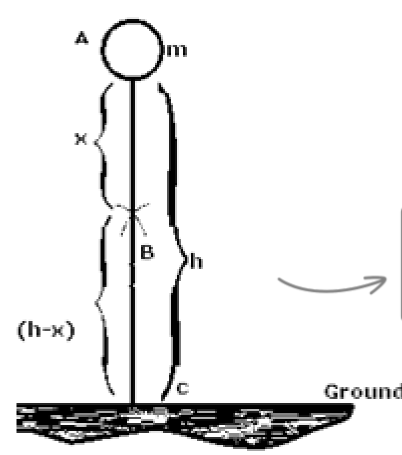
As we know that potential energy at point B is less than the potential energy at point A,i.e.

$$mg(h-x) < mgh$$

or $$mgh - mgx < mgh$$

The loss in potential energy at point B is mgx.

The Kinetic Energy at point A is equal to zero because the body is at rest.During its downward motion its velocity increases ,so its kinetic energy also increases.If there is no air friction then the loss of P.E is equal to the gain in K.E, means P.E is converted into K.E.



When the body reaches at point C its P.E becomes zero which means all of its P.E is converted into K.E,so we can write as

Loss in P.E = Gain in K.E

Practically there is always a force of friction which opposes the downward motion of the body.Let if friction f is present in this case then some amount of P.E is lost in work done against friction.Now, the modified equation can be written as,

Loss in P.E = Gain in K.E + Work done against friction

$$mgx = 1/2\, mv^2 + fx$$

or $$1/2\, mv^2 = mgx - fx$$

In terms of "h"

$$1/2\ mv^2 = mgh - fh$$

The above equation is known as "Work Energy Equation".

ABSOLUTE GRAVITATIONAL POTENTIAL ENERGY (A.G.P.E.)

DEFINITION:

The potential energy of a body at height "h" from centre of earth w.r.t. a point at which the gravitational field is zero i.e. a point which has no potential is called absolute gravitational potential energy.

1. Gravitational Potential Energy(G.P.E.):

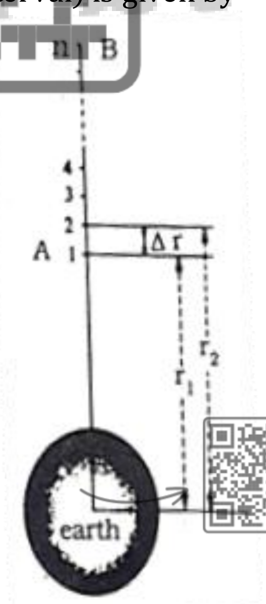
In order to derive formula for gravitational potential energy ,we have assumed that throughout the displacement of the body from the initial position to the final position force of gravity remains constant. But for large displacements (height h) as measured from the surface of the earth. e.g in spa ce flights we cannot take the gravitational force as constant. In fact. it decreases with the increase of height. Hence to calculate the work done (which is a measure of R_e) against the force of gravity the simple formula F .S cannot be applied.

To overcome this difficulty we divide the entire displacement into a large number of small displacement intervals and applying Newton's Law of Gravitation.

A point B is situated at large distance from the surface of earth. In order to find work done in bring ing the mass "m" fro initial position A or 1 to final position B or n. We divide the distance between A and B into large number say, n of intervals of equal width Δreach.

Since Δr is small,the force of gravity throughout this in can be assumed to be constant. This value of constant force may taken as the average of the forces acting at the two ends of an interval. The magnitude F_1, of the force $\vec{F}_1$ acting at the point 1 (first end of the first interval) is given by

$$F_1 = \frac{Gm\,M_e}{{r_1}^2}$$



Here M_e is the mass of earth. G is universal gravitational constant and r_1 is the distance of the point 1 from the centre of the earth.Similarly the magnitude F_2 of the force F_2 acting at point 2 is given by

$$F_2 = \frac{Gm\,M_e}{{r_2}^2}$$

The average force acting throughout the first interval

$$F = \frac{F_1 + F_2}{2}$$

where F represents the magnitude of the average force ,therefore

$$F = \frac{Gm\,M_e}{2}\left[\frac{1}{{r_1}^2} + \frac{1}{{r_2}^2}\right]$$

$$F = \frac{Gm\,M_e}{2}\left[\frac{{r_2}^2 + {r_1}^2}{{r_1}^2\ {r_2}^2}\right]$$

$$F = \frac{GmM_e}{2}\left[\frac{(r_1+\Delta r)^2 + r_1^2}{r_1^2\ r_2^2}\right] \qquad [\because r_2 = r_1 + \Delta r]$$

$$F = \frac{GmM_e}{2}\left[\frac{r_1^2 + 2r_1\Delta r + \Delta r^2 + r_1^2}{r_1^2\ r_2^2}\right]$$

as Δr is very small, Δr^2 is negligibly small,

$$F = \frac{GmM_e}{2}\left[\frac{2r_1^2 + 2r_1\Delta r}{r_1^2\ r_2^2}\right] = F = \frac{GmM_e}{2}\left[\frac{2r_1(r_1+\Delta r)}{r_1^2\ r_2^2}\right]$$

$$F = \frac{GmM_e}{r_1 r_2}$$

The work done in lifting the body from point 1 (position A) to point 2 by an applied force. which is equal and opposite to the average gravitational force is given by

$$W_{12} = \vec{F}.\overrightarrow{\Delta r}$$

Since the applied force F and displacement Δr are in the same direction.

$$W_{12} = F\Delta r$$

substituting for F, Δr in the above equation. we get

$$W_{12} = \frac{GmM_e}{r_1 r_2}(r_2 - r_1)$$

$$W_{12} = GmM_e\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Similarly the work done in lifting the body from point 2 to 3, 3 to 4,…and so on

$$W_{23} = GmM_e\left(\frac{1}{r_2} - \frac{1}{r_3}\right)$$

$$W_{(n-1)n} = GmM_e\left(\frac{1}{r_{n-1}} - \frac{1}{r_n}\right)$$

Hence the total work done by the applied force in lifting the body from initial position A to final position B,we get

$$W = W_{12} + W_{23} \ldots + W_{(n-1)n}$$

$$W = GmM_e\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + GmM_e\left(\frac{1}{r_2} - \frac{1}{r_3}\right) \ldots + GmM_e\left(\frac{1}{r_{n-1}} - \frac{1}{r_n}\right)$$

$$W = GmM_e\left(\frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{r_2} - \frac{1}{r_3} + \frac{1}{r_3} - \frac{1}{r_4} \ldots + \frac{1}{r_{n-2}} - \frac{1}{r_{n-1}} + \frac{1}{r_{n-1}} - \frac{1}{r_n}\right)$$

$$\boxed{W = GmM_e\left(\frac{1}{r_1} - \frac{1}{r_n}\right)}$$

This is the P.E represented by U of the body at the point B with respect to the point A. Hence the potential energy of the body at the point A with respect to that at the point B is ΔU = -W

$$\Delta U = -GmM_e\left(\frac{1}{r_1} - \frac{1}{r_n}\right)$$

or $$\boxed{\Delta U = GmM_e\left(\frac{1}{r_n} - \frac{1}{r_1}\right)}$$ => Gravitational Potential Energy

When the point B lies at an infinite distance i.e. r_n the P.E. at that point is zero (this point becomes reference point) then a

$$U = -\frac{GmM_e}{r_1}$$

or

$$U = -\frac{GmM_e}{r_1}$$

Therefore the absolute P.E of a body of mass m lying at the surface of the earth is given by

$$\boxed{P.E(abs) = U = -\frac{GmM_e}{R_E}}$$ => Absolute Gravitational Potential Energy

Where R_E is the radius of earth.

The minus sign indicates that the potential energy is “negative” at any finite distance that is the potential energy is zero at infinity and decreases as the separation distance decreases. This is due to the fact that the gravitational force acting on the particle by earth is attractive. As the particle moves in from infinity the Work is positive which means U is negative.

VARIOUS SOURCES OF ENERGY

There are many other forms of energy except K.E and P.E ,extracted from different sources.Some of them are given below.

(i) Wind Energy (Wind Power) :

The source of this energy is the wind. This energy is used in running flour mills. In Karachi near Suhrab Goth you can see a wind mill for drawing underground water.

(ii) Hydro electricity (Water Power):

Mangla dam. Tarbela dam and other dams in our country are used to produce electrical energy. Their prime function. is to retain river water so that it can be shuttled off to a water turbine that drives an electrical generator. The principle involves a way of supplying power to a generator other than by a steam turbine.

(iii) Fossil Fuel:

Fossil fuels are remnants of plants and animals which died millions of years ago. Depending on the conditions of formation the fuel can be liquid (crude oil), gaseous (natural gas), or solid (Coal,peat,lignite). Coal is being used by man since long as a source of energy in present age the main source of energy is gasoline. Fossil fuel is used for running machines for driving engines etc.

(iv) Nuclear Energy:

The nuclear energy is produced due to the fission of a heavy nucleus. If fission reaction occurs in a controlled manner (in a reactor) the nuclear energy is used to produce electrical power. A nuclear reactor is working in Karachi to generate electrical power.The energy thus produced is more economical and non polluting. If fission reaction is uncontrolled the enormous energy produced in the form of heat causes heavy destruction. The destruction of Japan due to it is a tragic example.

(v) Geothermal Energy:

Geothermal energy is the earth's natural heat. Heat ,in fact conducted out from the interior of the surface of planet (earth) at a rate of approximately 1.5 u cal/cm^2-s and over a time interval of a year this flux to the entire surface 1020 cal. Practically, heat must be concentrated in geothermal reservoirs where it is to be exploitable. It is interesting to observe, however ,that in the upper 10 km (when the temperature exceeds 100°C) the total stored geothermal energy exceeds by order of magnitude all thermal energy available in all nuclear and fossil fuel sources.

(vi) Solar Energy:

Solar energy is by far our most available energy source. Our lives absolutely depend on it for food production and we call on it for a multitude of things ranging from sun tanning to clothes drying. Solar energy could make a major impact on our energy economy.

(1) Providing space heating,space cooling and hot water building
(2) Providing clean fuels
(3) Generating electricity by solar cells.

(vii) Tidal Energy:

The thought of harnessing the enormous energy content of both the ocean and tides have pervaded the minds of human being for centuries. The tides have their origin in the gravitational force exerted on the earth by the moon and the sun. Water-powered mills operating from tidal motion were used in New England in the 18th century. Sewage pumps functioned in Germany and London by using tidal power. These systems were (replaced by the more economical and convenient electric motors.)Although no source exists that renders less environmental damage, tidal energy is difficult to harness and marginally economical.

For Pakistan:

The fossil fuel is used as the main source of energy in Pakistan. It requires a huge amount of foreign exchange to import it. Due to its burning environmental damage is done on a very high scale. The hydro electric generation is also limited and also costly. For our present and future, needs we must provide indigenous atomic reactor to generate electrical power. Along with solar energy should be exploited to a greater extent. Solar emery is ideal source of energy to get rid of pollution. Solar energy is available in most of the parts of Pakistan throughout the year.

M.C.Qs.

1. All of them are true except:
(a) Work is defined as the product of force and displacement
(b) Joule is the unit of work
(c) Force moves in its direction or in opposite directions
(d) The resultant force on it is zero

2. Work is defined as:
(a) Scalar product of force and displacement
(b) Vector product of force and displacement
(c) Scalar product of force and velocity
(d) Vector product of force and velocity

3. Work done will be zero when force and displacement are:
(a) In the same direction
(b) In opposite direction
(c) Perpendicular to each other
(d) Not zero

4. The work done on a body undergoing a certain displacement is given by:
(a) The area under a force vs. time curve
(b) The area under a force vs. distance curve
(c) The area under a velocity vs time curve
(d) The area under an acceleration vs time curve

5. If F=3i and d=6j, the work done will be:
(a) Zero (b) 2 (c) 9 (d)18

6. Power is the dot product of:
(a) Mass & velocity (b) Force & velocity
(c) Force & Energy (d) Force & mass

7. The power required to lift a 40 kg weight, up to the height of 5 m in 10 sec will be:
(a) 80 watts (b) 200 watts
(c) 28 watts (d) 14000 watts

8. A 600 N man runs up a stair of 4m height,in 3 seconds. The power needed is:
(a) 24W (b) 350W
(c) 450W (d) 800W

9. One kilo watt hour is equal to:
(a) 3.6×10^6 J (b) 3.3×10^9 J
(c) 3.9×10^6 J (d) 3.6×10^9 J

10. This one of the following is not the unit of power:
(a) horse power (b) joule/sec
(c) kilowatt hour (d) foot-pound/sec

11. The K. E of a 1000 kg car moving at a speed of 80 km/hr will be:
(a) $2.47\text{x}10^8$ J (b) $2.47\text{x}10^5$ J
(c) $24.7\text{x}10^7$ J (d) $24.7\text{x}10^3$ J

12. The energy due the motion of a mass is known as:
(a) Potential energy (b) Motion energy
(c) Mobile energy (d) Kinetic energy

13. If the velocity of the moving particle is double the factor by, which the K. E is increased is:
(a) 4 (b) 1/2 (c) 2
(d) 6

14. The velocity of a body is doubled and mass is reduced to one fourth of its initial value, the K.E is:
(a) doubled (b) fourfold

(c) same (d) halved

15. A bucket of mass 10 kg is moved downwards in the gravitational field through a distance of 1 m. The work done in this case is equal to:

(a) 10 J (b) 98 J

(c) -98 J (d) 0.1 J

16. The work done by a conservative force along a closed path is:

a) positive (b) negative

(c) zero (d) none of these

17. When a body moves vertically upward, the work done will be:

(a) positive (b) negative

(c) zero (d) maximum

18. A body of mass 10 kg moving at a height of 2 m, with uniform speed of 2 m/s. Its total energy is

(a) 316 J (b) 216 J

(c) 116 J (d) 392 J

19. Which one has higher kinetic energy? Both light and heavy bodies have equal momentum.

(a) Heavy body (b) Light body

(c) Both (d) None of the option

20. Power is

a) Rate of doing work b) Ability to do work

c) Rate of energy creation d) Equivalent to work

21. Which of the following is true?

(a) Potential energy decreases as altitude increases

(b) Potential energy increases as altitude increases

(c) Potential energy first increases and then decreases as altitude increases

(d) Potential energy first decreases and then increases as altitude increases

22. What happens to the total energy of a moving object if all the applied forces are conserved?

(a) It increases (b) It decreases

(c) It remains constant (d) none of these

23. What happens to the kinetic energy of a moving object if the net work done is positive?

(a) The kinetic energy increases

(b) The kinetic energy decreases

(c) The kinetic energy remains the same

(d) The kinetic energy is zero

24. A spacecraft moves around Earth in a circular orbit with a constant radius. How much work is done by the gravitational force on the spacecraft during one revolution?

(a) FGd (b) - FGd

(c) mgh (d) Zero

25. When a body falls freely under gravity, then the work done by the gravity is ________

(a) Positive (b) Negative

(c) Zero d) Infinity

26. When a body slides against a rough horizontal surface, the work done by friction is ________

(a) Positive (b) Zero

(c) Negative (d) Constant

27. When a coolie walks on a horizontal platform with a load on his head, the work done by the coolie on the load is zero.

(a) Positive (b) Zero

(c) Negative (d) Constant

28. When a body slides against a rough horizontal surface, the work done by friction is ________

(a) Positive (b) Zero

(c) Negative (d) Constant

29. The tidal energy is due to:

(a) The rotation of earth about sun
(b) The rotation of earth relative moon
(c) The radioactive decay inside earth
(d) Attraction of sun and moon

30. The work done in moving a object along a vector=3i + 2j – 5k. If the applied force is F = 2i–j - k:
(a) 10j (b) 6i – 2j – 5k
(c) 0j (d) 9j

PAST PAPER M.C.Qs.

2022

2.If velocity of a body is doubled and mass is reduced to one forth, kinetic energy will be
*doubled **_* unchanged_** * halved * fourfold

8. One kilo watt hour is equal to:
_*3.6 x 10^6 J_ *3.3 x 10^9 J *3.9 x 10^6 J *3.6 x 10^9 J

10.The dot product of force and velocity is called
* work **_*power_** *momentum *energy

12. If F= 3i and d=6j the work done will be:
*zero *2 *9 *18

22.A body pushes a toy car , on a horizontal floor with a force of 10 N up toa displacement of 2m then work done by gravity on the car is:
* 20J * 10J * 5J **_* 0 J_**

31.A force acting on a body is perpendicular to displacement , the work done is equal to:
*positive *negative **_*zero_** *infinite

2021

(xiv) Work energy equation is called:
* Law of conservation of mass * Law of conservation of momentum
* **_Law of conservation of energy_** * Law of conservation of angular momentum

(xxii) A weight lifter consumes 500 J of energy to lift a load in 2 seconds, the power consumed is:
*125 watt *500 watt **_*250watt_** *1000watt

(xxv) Both Kilowatt hour and electron volt are the units of:
*Power *Charge **_*Energy_** *Angular momentum

(xxxii)If the speed of moving body is to be halved, its kinetic energy becomes

***One fourth** *double * Half *Four times

(xxxv) If F= 3i and d 6j the work done will be:

***zero** *2 *9 *18

(xxxvii) The ocean tides are caused by:

*Earth's gravitational force only *Moon's gravitational force only

* Sun's gravitational force only ***Gravitational force of both the sun and moon**

(xli) The work done by the force of 10N applied to the direction of motion up to 20m is:

*10J ***200J** *2000J *20J

2019

1.If F=3i and d=6j, the work done will be:

*** Zero** *2 *9 *18

2018

4. A 600 N man runs up a stair of 4m height,in 3 seconds. The power needed is:

*24W *350W *450W ***800W**

15.If the velocity of a body is doubled and mass is reduced to one-fourth of its initial value ,the kinetic energy will be:

*be doubled *four fold *** remains same** * becomes zero

2017

6. The ocean tides are caused by:

*earth's gravitational force only * moon's gravitational force only

* sun's gravitational force only *** sun's and moon's gravitational force**

8. Both kilowatt hour and electron volt are the unit of :

*power ***energy** *charge *angular momentum

2016

14. One kilo watt hour is equal to:

***3.6×10^6 J** *3.3×10^9 J *3.9×10^6 J *3.6×10^9 J

2015

1. Electron volt is the unit of:

*Power *voltage ***energy** *charge

17. This one of the following is not the unit of power:

* horse power * joule/sec ***kilowatt hour** *foot-pound/sec

2014

2. Kilowatt hour is a unit of:

<u>*Energy</u>   * Power   *Time   *Force

12. If mass and speed both are doubled, the kinetic energy will be:

* double   *four times   *six times   **<u>*eight times</u>**

2013

17. The weight lifter consumes 500J of energy to lift a load in 2 seconds, the power used by him is:

*125 watt   **<u>*250 watt</u>**   *500 watt   *1000 watt

2012

14. The rate of doing work is zero when the angle between force and velocity is:

*0^0 * 45^0 * 180^0 **<u>* 90^0</u>**

16. The velocity of a body is doubled and mass is reduced to one fourth of its initial value, the K.E is:

*doubled   * fourfold   **<u>* same</u>**   * halved

11. A bucket of mass 10 kg is moved downwards in the gravitational field through a distance of 1 m. The work done in this case is equal to:

*10 J   **<u>* 98 J</u>**   * -98 J   * 0.1 J

2011

7. If the speed of moving body is halved, its kinetic energy becomes:

<u>*one fourth</u>   *half   *three times   *four times

15. The work done by a conservative force along a closed path is:

*positive   * negative   **<u>*zero</u>**   *none of these

2010

2. When a body moves vertically upward, the work done will be:

*positive   **<u>* negative</u>**   * zero   * maximum

TEXTBOOK NUMERICALS

Q.1: Calculate the work done by a force F specified by F = 3i + 4j + 5k in displacing a body from position B to position A along a straight path. The position vectors A & B are respectively given as r_A = 2i + 5j – 2k & r_B = 7i + 3j – 5k

<u>Data:</u>

Initial position = r_A = 2i + 5j -2k

Final position = r_B= 7i + 3j -5k

Force = F = 3i +4 j + 5k

Work = W =?

<u>Solution:</u>

According to the def. of work

$W = \vec{F}.\vec{S}$---- (i)

$$\vec{S} = r_A - r_B$$

$$\vec{S} = 2i + 5j - 2k - (7i + 3j - 5k)$$

$$\vec{S} = 2i + 5j - 2k - 7i - 3j + 5k$$

$$\boxed{\vec{S} = -5i + 2j + 3k}$$

Putting in eq (i)

$$W = \vec{F}.\vec{S}$$

$$W = (3i + 4j + 5k).(-5i + 2j + 3k)$$

$$W = -15 + 8 + 15$$

$$\boxed{W = 8\ units}$$

Result: The work done is 8 units.

Q.2: A 2000 kg car traveling at 20 m/s comes to rest on a level ground in a distance of 100 m. How large is the average frictional force tending to stop it?

Data:

Mass of Car $= m = 2000\ kg$

Initial Speed of Car $= v_i = 20\ m/s$

Final Speed of Car $= v_f = 0$

Distance covered $= S = 100\ m$

Frictional Force $= F = ?$

Solution:

As we know that

$$F = ma \text{ --- (i)}$$

For Acceleration:

$$2aS = V_f^2 - V_i^2$$

$$2a(100) = (0)^2 - (20)^2$$

$$a(200) = -400$$

$$\boxed{a = -2\ m/s^2}$$

Putting values in eq (i)

$$F = ma$$

$$F = 2000 \times (-2)$$

$$\boxed{F = -4000\ N}$$

Result: The average frictional force tending to stop car is 4000 N.

Q.3: A 100-kg man is in a car traveling at 20 m/s. (a) Find his kinetic energy. (b) The car strikes a concrete wall and comes to rest after the front of the car has collapsed 1 m. The man is wearing a seat bell and harness. What is the average force exerted by the belt and harness during the crash?

Data:

Mass of Man $= m = 100\ kg$

Initial Speed of Car $= v_i = 20\ m/s$

(a) Kinetic Energy of Man $= K.E = ?$

Final Speed of Car $= v_f = 0$

Distance covered $= S = 1\ m$

(b)Force exerted by the belt $= F = ?$

Solution:

(a) Kinetic Energy is given by

$$K.E = \frac{1}{2}mv_i^{\ 2}$$

$$K.E = \frac{1}{2} \times 100 \times (20)^2$$

$$\boxed{K.E = 20000\ J}$$

(b) As we know that

$$F = ma \text{ --- (i)}$$

For Acceleration:

$$2aS = V_f^2 - V_i^2$$

$$2a(1) = (0)^2 - (20)^2$$

$$a(2) = -400$$

$$\boxed{a = -200\ m/s^2}$$

Putting values in eq (i)

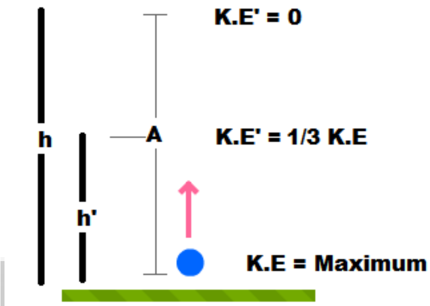
$$F = ma$$

$$F = 100 \times (-200)$$

$$\boxed{F = -20000\ N}$$

Result: The Kinetic Energy of Man is 20000 J and the force exerted by the belt is $-20000\ N$

Q.4: When an object is thrown upward, it rises to a height 'h'. How high is the object, in terms of h, when it has lost one-third of its original kinetic energy?



Data:

Final Height $= h$

Height where 1/3 K.E lost $= h' = ?$

Solution:

According to the Law of conservation of Energy

$$Change\ in\ K.E = Change\ in\ Potential\ Energy$$

$$\boxed{K.E = mgh}$$

Similarly at point A

$$K.E' = mgh' \text{ ---(i)}$$

According to the given condition

$$K.E' = \frac{1}{3}K.E$$

so Eq (i) =>

$$\frac{1}{3}K.E = mgh'$$

Or

$$\frac{1}{3}(mgh) = mgh'$$

$$\boxed{h' = \frac{1}{3}h}$$

Result: The object will loose its One Third K.E when it has covered one-third of its height.

Q.5: A pump is needed to lift water through a height of 2.5 m at the rate of 500 g/min. What must the minimum horse power of the pump be?

Data:

$$Height = h = 2.5\ m$$

$$Rate\ of\ flow = \frac{m}{t} = 500\frac{g}{min} = \frac{500}{(1000 \times 60)}$$

$$= 0.0083\ kg/sec$$

$$Power\ of\ pump = P = ?\,(in\ hp)$$

Solution:

According to the definition of Power

$$P = \frac{W}{t}$$

$$P = \frac{mgh}{t}$$

$$P = \frac{m}{t} \times g \times h$$

$$P = 0.0083 \times 9.8 \times 2.5$$

$$P = 0.204\ W$$

$$Since \quad 1\ hp = 726\ watt$$

$$P = \frac{0.204}{746}$$

$$\boxed{P = 2.7 \times 10^{-4} hp}$$

Result: The minimum horse power required is $2.7 \times 10^{-4} hp$.

Q.6: A horse pulls a cart horizontally with a force of 40 lb at an angle 30° above the horizontal and moves along at a speed of 6.0 miles/hr. (a) How much work does the horse do in 10 minutes? (b) What is the power out put of the horse?

Data:

Force applied by horse = F = 40 lb

Angle b/w force and displacement $= \theta = 30^0$

Speed = v = 6 mi / hr $= \frac{6 \times 5280}{3600} = 8.8\ ft/sec$

Work done by the horse = W = ?

Time = t = 10 minutes = 10 x 60 = 600 s

Power output of the horse = p = ?

Solution:

$$P = Fv\ Cos\theta$$

$$P = 40 \times 8.8 \times\ Cos30$$

$$\boxed{P = 304.8\ lb - ft\ /sec}$$

In hp :

$$1\text{ hp} = 550\ lb - ft\ /sec$$

So,

$$\boxed{P = \frac{304.8}{550} = 0.55\ hp}$$

According to the definition of Power

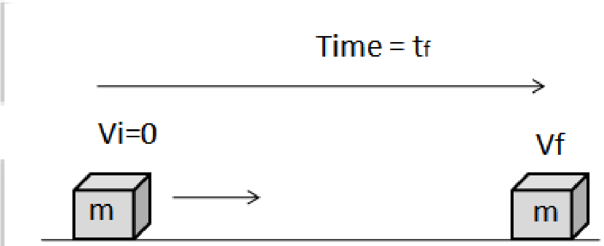
$$P = \frac{W}{t}$$

$W = P \times t = 304.8 \times 600 = 1.82 \times 10^5 lb - ft$

Result: The work done by the horse is $1.82 \times 10^5 lb - ft$ and power output is 0.55 hp.

Q.7: A body of mass 'm' accelerates uniformly from rest to a speed V_f in time t_f. Show that the work done on the body as a function of time 't', in terms of V_f and t_f, is $\frac{1}{2} m \frac{v_f^2}{t_f^2} t^2$.

Proof:



The work done is given by

$W = FS$ ---(i)

First we will find "F" Using newton's 2nd Law of Motion

$$F = ma$$

$$F = m\left(\frac{v_f - v_i}{t_f}\right)$$

$$F = m\left(\frac{v_f - 0}{t_f}\right)$$

$$\boxed{F = \frac{m v_f}{t_f}}$$

Now, we will calculate "S" using 2nd equation of Motion

$$S = v_i t + \frac{1}{2} a t^2$$

$$S = 0 \times t + \frac{1}{2}\left(\frac{v_f}{t_f}\right) t^2$$

$$\boxed{S = \frac{1}{2}\left(\frac{v_f}{t_f}\right) t^2}$$

Putting values of "F" and "S" in eq (i)

$$W = \frac{m v_f}{t_f} \times \frac{1}{2}\left(\frac{v_f}{t_f}\right) t^2$$

$$\boxed{W = \frac{1}{2} m \frac{v_f^2}{t_f^2} \times t^2}$$

Hence proved that the work done on the body as a function of time 't', in terms of V_f and t_f, is $\frac{1}{2} m \frac{v_f^2}{t_f^2} t^2$.

Q.8: A rocket of mass 0.200 kg is launched from rest. It reaches a point p lying at a height 30.0 m above the surface of the earth from the starting point. In the process + 425 J of work is done on the rocket by the burning chemical propellant. Ignoring air-resistance and the amount of mass lost due to the burning propellant, find the speed V_f of the rocket at the point P.

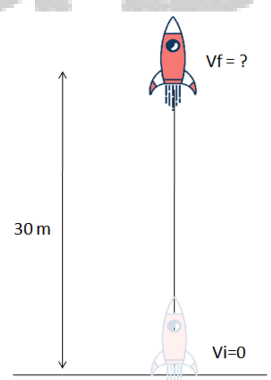
Data:

Mass of Rocket = $m = 0.200\ kg$

Height Covered by Rocket = $h = 30.0\ m$

Work done on the Rocket = $W = +425\ J$

Final Speed of Rocket = v_f =?



Solution:

In this case the total work done by the fuel will be conerted into Kinetic Energy and Potential Energy.Therefore,

$W = K.E + P.E$
$W = \frac{1}{2}m{v_f}^2 + mgh$
$425 = \frac{1}{2}(0.200) \times {v_f}^2 + 0.200 \times 9.8 \times 30$
$425 = 0.1 \times {v_f}^2 + 58.8$
${v_f}^2 = \frac{425 - 58.8}{0.1} = 3662$
Taking Square root on both sides
$v_f = \sqrt{3662}$
$\boxed{v_f = 60.5\ m/s}$

Result: The speed V_f of the rocket will be 60.5 m/s

PAST PAPER NUMERICALS

2022

viii) Two forces F_1 = 3i-2j+ 5k and F_2 = i +6 j +2k act on a body which is displaced along a vector r = 4i - j + 3k . Calculate the work done on the body.

Data:
$F_1 = 3i - 2j + 5k$
$F_2 = i + 6j + 2k$
$r = 4i - j + 3k$
Work done = w =?
Solution:
First we have to find the net force
$F = F_1 + F_2 = 3i - 2j + 5k + i + 6j + 2k$
$\boxed{F = 4i + 4j + 7k}$
Now, according to the definition of work
$w = F.r = (4i + 4j + 7k).(4i - j + 3k)$
$w = 16 - 4 + 21$
$\boxed{w = 33\ joules}$

Result: The work done by forces is 33 J.

2019

No Numerical

2018

Q.2(ix) A crane lifts a load of 6000 N through a vertical distance of 15 m in 30 s. What is the potential energy at the highest point of this operation?

Data:
Load = W =6000 N
Height covered = h = 15 m
Time = t = 30 s
Potential Energy = P.E = ?
Solution:
According to the definition of Potential Energy
$P.E = mgh$
$P.E = (mg)h$
$Since\ W = mg$
$\therefore P.E = Wh$
$P.E = 6000 \times 15$
$\boxed{P.E = 90000\ J}$

Result: The potential energy at the highest point is 90000 J.

2017

2(v) A horse pulls a cart horizontally with a force of 60 lb at an angle of 30 degrees above the horizontal and moves along at a speed of 8 miles per hour. How much work does the horse do in 15 minutes and what

is the power output of the horse.(1 hp=550 lb-ft / sec) and (1 mile=5280 ft).

Data:

Force applied by horse = F = 60 lb

Angle b/w force and displacement= $\theta = 30^0$

Speed = v = 6 mi / hr = $\frac{8\times 5280}{3600} = 11.73\ ft/sec$

Work done by the horse = W = ?

Time = t = 15 minutes = 15 x 60 = 900 s

Power output of the horse = p = ?

Solution:

$$P = Fv\ Cos\theta$$

$$P = 60 \times 11.73 \times\ Cos30$$

$$\boxed{P = 609.6\ lb - ft\ /sec}$$

In hp :

$$1\ \text{hp} = 550\ lb - ft\ /sec$$

So, $\boxed{P = \frac{609.6}{550} = 1.10\ hp}$

According to the definition of Power

$$P = \frac{W}{t}$$

$$\boxed{W = P \times t = 609.6 \times 900 = 5.48 \times 10^5 lb - ft}$$

Result: The work done by the horse is $5.48 \times 10^5 lb - ft$ and power output is 1.10 hp.

2016

Q.2 (vi) Textbook Numerical 5

2015

Q.2 iii) A 80 kg man runs up a hill through a height of 3m in 2 sec. What is his average power output?

Data:

Mass of man = m= 80 kg

Height = h = 3 m

Time = t = 2 sec

Average Power = P =?

Solution:

The Average power is given by

$$P = \frac{W}{t}$$

$$P = \frac{mg\,h}{t}$$

$$P = \frac{80\times 9.8\times 3}{2} = \frac{2352}{2}$$

$$\boxed{P = 1176\ W}$$

Result:

His average output power is 1176 W.

Q.2 v) An object moves along a straight line in a force field from (3, 2, -6) to (14, 13, 9) when a uniform force F=4i + j + 3k acts on it. Find the work done.

Data:

Initial position =r_1= (3, 2,-6) = 3i + 2j -6k

Final position =r_2= (14, 13, 9) = 14i + 13j + 9k

Force = F = 4i + j + 3k

Work = W =?

Solution:

According to the def. of work

$W = \vec{F}.\vec{S}$---- (i)

$$\vec{S} = r_2 - r_1 = 14i + 13j + 9k - (3i + 2j - 6k)$$

$$\vec{S} = 14i + 13j + 9k - 3i - 2j + 6k$$

$$\boxed{\vec{S} = 11i + 11j + 15k}$$

Putting in eq (i)

$$W = \vec{F}.\vec{S}$$

$$W = (4i + j + 3k).(11i + 11j + 15k)$$

$$W = 44 + 11 + 45$$

$$\boxed{W = 100\ units}$$

Result:

The work done is 100 units.

2014

Q.2 (xiv) Textbook Numerical 5

2013

Q.2 (v) A horse pulls a cart horizontally with a force of 40 N at an angle of 25 degrees above the horizontal

and moves along at a speed of 15 m/s . How much work does the horse do in 5 minutes and what is the power output of the horse. Give your answer in horse power (1 hp=746 W)

Data:

Force applied by horse = F = 40 N

Angle b/w force and displacement= $\theta = 25^0$

Speed = v = 15 m/s

Work done by the horse = W = ?

Time = t = 5 minutes = 5 x 60 = 300 s

Power output of the horse = p = ?

Solution:

$$P = Fv\ Cos\theta$$

$$P = 40 \times 15 \times\ Cos 25^0$$

$$\boxed{P = 543.7\ W}$$

In hp :

$$1\text{ hp} = 746\,W$$

So, $\boxed{P = \frac{543.7}{746} = 0.72\ hp}$

According to the definition of Power

$$P = \frac{W}{t}$$

$$\boxed{W = P \times t = 543.7 \times 300 = 1.63 \times 10^5 J}$$

Result: The work done by the horse is $1.63 \times 10^5\ J$ and power output is $0.72\ hp$.

2012

Q.2 (iv) An object weighing 98N is dropped from a height of 10m. It is found to be moving with a velocity 12m/sec just before it hits the ground. How large was the frictional force acting upon it?

Data:

Weight of Object = W = 98 n

Height of object = h = 10 m

Initial Velocity = vi = 0

Final velocity = vf = 12 m/s

Frictional Force = f =?

Solution:

A/c Work – Energy Equation

$$mgh = \frac{1}{2}mv^2 + fh \text{ --- (i)}$$

$$m = \frac{W}{g} = \frac{98}{9.8} = 10\ kg$$

Putting in eq (i)

$$10 \times 9.8 \times 10 = \frac{1}{2} \times 10 \times 12^2 + f \times 10$$

$$980 = 720 + f \times 10$$

$$f = \frac{980 - 720}{10}$$

$$\boxed{f = 26\ N}$$

Result: The acting frictional force is 26 N.

2011

Q.2 (xii) **Textbook Numerical 5**

2010

Q.2 (x) **Textbook Numerical 5**

THEORY NOTES

SIMPLE HARMONIC MOTION:

DEFINITION:

If a body moves in a straight line such that its acceleration is always directed towards a fixed point on that line and its magnitude is proportional to the displacement from that point, then the body is said to execute simple harmonic motion.

EXAMPLES: Motion of a mass attached to one end of a spring.
Vibration of a string of sitar.
Motion of the bob of a simple pendulum
Motion of the pendulum of a clock.

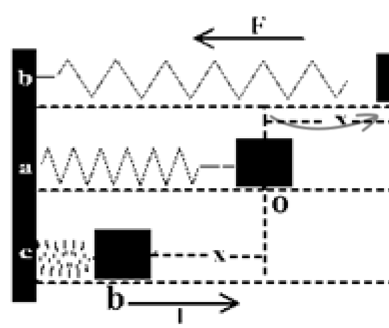
CONDITIONS OF A BODY TO EXECUTE SIMPLE HARMONIC MOTION

(i) The motion of the body must be under elastic restoring force.
(ii) The acceleration of the body must be s proportional to the displacement and directed onwards the mean position.
(iii) The body must have inertia i.e. mass.
If these conditions are satisfied the body will execute simple harmonic motion.

MOTION UNDER ELASTIC RESORING FORCE

CASE I:-

As shown in the figure a mass 'm' is attached to one end of a spring placed on horizontal smooth surface, the other end of which is rigidly fixed. If the mass 'm' is pulled to the right through a distance x and then released, the mass 'm' will vibrate about its mean position.



The force exerted on the spring will be proportional to the displacement.

$F \propto x$

or

$F = kx$

This is known as Hook's law.

Where k is a constant of proportionality, called force constant of spring. The spring also exerts an equal and opposite force to restore its shape. This force is called restoring force.

Restoring force = - Kx

EXPLANATION:

Consider the motion of mass 'm' in fig (a) there is no force on the mass 'm' because spring suffers no extension. In fig (b), the spring is pulled to the right through a distance x_0 the restoring force of the spring is F = - Kx

The work done in pulling the spring through a distance x_0 is stored in the form of potential energy of the spring. In return the spring applies a force to restore its position and the mass 'm' moves to the left. Thus potential energy changes into kinetic energy. At its mean position, the mass 'm' has a maximum speed and because of inertia the mass moves to the left. The spring then compresses and the motion of the mass 'm' retards. At its extreme left position all the energy is potential. This process repeats and the energy of the spring oscillates between potential and kinetic energy.

Let x be the displacement of the mass 'm' at any instant then.

$$F = -Kx$$

From Newton's second law of motion

$$F = ma$$

where a = acceleration

Hence

Or $\quad a = -(K/m)\,x$

Or $\quad \boxed{a \propto -x}$

Where k/m is a constant

The equation shows that the motion 'm' is vibratory, and its acceleration is directly proportional to the displacement and directed towards the mean position. This type of motion is called simple harmonic motion.

CONNECTION BETWEEN S.H.M. AND UNIFORM CIRCULAR MOTION

CASE II:-

Let us consider a particle of mass 'm' moving around a vertical circle of radius 'x_0' with constant angular velocity "ω". If 'θ' is the angular displacement swept during time 't' then $\theta = \omega t$

The projection 'Q' of particle 'P' on the diameter AB of the circle vibrates to and fro about the centre of the circle as 'P' moves along circular path. It is also observed that the projection 'Q' speeds up when it moves towards the centre 'O' and slows down when it moves away from the centre.Thus the instantaneous acceleration of projection 'Q' is directed towards the centre and it is in vibratory motion.

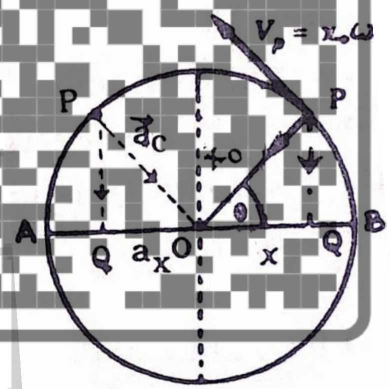
The motion of 'Q' is associated with the motion of 'P' hence the acceleration of 'Q' must be a component of the acceleration of the motion of particle P.The acceleration of the particle P is Centripetal acceleration i.e. directed towards centre of the circle along the line PO and it given by,

$$a_c = \frac{v^2}{r}$$

or $$a_c = -\frac{{v_p}^2}{x_o}$$ (x_0 and a_c are in opposite direction)

$$a_c = -\frac{(x_o\omega)^2}{x_o} \qquad \because v = r\omega$$

or $$a_c = -x_0\omega^2 \text{--------(i)} \quad \therefore v_p = x_0\omega$$



The acceleration of projection Q is equal to the component of acceleration of particle P along x axis .i.e.

$$a_x = a_c \cos\theta \text{ --------(ii)}$$

In triangle POQ

$$\cos\theta = \frac{\overline{OQ}}{\overline{OP}}$$

$$\cos\theta = \frac{x}{x_o} \text{ -------(iii)}$$

putting values from eq(i) and (iii) in eq(ii) we get ,

$$a_x = -x_0\omega^2 \frac{x}{x_o}$$

or $$a_x = -\omega^2 x$$

As we know that ω is constant, Therefore,

$$\boxed{a \propto -x}$$

Hence it is proved that the motion of projection Q of particle p executing uniform circular motion is S.H.M.

EQUATION OF DISPLACEMENT:

At some instant of time t, the angle between OP and x axis is ω t+ φ , where φ is the angle which

OP makes with the x axis at time t=0.This angle is known as initial phase angle.

In right angled triangle OPQ

$\cos\theta = \frac{OQ}{OP}$

$\cos(\omega\, t + \varphi) = \frac{x}{x_o}$ $\quad\because\ \theta = \varphi + \omega t$

or

$$x = x_o \cos(\omega\, t + \varphi)$$

Where x_0=amplitude of S.H.M of Q and x =instantaneous displacement.

EQUATION FOR ACCELERATION:

As shown in the figure, a particle p is moving along the circumference of a circle of radius 'r; with its linear velocity V_p, its angular velocity ω is given by

$$\omega = \frac{V_p}{r}$$

or $v_p = r\omega$ ---------------- (1)

let the particle starts from 'A' and in time 't' it sweeps and angle θ then

$$\theta = \omega\, t$$

When the particle 'P' is at 'B' its projection 'Q' is at O. As the particle moves and reaches 'C', the projection of P (i.e.Q) along reaches C. When the particle is at D,Q again at O. And when P is at A, Q is also at A. Thus when the particle P moves along circular path. Q moves along AOC and back to COA. Hence the motion of O is along a straight line.

Since the particle P is moving along a circular path it centripetal acceleration a_c is given by

$$a_c = -r\omega^2$$

The acceleration of Q is along AOC, hence the component a_c along AOC will give the acceleration of O.The competent of a_c along AOC is given by

$$a_o = a_c \cos\theta = \omega^2 r \cos\theta$$

From figure $r \cos\theta = x$

$$a_o = -\omega^2 x$$

The negative sing shows that the acceleration of Q is directed towards the center and is proportional to x.

EQUATION OF VELOCITY:

The velocity of Q is equal to the X component of the velocity of P directed along AOC. Let Vo be the velocity of Q along AOC and Vp that of P along the tangent at any point on the circumference of the circle then

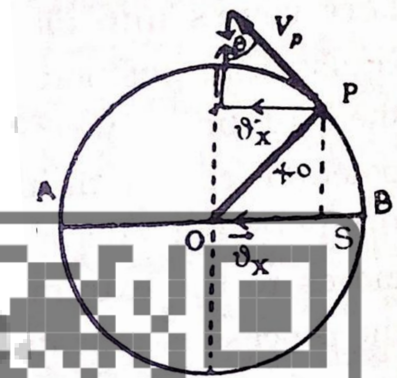
$V_Q = V_P \sin\theta = x_o\,\omega \sin\theta$ --------- (1)

From the relation $\sin^2\theta + \cos^2\theta = 1$, we get

$\sin^2\theta = 1 - \cos^2\theta$

$$sin\theta = \sqrt{1 - cos^2\,\theta}$$

putting in equation (i)



$$V_Q = x_0\omega\sqrt{1 - \left(\frac{x}{x_0}\right)^2}$$

$$V_Q = x_0\omega\sqrt{\frac{x_0^2 - x^2}{x_0^2}}$$

$$V_Q = \frac{\cancel{x_0}\omega}{\cancel{x_0}}\sqrt{x_0^2 - x^2}$$

so,

$$\boxed{V_Q = \omega\sqrt{x_0^2 - x^2}}$$ ---------(ii)

<u>i) At Extreme Position:</u>

x=x0,putting in eq (ii)

$$V_Q = \omega\sqrt{x_0^2 - x_0^2}$$

$$V_Q = \omega\sqrt{0}$$

$$\boxed{V_Q = 0}$$

The velocity of projection at the extreme position is equal to zero.

<u>ii) At Mean Position:</u>

x=0,putting in eq (ii)

$$V_Q = \omega\sqrt{x_0^2 - 0}$$

$$V_Q = \omega\sqrt{x_0^2}$$

$$V_Q = \omega x_0$$

The velocity of projection at the extreme position is Maximum.

TIME PERIOD:

The time required to complete one cycle of motion is called time period. Denoted by "T".

A/c to the definition of angular velocity ω

$$\omega = \frac{\Delta\theta}{\Delta t}$$

For one complete cycle, $\Delta\theta = 360^0 = 2\pi, \Delta t = T$

$$\omega = \frac{2\pi}{T}$$

or

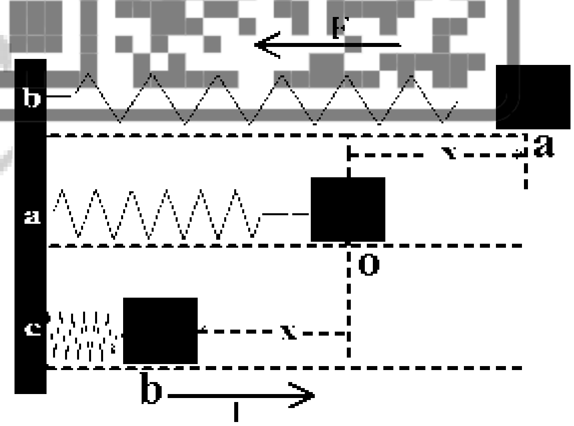
$$\boxed{T = \frac{2\pi}{\omega}}$$

ENERGY OF A BODY EXECUTION SIMPLE HARMONIC MOTION

Let us consider a mass "m" connected with one end of a string whose other end is connected with a rigid wall and it can execute SHM on friction less surface as shown in fig.

(I) KINETIC ENERGY:

The instantaneous velocity of the body when its displacement is x is given by,



$$V = \omega\sqrt{x_0^2 - x^2}$$

In case of Spring mass system $\omega = \sqrt{\frac{K}{m}}$

$$=> V = \sqrt{\frac{K}{m}}\sqrt{x_0^2 - x^2}$$

As we know that ,

K.E $=1/2\ mv^2$

K.E $=1/2\ m(\sqrt{\frac{K}{m}}\sqrt{x_0^2 - x^2})^2$

K.E $=1/2\ m\frac{K}{m}(x_0^2 - x^2)$

$\boxed{\text{K.E} = 1/2\ K(x_0^2 - x^2)}$ --------(i)

(I) POTENTIAL ENERGY:

A/c to Hooke's law

$F = Kx$

At mean Position=> $F = 0$

At extreme position => $F = kx$

Therefore the average force on mass "m" during the displacement x is

$$F = \frac{0 + Kx}{2}$$

$$F = \frac{1}{2}kx$$

Now, A/c definition of P.E

P.E=avg. force x displacement

$$P.E = \frac{1}{2}kx \times x$$

$$\boxed{P.E = \frac{1}{2}kx^2} \text{------------(ii)}$$

(III) TOTAL ENERGY:

The energy of a body executing simple harmonic motion is the sum of potential energy and kinetic energy at that instant at a displacement x from the mean position.

$E = K.E + P.E$

putting values from eq(i) and eq(ii)

$$E = 1/2\ K\ (x_0^2 - x^2) + \frac{1}{2}kx^2$$

$$E = \frac{1}{2}kx^2 + \frac{1}{2}kx_0^{\ 2} - \frac{1}{2}kx^2$$

$$\boxed{E = \frac{1}{2}kx_0^{\ 2}}$$

The above equation shows that the total energy of a particle executing simple harmonic motion is proportional to the square of its amplitude of vibration.

SIMPLE PENDULUM

DEFINITION:

An ideal simple pendulum consists of a point mass suspended from a light inextensible string tied to a grid and friction less support when the bob of the pendulum is vertical, the gravitational force W acts vertically downward and the tension T acts vertically upwards in this case the force W is balanced by the tension 'T' in the string.

CASE III:

EXPLANATION:

When the bob is slightly displaced from its mean position, it beings to perform oscillatory motion. Let us see the motion of the bob.

The bob of the pendulum is under the action of two force.

(i) The gravitational force W = mg acting vertically downwards

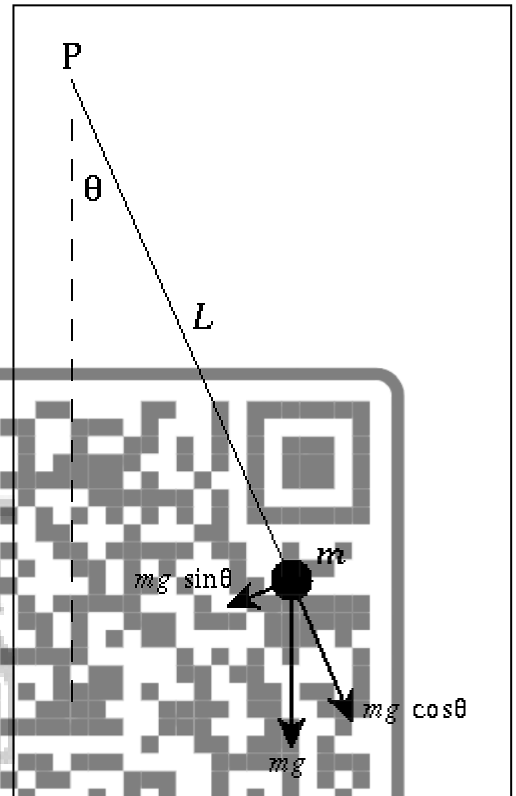
(ii) The tension T acts along the string.

The net force acting on the bob is F_{Net} = W – T

Resolving W into two components

(i) along the length of string (parallel) and

(ii) perpendicular to the string.



$W_{\parallel}$ = mg cosθ and $W_{\perp}$ = mg sinθ

Since there is no motion of the bob along the string, a net force in the direction of string is zero. in this case.

mg cosθ = T

Hence the magnitude of the net force is F_{Net} = mg sinθ brings the bob to its mean position.

From Newton's second law of motion F_{Net} = ma

~~m~~a = ~~m~~g sin θ (since the force is directed towards mean position, hence,)

a = - g sin θ

If θ is small sin θ ≈θ

then a = - g θ -------------(i)

As we know that

S=r θ => θ=S/r

In this case S=x and r=l (length of string)

so,

θ=x /l

Putting in eq(i)

a=-g (x / l)

or a=- (g/l) x

where g and l are constant,therefore,

$$a \propto -x$$

Hence it is proved that the motion of pendulum is S.H.M.The acceleration of the simple pendulum is directly proportional to the displacement and directed towards the mean position. Thus the motion of simple pendulum is simple harmonic.

TIME PERIOD OF SIMPLE PENDULUM:

A/C to the definition of time period

$$T = \frac{2\pi}{\omega} \text{ ---------(ii)}$$

In case of circular motion

$$a = -\omega^2 x \text{ -----------(iii)}$$

and

In case of Pendulum

$$a = -\frac{g}{l}x \text{ -----------(iv)}$$

comparing eq(iii) and eq(iv), we get

$$\boxed{\omega = \sqrt{\frac{g}{l}}}$$

putting in eq(ii),we get

$$T = \frac{2\pi}{\sqrt{\frac{g}{l}}}$$

or

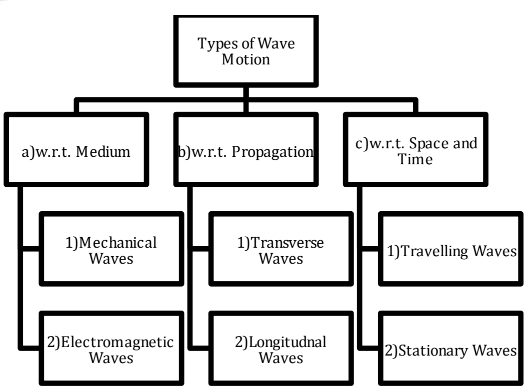
$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

WAVE MOTION

DEFINITION:

The mechanism by which energy transfers from one point to another is known as wave motion. Wave motion is a form of disturbance, which travels through a medium due to periodic motion of particles of the medium about their mean position.

TYPES OF WAVES:



A) W.R.T. MEDIUM:

1.MECHANICAL WAVES:

Those waves which require a particular medium for their propagation are known as mechanical waves.

For Example: Waves in water, Waves in String, Sound Waves e.t.c.

2. ELECTROMAGNETIC WAVES:

Those waves which do not require a particular medium for their propagation are known as electromagnetic waves.

For Example: Light waves, X rays, Microwaves e.t.c.

B) W.R.T. PROPAGATION:

1.TRANSVERSE WAVES:

Those waves in which particles of medium oscillate perpendicular to the direction of propagation of waves are called transverse waves. These waves consists of "Crests" and "Troughs". These transverse waves can be mechanical or electromagnetic in nature, water waves, waves in string are the examples of mechanical and transverse waves, while the x rays, radio waves, light waves are the example of electromagnetic transverse waves.

2.LONGITUDNAL WAVES:

Those waves in which particles of medium oscillate parallel to the direction of propagation of waves are called transverse waves. These waves consists of "Compressions" and "Rarefactions". These waves are produced in elastic materials like gases and springs. These waves are produced due to the high and low pressure zones in the medium. For Example: sound waves are longitudinal.

C) W.R.T. SPACE AND TIME:

1.TRAVELLING WAVES:

Travelling waves are those waves in which the displacement depends on both space and time. In case of travelling wave all particles of the medium vibrate simple harmonically with the frequency equal to the frequency of vibration of source that drives the wave into medium.

A travelling wave can be produced in a medium by disturbing its one end.
Mathematically the wave function is given by,

$$y=f(x)$$

Here y is the vertical displacement of particle from its mean position and x is the horizontal displacement. As we know that when the wave travels though a medium then its position changes w.r.t.

time also. So we can give the location of wave by the following equation.

$$y=f(x,t)$$

This is called wave function.

For travelling wave, moving along + x-axis the function will be, $y=f(x - vt)$

and

For travelling wave, moving along - x-axis the function will be, $y=f(x + vt)$

2.STATIONARY OR STANDING WAVES:

The waves which are formed due to the overlapping of two travelling waves of same amplitude and frequency moving in opposite direction in the same medium are called stationary waves. These waves are only the function of time not space.

STANDING OR STATIONARY WAVES IN A STRING:

When a stretched is fixed between two supports and is then plucked from the middle, the crest extends the whole length between the supports. At each end, the wave reflects from the denser medium and hence it suffers a phase change. The crest returns as a crest. Thus a wave is set up between two fixed points which lasts for a long time.At the end P and Q, the incident and reflected waves are always equal in amplitude and opposite in phase and hence the ends are stationery. Such waves are called stationery waves.

In a stationery waves the points of minimum displacement are called NODES (N) and the points of maximum displacement are called ANTI NODES (A).

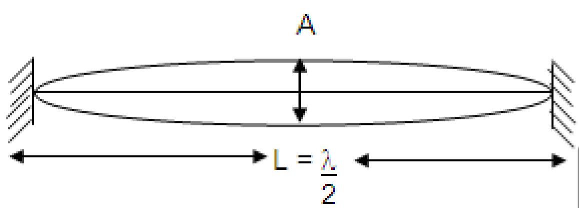
FUNDAMENTAL FREQUENCY AND HARMONICS:

When a stretched string is fixed between two ends and then plucked, stationery waves are produced due to superposition of wave's incident and reflected from the ends. The string can vibrate into several segments. There vibrations are called normal modes. Each mode has its characteristic frequency.

(i) **For ONE loop:**

When the string is plucked from the center it vibrates in one loop. The frequency of such a vibration is called fundamental frequency or first fundamental frequency or first harmonics and it is the lowest frequency with which the string can vibrate. In this case.

$$L = \frac{\lambda}{2} \quad or\ \lambda = 2L$$



Let ν_1, the frequency of first harmonics is given by

$$\because V = \nu\,\lambda \qquad \text{or}\ \nu = \frac{V}{\lambda}$$

or

$$\boxed{\nu_1 = \frac{V}{2L}}$$

(ii) **For TWO loops:** When the string vibrates in two loops, the frequency is called second harmonics, In this case.

$$\lambda = L$$

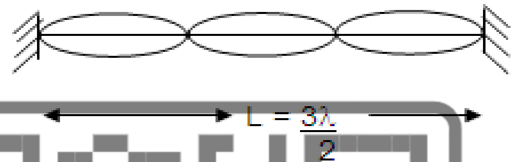
Let ν_2, the frequency of second harmonics is given by

$$\because V = \nu\,\lambda \quad \text{or } \nu = \frac{V}{\lambda}$$

or

$$\boxed{\nu_2 = \frac{V}{L}}$$

(iii) **<u>For THREE loops:</u>** When the string vibrates in two loops, the frequency is called second harmonics, In this case.



$$L = 3\frac{\lambda}{2} \quad or\ \lambda = \frac{2L}{3}$$

Let ν_3, the frequency of third harmonics is given by

$$\because V = \nu\,\lambda \quad \text{or } \nu = \frac{V}{\lambda}$$

or

$$\boxed{\nu_3 = \frac{3V}{2L}}$$

(iv) **<u>For "n" loops:</u>** When the string vibrates in two loops, the frequency is called second harmonics, In this case.

$$L = n\frac{\lambda}{2} \quad or\ \lambda = \frac{2L}{n}$$

Let ν_3, the frequency of third harmonics is given by

$$\because V = \nu\,\lambda \quad \text{or } \nu = \frac{V}{\lambda}$$

or

$$\boxed{\nu_n = \frac{nV}{2L}}$$

$$\because \nu_1 = \frac{V}{2L}$$

so,

$$\boxed{\nu_n = n\nu_1}$$

Thus we see that in case of a string fixed at both ends the harmonics are integral multiple of the fundamental frequency.

SUPERPOSITION PRINCIPLE

PRINCIPLE:

When two or more than two waves overlap each other, then a resultant wave is formed. The net wave displacement caused by the resultant wave is found equal to the algebraic sum of the individual wave displacements of all given waves. Mathematically we can write as,

$$Y = y_1 + y_2 + y_3 + \cdots + y_n$$

EXPLANATION:

Let us consider two sinusoidal waves with the same amplitude, frequency and wavelength and travelling in opposite direction. i.e.

$y_1 = A_0\ Sin(kx - \omega t)$ and
$y_2 = A_0\ Sin(kx + \omega t)$

where,

A_0=Amplitude of wave
$k\ = \frac{2\pi}{\lambda}$ = Angular wave number
ω=2 πf
x= Position
t= Time

According to the superposition principle the resultant wave is given by,

$$Y = y_1 + y_2$$

By putting values ,

$$Y = A_0\ Sin(kx - \omega t)\ \ + A_0\ Sin(kx + \omega t)$$
$$Y = A_0[\ Sin(kx - \omega t)\ \ +\ \ Sin(kx + \omega t)\ ]$$
$$Y = A_0\ [2Sin(kx)Cos(\omega t)\ ] \qquad \because Sin(\alpha - \beta)\ +\ Sin(\alpha + \beta) = Sin \propto Cos\ \beta$$

This equation represents the wave function of the stationary wave in which the amplitude is equal to 2 $A_0\ Sin(kx)$.

POSITIONS OF NODES:

As we know that the nodes are the point of minimum amplitude or intensity ,therefore

$$Sin(kx) = 0$$
$$kx = \sin^{-1} 0$$
$$\frac{2\pi}{\lambda}x = 0^0, 180^0, 360^0, 540^0, \ldots$$

or $\quad \frac{2\pi}{\lambda}x = 0, \pi, 2\pi, 3\pi, 4\pi, \ldots$

or $\quad x = \frac{\lambda}{2\pi}(0, \pi, 2\pi, 3\pi, 4\pi, \ldots)$

or

$$\boxed{x = 0, \frac{\lambda}{2}, \lambda, 3\frac{\lambda}{2}, 2\lambda, \ldots}$$

These are the positions of Nodes in standing waves, which has the minimum amplitudes.

POSITIONS OF ANTINODES:

As we know that the antinodes are the point of maximum amplitude or intensity ,therefore

$$Sin(kx) = \pm 1$$

$$kx = \sin^{-1} \pm 1$$

$$\frac{2\pi}{\lambda} x = 90^0, 270^0, 450^0, 630^0, \ldots$$

or $$\frac{2\pi}{\lambda} x = \frac{\pi}{2}, 3\frac{\pi}{2}, 5\frac{\pi}{2}, 7\frac{\pi}{2}, \ldots$$

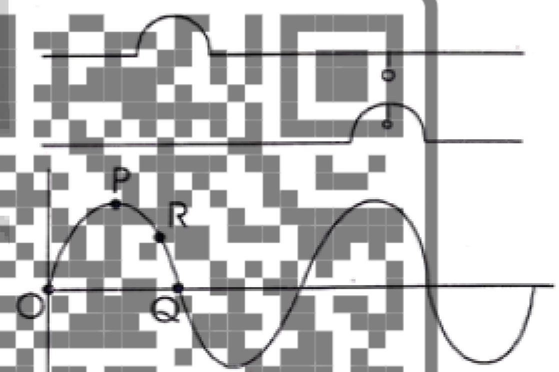
or $$x = \frac{\lambda}{2\pi}\left(\frac{\pi}{2}, 3\frac{\pi}{2}, 5\frac{\pi}{2}, 7\frac{\pi}{2}, \ldots\right)$$

or

$$\boxed{x = \frac{\lambda}{4}, 3\frac{\lambda}{4}, 5\frac{\lambda}{4}, 7\frac{\lambda}{4}, \ldots}$$

These are the positions of Antinodes in standing waves, which has the maximum amplitudes.

ENERGY IN WAVES



Let a harmonic wave travelling along a string. The points P,Q and R represent various segments of the string which move vertically. The wave moves a distance equal to one wavelength 'λ' in time period 'T'. We know that every point of the string moves vertically up or down.Thus every segment of equal mass has the same total energy. The energy of the segment P is entirely potential energy since the segment is momentarily stationary. The energy of the segment Q is entirely kinetic energy and segment R has both kinetic & potential energies. Suppose at point Q, the mass of the segment of the string is Δm and it has maximum transverse velocity $V_{y\,MAX}$. Then the total energy of the segment is.

$$\Delta E = K.E$$

$$\Delta E = \tfrac{1}{2}\Delta m\,(V_{y\,MAX})^2$$

$$\Delta E = \tfrac{1}{2}\Delta m\,(y_0\omega)^2$$

$$\Delta E = \tfrac{1}{2}\Delta m\,y_0^2\omega^2$$

Now , power is defined as

$$P = \frac{\Delta E}{T}$$

$$P = \frac{\tfrac{1}{2}\Delta m\,y_0^2\omega^2}{T}$$

$$P = \frac{1}{T} \times \frac{1}{2}\,\Delta m\,y_0^2\omega^2$$

$$P = \frac{\lambda}{T} \times \frac{1}{2}\frac{\Delta m}{\lambda}\,y_0^2\omega^2$$

$$P = \lambda\nu \times \frac{1}{2}\frac{\Delta m}{\lambda}\,y_0^2\omega^2 \qquad \because \nu = \frac{1}{T}$$

$$P = v \times \frac{1}{2}\frac{\Delta m}{\lambda}\,y_0^2\omega^2 \qquad \because v = \lambda\nu$$

$$P = v \times \frac{1}{2}\mu\,y_0^2\omega^2 \qquad \because \mu = \frac{\Delta m}{\lambda}$$

or

$$\boxed{P = \frac{1}{2} v\, \mu\, {y_0}^2 \omega^2}$$

This result shows that the power transmitted by harmonic waves produced 0“ a string is proportional to,

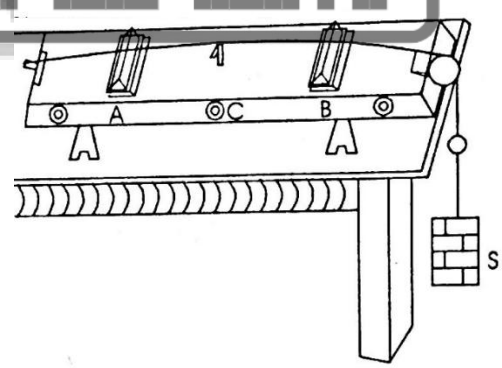
(i) The Velocity of the waves ‘V’
(ii) The square of the frequency ω
(iii) The square of the amplitude y_o
(iv) The linear density of the medium (String) μ

SONOMETER

A Sonometer consists of a hollow wooden box over which steel wire a stretched. One end of the wire is tied to a peg and the other end passes over a friction – less pulley. A hanger is tied at the other end. The hanger carries slotted weights to change the tension ‘T’ in the wire. Two bridges C and D are placed below the wire. The length of the vibrating wire can be changed by changing the distance between the bridges C and D. The Sonometer is used for determining the frequency of a turning fork.

In order to find the frequency of a given tuning fork, a tension T = Mg is produced in the wire by keeping a mass M in the hanger. A thin piece of paper called rider is placed on the wire between C and D. The stem of the vibrating tuning fork is placed against the board of the Sonometer and the distance between the bridges is adjusted, when the frequency of the vibrating wire is in unison with the frequency of the tuning fork, the rider will jump off the wire. Let L be the distance between the bridges at this, position, and μ be mass per unit length (Linear density) of the wire, then the fundamental frequency of the tuning for will be,

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \text{ --------------(i)}$$



Where $\sqrt{\frac{T}{\mu}}$ speed of a transverse wave in the wire and L is half the wavelength

Equation (1) also gives the three laws of transverse vibration of string.

(i) The frequency produced in a stretched string is inversely proportion to its length,

$$f \propto \frac{1}{L}$$

(ii) For a given mass per unit length and length of string the frequency is directly proportional to the square root of its tension.

$$f \propto \sqrt{T}$$

(iii) For a given tension and length of string the frequency is inversely proportional to the square root of its mass per unit length.

$$f \propto \frac{1}{\sqrt{\mu}}$$

These three laws can be verified by a Sonometer.

SOUND

DEFINITION:

"A vibration transmitted by air or other medium in the form of alternate compressions and rarefactions of the medium is known as Sound."

PRODUCTION OF SOUND:

Sound is produced by a vibrating body like a drum, bell, etc, when a body vibrates. due to the to and fro motion of the drum, compressions and rarefactions are produced and transmitted or pro pagated in air. When a body vibrates in air, it produces longitudinal waves by compressions and rarefactions. These compressions and rarefactions are traveled by the particles of the medium and transferred into the next particles. Due to this transference, sound propagates in a medium.

1. INFRA SONIC SOUND: The term "infrasonic" applied to sound refers to sound waves below the frequencies of audible sound, and nominally includes anything under 20 Hz.

2. AUDIBLE FREQUENCY RANGE: Usually "sound" is used to mean sound which can be perceived by the human ear, i.e., "sound" refers to audible sound unless otherwise classified. A reasonably standard definition of audible sound is that it is a pressure wave with frequency between 20 Hz and 20,000 Hz

3. ULTRA SONIC SOUND: The term "ultrasonic" applied to sound refers to anything above the frequencies of audible sound, and nominally includes anything over 20,000 Hz. Frequencies used for medical diagnostic ultrasound scans extend to 10 MHz and beyond.

SPEED OF SOUND WAVES

NEWTON'S FORMULA FOR THE SPEED OF SOUND WAVE:

In case of mechanical waves, the velocity of propagation depends upon the ratio between the elastic property of the medium (bulk modulus), and the inertial property of the medium (density).

Sound waves are compression waves which propagate through compressible medium such as air. For compression waves, the elastic property describes how the medium responds to changes in pressure with a change in volume. This is known as bulk modulus B .

$$B = \frac{-\Delta P}{\Delta V/V}$$

Where ΔP is the change in pressure and Δv is the change in volume V. The negative sign ensures that an increase in pressure (ΔP > o) causes a decreases in volume (Δv < o).

The inertial property of a medium is gives by its density "ρ" .Hence the speed of sound wave in a medium is given by

$$v = \sqrt{\frac{B}{\rho}}$$

Newton's formula was based on the assumption that when compressions and refractions travel through medium (air or gas), the temperature of the air remains constant and Boyle's law is obtained under this isothermal process. So the Bulk modulus B is equal to the pressure P.

$$\boxed{v = \sqrt{\frac{P}{\rho}}}$$

The above formula is known as Newton's formula for the speed of sound.The speed of sound in air as obtained by Newton's formula was not in good agreement with the experimental results. Theoretical value was less than the experimental value.

LAPLACE CORRECTION:-

Laplace suggested that when compressions and rarefaction travel through air, the temperature falls. Therefore, the compression and rarefactions occur adiabatically.

In such a case the bulk modulus of the gas is not equal to the pressure of the gas but, 'γ' times the pressure of the gas. Where, 'γ' is the ratio of the specific heat of the gas at constant pressure to the specific heat at constant volume, For air, $\gamma = 1.4$.

$$\boxed{v = \sqrt{\frac{\gamma P}{\rho}}}$$

This is known as Laplace correction.

For an ideal gas PV = nRT

$$v = \sqrt{\frac{\gamma nRT}{V\rho}}$$

As we know that, $V\rho$=m (mass of gas) so,

$$v = \sqrt{\frac{\gamma nRT}{m}}$$

or,

$$\boxed{v = \sqrt{\frac{\gamma RT}{M}}} \qquad \because M=n/m$$

The speed of sound wave is directly proportional to the square root of the temperature in Kelvin Scale.

BEATS

The periodic alternation of sound between a maximum and minimum loudness caused by the super position of two waves of nearly the same frequency are called beats.

PRODUCTION OF BEATS:

Take two tuning forks A and B of nearly the same frequencies say 32 and 30 hz. Respectively place them at equal distance from the ear. Let at time t = 0, the two forks are in phase and the right hand prongs of both the forks are sending compressions towards right. There two compressions will arrive at the ear together and thus a loud sound in heard.

As time goes on the fork B, vibrating at slightly lower frequency than A will begin to fall behind. After ¼ second the fork A will complete 8 vibrations and will just be sending out compressions. On the other hand the fork B will complete 71/2 vibrations and will sending out a rarefaction from B will reach the ear at the same time. They will cancel each other. Hence no sound will be heard.

As time passes the fork B will still fall behind A. After half a second the fork A will complete 16 vibrations while the fork B will complete 15 vibrations. Both the forks will be sending out compressions together and thus again a loud sound will be heard.

After ¾ second – fork A will complete 24 vibrations and fork B will complete 22½ vibrations. At this compression and fork B will be sending a rarefaction. B will be sending a rarefaction. Thus no sound will be heard.

After 1 second the fork A will complete 32 vibrations. Both the forks will be sending out compressions together and thus again a loud sound will be heard.

We have seen that in one second two beat are produced. The difference between the frequencies is also two. Thus we conclude that the number of beats per second is equal to the difference between the frequencies of the forks. The maximum beat frequency that the human ear can detect is 7 beats per second. When the beat frequency is number of beats produced per second) is greater than 7 we cannot hear them clearly.

ANALYTICAL TREATMENT OF BEATS:

Consider two waves with equal amplitude traveling through a medium in the same direction having slightly different frequencies f_1 and f_2. The displacement that each wave produces can be represented by the equation.

$$Y_1 = A_o \cos 2\pi f_1 t \quad \text{------------} \quad (1)$$

$$Y_2 = A_o \cos 2\pi f_2 t \quad \text{------------} \quad (2)$$

Where y1 and y2 are the instantaneous displacement of the waves (1)and (2). Let Y be the instantaneous displacement of the resultant wave, then by the principle of superposition of waves, the displacement Y of the new wave can be found out by adding the two displacements.

$$Y = Y_1 + Y_2$$

$$Y = A_o (\cos 2\pi f_2 t + \cos 2\pi f_2 t) \qquad \because \cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$Y = 2A_o \cos 2\pi \left(\frac{f_1 - f_2}{2}\right) t \times \cos 2\pi \left(\frac{f_1 + f_2}{2}\right) t \quad \text{---------} \quad (3)$$

The resultant displacement y as expressed by equation (3) has an affective frequency equal to the average $(f_1 + f_2)$ and amplitude is given by

$$A = 2A_o \cos 2\pi \left(\frac{f_1 - f_2}{2}\right) t \quad \text{------------} \quad (4)$$

Equation (4) shows that the amplitude varies in time with frequency given by $(\underline{f_1 - f_2})$ when f_1 close to f_2 the amplitude variation is shown by the dotted line of the resultant wave. A beat of maximum amplitude is detected whenever.

$$\cos 2\pi \left(\frac{f_1 - f_2}{2}\right) t = \pm 1$$

There will be two maxima in each cycle. Since the amplitude varies with the frequency as, the number of beats per second. Hence the beat frequency f_b is twice this value.

$$\boxed{f_b = f_1 - f_2} \quad \text{---------------} \quad (5)$$

CHARACTERISTICS OF MUSICAL SOUND

Musical sounds of tones can be distinguished from one other by the following characteristics.

(i) Intensity or loudness (ii) Pitch or frequency (iii) Quality.

INTENSITY AND LOUDNESS:

The characteristics of sound by which we can distinguish between Loud and Faint sound is called Loudness. The intensity of sound is defined as the amount of energy transmitted per sound through unit area held perpendicular to the direction of propagation of sound wave. It is denoted by I.

$$Intensity = \frac{Energy \;\; transmitted}{Time \;\times area}$$

or $$\boxed{I = \frac{E}{t \times A}}$$

Unit: It's S.I unit is $\frac{Joule}{sec \times m^2}$ or $\frac{watt}{m^2}$

WEBER – FETCHNER LAW:

Weber and Fetchner found experimentally that the loudness depends upon the intensity as well as on ear. The ear operates on logarithmic scale rather than responding linearly to the intensity of sound.

The loudness (L) of the sound as received by the ear, is proportional to the logarithm of its intensity 'I'.

$$L \propto Log\ I$$

or $$\boxed{L = K\ Log\ I}$$

Where k is a constant of proportionally.

INTENSITY LEVEL:

If Io and I be intensities of two sounds waves, then the difference in the loudness of sound ($L - L_0$) is known as the intensity level between them.

Intensity level $= L - L_o = K \log I - K \log I_o$

$$\boxed{S_L = K \log [\ I/I_o]}$$

Unit of intensity level (loudness) and is called as bel after the name of Alexandar Graham Bell. Since bel is a big unit of loudness and its submultiples are used for practical purposes.

1 deci bel = 0.1 bel.

1 centi bel = 0.01 bel.

PITCH OF SOUND:

The characteristics of sound by which a shrill sound can be distinguished from a grave sound is called pitch of sound.It depends upon the frequency of sound. More the frequency higher the pitch lowers the frequency lowers the pitch of sound. For example the pitch of sound produced by rats, bats, cats is higher than that of frog, dogs beating drums.

QUALITY OF SOUND:

The quality of sound is that characteristic of sound which enables the ear to recognize a sound also assigns its source.

The note played at piano to be different from the note played at the violin through both has same frequency and loudness it is because of the quality of two notes is different. A wave form which is a combination of fundamental frequency and second harmonic and a wave from which is a combination of the fundamental frequency and third harmonic will produce sound of different qualities though their pitch and loudness may be same.

DOPPLER'S EFFECT

The change in the pitch (frequency) of sound caused by the relative motion between the source and observer is called Doppler's Effect.

<u>CASE 1(A):</u> When listener moves towards the stationary source of sound.

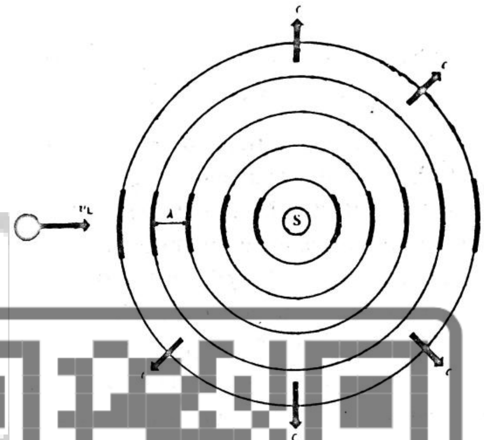
Let us consider the listener is moving towards the stationary source of sound emitting sound of frequency ν ,with velocity V_L.The speed of sound is V.In this case the apparent frequency heard by listener is ν'.

As we know that

$$V = \nu\,\lambda \qquad => \lambda = \frac{V}{\nu} \text{----(i)}$$

and

$$\nu = \frac{V}{\lambda} \text{ (Real Frequency)}$$



But the relative velocity of sound for the listener will be $V+V_L$, then apparent frequency will be

$$\nu' = \frac{V+V_L}{\lambda}$$

putting the value of λ from eq(i),we get

$$\nu' = \frac{V+V_L}{\frac{V}{\nu}}$$

or

$$\boxed{\nu' = \left(\frac{V+V_L}{V}\right)\nu}$$

This expression shows that the apparent frequency will be greater than real frequency.

<u>CASE 1(B):</u> When listener moves away from the stationary source of sound.

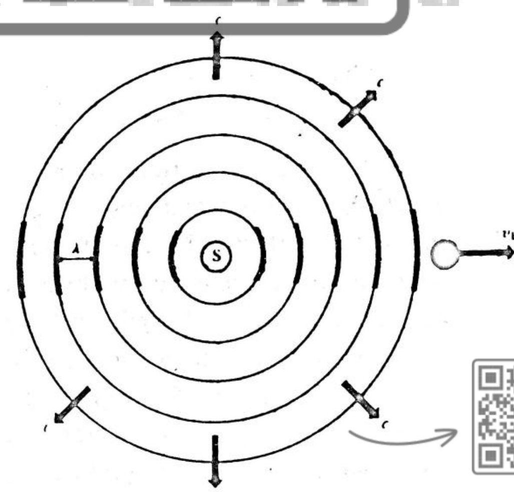
Let us consider the listener is moving towards the stationary source of sound emitting sound of frequency ν ,with velocity V_L.The speed of sound is V.In this case the apparent frequency heard by listener is ν'.

As we know that

$$V = \nu\,\lambda \qquad => \lambda = \frac{V}{\nu} \text{----(ii)}$$

and

$$\nu = \frac{V}{\lambda} \text{ (Real Frequency)}$$



But the relative velocity of sound for the listener will be $V-V_L$, then apparent frequency will be

$$\nu' = \frac{V-V_L}{\lambda}$$

putting the value of λ from eq(ii),we get

$$\nu' = \frac{V-V_L}{\frac{V}{\nu}}$$

or

$$\boxed{\nu' = \left(\frac{V-V_L}{V}\right)\nu}$$

This expression shows that the apparent frequency will be less than real frequency.

<u>CASE 2(A):</u> When source of sound moves towards the stationary listener.

Let us consider a source of sound moving with velocity Vs towards stationary listener. The wave crests detected by the listener are closer together because the source is moving in the direction of outgoing wave resulting the shortening of wavelength.

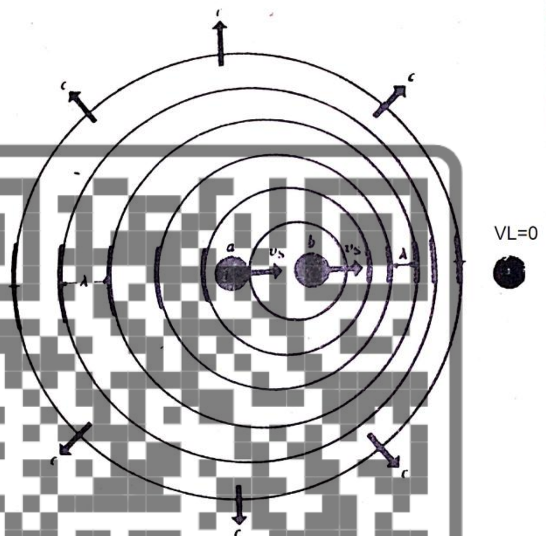
As we know that

$V = \nu\,\lambda \qquad => \lambda = \frac{V}{\nu}$ (Distance occupied by one wave)

and

$\nu = \frac{V}{\lambda}$ (Real Frequency) ------(i)

During each vibration source travels a distance equal to $\frac{V_S}{\nu}$ towards the listener then apparent wavelength is shortened.



$$\lambda' = \frac{V}{\nu} - \frac{V_S}{\nu}$$

or

$$\lambda' = \frac{V - V_S}{\nu} \text{ ---------(ii)}$$

The apparent frequency from eq(i) is given as,

$$\nu' = \frac{V}{\lambda'}$$

Putting value of λ' from eq(ii),we get

$$\nu' = \frac{V}{\frac{V - V_S}{\nu}}$$

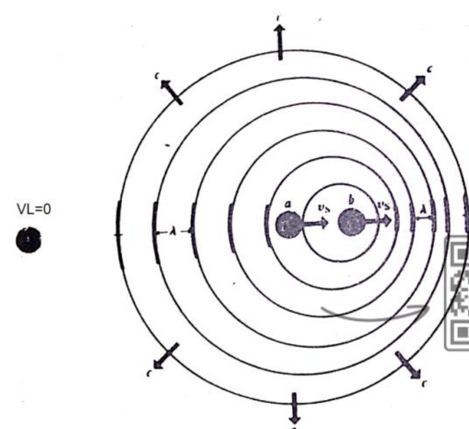
or

$$\boxed{\nu' = \left(\frac{V}{V - V_S}\right)\nu}$$

This expression shows that the apparent frequency will be greater than real frequency.

<u>CASE 2(B):</u> When source of sound moves away from the stationary listener.

Let us consider a source of sound moving with velocity Vs away from stationary listener. The wave crests detected by the listener are farther together because the source is moving in the opposite direction of outgoing wave resulting the increasing of wavelength.



As we know that

$V = \nu\,\lambda \qquad => \lambda = \frac{V}{\nu}$ (Distance occupied by one wave)

and

$\nu = \frac{V}{\lambda}$ (Real Frequency) ------(i)

During each vibration source travels a distance equal to $\frac{V_S}{\nu}$ away from the listener then apparent wavelength is increased.

$$\lambda' = \frac{V}{\nu} + \frac{V_S}{\nu}$$

or

$$\lambda' = \frac{V+V_S}{\nu} \text{ ----------(ii)}$$

The apparent frequency from eq(i) is given as,

$$\nu' = \frac{V}{\lambda'}$$

Putting value of λ' from eq(ii),we get

$$\nu' = \frac{V}{\frac{V+V_S}{\nu}}$$

or

$$\boxed{\nu' = \left(\frac{V}{V+V_S}\right)\nu}$$

This expression shows that the apparent frequency will be less than real frequency.

CASE 3(A): When both source of sound and listener move towards each other.

In this case,

$$\boxed{\nu' = \left(\frac{V+V_L}{V-V_S}\right)\nu}$$

This expression shows that the apparent frequency will rapidly increase.

CASE 3(B): When both source of sound and listener move away from each other.

In this case,

$$\boxed{\nu' = \left(\frac{V-V_L}{V+V_S}\right)\nu}$$

This expression shows that the apparent frequency will rapidly decrease.

ACOUSTICS

In the recording and reproduction of sound, one must avoid all possible sources of distortion. The size, shape of room, a studio or an auditorium the material used in the construction of the floor, ceiling walls, doors and windows etc. Also the number of chairs produced or reproduced in it.

A careful study of all these factors and their effect in the quality of sound produced in a room a studio or an auditorium is an important branch of radio engineering and is called ACOUSTICS.

Sound starting from a source S will reach a listener only along one path SL where as all other going spherical along all other path are absorbed all other path are absorbed or reflected by the walls, floor, ceiling etc. At the room ,the material of walls etc. absorbs sound waves of different frequencies by different amounts and remaining components of frequencies are reflected and approach to listener by several paths are shown.

REVERBERATION:

As some of sound wave energy is absorbed and some is reflected. The reflected portion travels back to hall and re-unite to form "echoes' which interfere to produce desirable or undesirable hearing. Even when the direct plus from source has ceased, some of the energy due to multi reflections from the walls, floor, ceiling etc is still on it way to ear. This cause prolongation or persistence of sound for some time.

The persistence of audible sound after the sources has ceased to operate is called reverberation and the time during which sound persists is called reverberation times.All the above sources which affect the quality of sound can be minimized by careful choice.

M.C.Qs.

1. The oscillatory motion in which the instantaneous acceleration is proportional to the displacement of the displacement of the oscillating bodies is called:
(a) Elastic motion (b) Translatory motion
(c) Transverse motion (d) Harmonic motion

2. Total energy of a particle performing SHM is directly proportional to:
(a) The amplitude
(b) The square root of amplitude
(c) Square of amplitude
(d) The reciprocal of amplitude

3. When a particle is executing SHM it is found that:
(a) The frequency depends upon the amplitude
(b) The periods depend on the amplitude.
(c) The period and frequency depend upon the amplitude
(d) The period and frequency are independent of the amplitude.

4. Beats are produced due to:
(a) diffraction of waves in time
(b) reflection of waves in time
(c) interference of waves in time
(d) polarization of waves in time

5. Which one of the following not undergoing a simple harmonic motion?
(a) Motion of a pendulum
(b) vibration of a violin string
(c) Motion of body in a rectilinear Path
(d) Oscillation of mass on a string

6.The product of frequency and time period is:
(a) 1 (b) 2 (c) 3 (d) 4

7. If a second pendulum is taken up on the moon, in order to have its time period same:
(a) The length of the pendulum must be increased (b) The length of the pendulum must be decreases
(c) The length of the pendulum must be kept the same
(d) None of the above

8. An ordinary clock loses time in summer this is because:

(a) The length of the pendulum increases
(b) The length of the pendulum decreases
(c) The length of the pendulum decreases and time period increases.
(d) The length the pendulum decreases and time period increases.

9.Weber Fechner law is:

(a) I α Log L (b) L α Log I
(c) I α 1/ Log L (d) I α Log L

10. Which one of the following contains a pair of transverse and longitudinal wave?

(a) Radio & X - rays
(b) Infra - red & ultra- violet
(c) Sound & radio wave
(d) Wave in a ripple tank & light

11. The velocity of a particle moving with a frequency 'f' and wave length 'λ' is:

(a) f λ (b) f/λ (c) λ/f
(d) fλ/2

12. Intensity of sound is measured in :

(a) watt/ m^2 (b) joule /m
(c) watt/ sec (d) watt/ m

13. Then temperature of air rises, the speed of sound waves increase because:

(a) wavelength of wave increases
(b) the frequency of wave increase
(c) both frequency and wavelength increases
(d) neither frequency nor wavelength increase

14. If the frequency of fifth harmonic of a vibrating string is 200 Hz ,its fundamental frequency is:

(a) 5 Hz (b) 25 Hz (c) 40 Hz (d) 100 Hz

15. It is common characteristics of all types of wave motion that without the transport of particles what transfers?

(a) Gravity (b) X rays
(c) Energy (d) Mass

16. The wave length of a radio wave when transmitted as a frequency of 150 MHz, will be:

(a) 20 m (b) 2 m (c) 10 m (d) 0.75 m

17. A simple pendulum completes one vibration in one second. If g = 981 cm/s² its length will be:

(a) 24.8 m (b) 24.8 cm
(c) 2.48 cm (d) 2.48 m

18. The range of audible sound is:

(a) 1 Hz – 10 Hz
(b) 20Hz – 20000Hz
(c) 21000Hz – 24000Hz
(d) 25000Hz – 50000Hz

19. When a string which is tied at both the ends is plucked from the centre the wave produced is:

(a) Transverse wave (b) Longitudinal wave
(c) Standing wave (d) Electromagnetic wave

20. Pitch depends upon:

(a) frequency (b) loudness
(c) time period (d) distance

21. Which of the following is not a transverse waves?

(a) x-rays (b) sound
(c) γ-rays (d) infrared

22. The distance between adjacent nodes or antinodes is:

(a) λ (b) λ/2 (c) λ/4 (d) 2λ

23. The velocity of sound in space is:
(a) zero m/s (b) 332 m/s
(c) 33200 cm/s (d) 3×10^8 m/s

24. The traveling wave in which particle of the disturbed medium move perpendicular to the direction of propagation of the wave is called:
(a) Longitudinal wave (b) Transverse wave

(c) Standing wave (d) Stationary wave

25. Earthquake waves are the example of:
(a) audio waves (b) infrasonic waves
(c) Ultrasonic waves (d) shock waves

26. In a stretched string, if the speed of the wave is four times, the tension will be ____ times the original:
(a) 2 (b) 4 (c) 8 (d) 16

27. Frequency of a stretched string is proportional to the:
(a) Tension (b) linear density
(c) reciprocal of the length d) Square of the tension

28. For a stationary wave in a string the points at which the particle is at maximum displacement from the mean position are called:
(a) Nodes (b) Anti nodes
(c) Compression (d) Rarefaction

29. A string fixed at two ends vibrates in two whole segment. The standing wave pattern set up is called:
(a) Fundamental (b) First harmonic
(c) Second harmonic (d) fourth harmonic

30. Sin θ= θ if θ is specially less than:
(a) 15^0 (b) 10^0 (c) 5^0 (d) 1 radian

PAST PAPER M.C.Qs.

2022

3.The range of audible sound is:
* 1 to 10 Hz **<u>* 20Hz to 20,000 Hz</u>** * 21000 to 24000 Hz * 25000 Hz onwards

19.The oscillatory of simple pendulum, the restoring force is
<u>* mg sinθ</u> * mg cosθ * mg tanθ * mg

29.Pitch of sound depends upon:
*Amplitude * Intensity * frequency **<u>* loudness</u>**

32.The distance between two consecutive nodes of a stationary wave is:
* $\frac{\lambda}{4}$ * λ * $\frac{\lambda}{2}$ * $\frac{\lambda}{3}$

2021

(x) The speed of sound in space (vaccum) is:
*332m/s * 344m/s *330m/s **<u>*Zero m/s</u>**

(xi) A simple pendulum is performing S.H.M with time period T. If its length is doubled. The new time period will be

*2T *0.5T *2.5T *** 1.414T**

(xv) A body is executing S.H.M with amplitude A. Its potential energy is maximum when its displacement from mean position is:

*Zero *A/2 ***A** * A/ 4

(xxvii) If the mass of the bob of a simple pendulum is doubled, its time period will:

*be doubled *becomes triple *** remain same** *behalved

(xxviii) The unit of Intensity of sound is:

*watt/m^2 * watt-s * watt/s *** watt/m**

(xxxiii) It does not exhibit simple harmonic motion:

*A hanging spring supporting a weight *The motion of the prongs of tunning fork

***The wheel of an automobile** *Motion of a string of a violin

2019

2.Beats are produced due to:

* diffraction of waves in time *reflection of waves in time

***interference of waves in time** *polarization of waves in time

10.The product of frequency and time period is:

***1** *2 *3 *4

17. Weber Fechner law is:

*I α Log L *** L α Log I** * I α 1/ Log L * I α Log L

2018

5. Intensity of sound is measured in :

*** watt/ m^2** *joule /m *watt/ sec *watt/ m

10. Then temperature of air rises, the speed of sound waves increase because:

***wavelength of wave increases** *the frequency of wave increase

* both frequency and wavelength increases * neither frequency nor wavelength increase

2017

9. If the frequency of fifth harmonic of a vibrating string is 200 Hz ,its fundamental frequency is:

*5 Hz *25 Hz ***40 Hz** *100 Hz

10. The speed of sound in vacuum is:

***zero m/s** *332 m/s *33200 cm/s` *3 x 10^8 m/s

11. The distance between two consecutive nodes of a transverse stationary wave is equal to:

*$\frac{\lambda}{4}$ *$\frac{\lambda}{2}$ *λ * 2λ

2016

5.The range of audible sound is:

*1 Hz – 10 Hz ***20Hz – 20000Hz**

*21000Hz – 24000Hz *25000Hz – 50000Hz

15. Two vibrating bodies, having slightly different frequencies, produce:

*Echo ***Beats** *Resonance *Polarization

2015

14. The velocity of a wave of wavelength ' λ ' and frequency 'ν' is given by:

* $\frac{\nu}{\lambda}$ * $\frac{\lambda}{\nu}$ ***$\nu\lambda$** * $\frac{1}{\lambda\nu}$

2014

3. The earth quake waves are the example of:

*Audible Waves *** Infrasonic waves** *Shock waves *Ultrasonic Waves

5. The distance between two consecutive nodes of a stationary wave will be:

* λ *** $\lambda/2$** * $\lambda/4$ * $\lambda/6$

15. If the mass of the bob of a simple pendulum is doubled, its time period will be:

*be doubled *become triple ***remain the same** *halved

2013

1. Power Law determines

*power *work *intensity ***loudness of sound**

10. The wave enters from one medium to another medium, no change occurs in its:

***frequency** *wavelength *amplitude *speed

15. The time period of simple pendulum depends upon:

*mass *length

*acceleration due to gravity ***both length and acceleration due to gravity**

3. The maximum number of beats per second which can be detected by the human ear is:

*2 *3 *5 ***7**

2012

2. The S.I unit of intensity level of sound is:

*watt * diopter * sone *** decibel**

6. The frequency of wave produced in a stretched string depends upon:

*length * tension * linear density *** all of these**

13. Sin θ= θ if θ is specially less than:

*15^0 * 10^0 * 5^0 *** 1 radian**

2011

1. Earthquake waves are the example of:

*audio waves *** infrasonic waves**

* Ultrasonic waves * shock waves

9. This is compression waves:
*light waves *x rays **<u>*sound waves</u>** *radio waves

10. If two tuning forks of frequencies 256 Hz and 260 Hz are sounded together, the number of beats per second will be:
*3 **<u>*4</u>** *5 *6

<u>2010</u>

4. Which of the following does not exhibit simple harmonic motion?
*A hanging spring supporting a weight
* The balance wheel of a watch
<u>* The wheel of an automobile</u>
*the string of violin

5. Pitch depends upon:
<u>*frequency</u> * loudness
*time period * distance

6. The velocity of sound in space is:
<u>*zero m/s</u> *332 m/s *33200 cm/s` *3 x 10^8 m/s

TEXTBOOK NUMERICALS

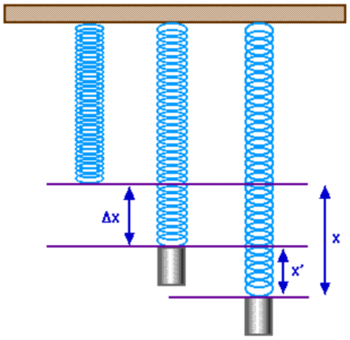
Q.1: An object is connected to one end of a horizontal spring whose other end is fixed. The object is pulled to the right (in the positive x-direction) by an externally applied force of magnitude 20 N causing the spring to stretch through a displacement of 1 cm (a) Determine the value of force constant if, the mass of the object is 4 kg (b) Determine the period of oscillation when the applied force is suddenly removed.

<u>Data:</u>
Applied Force $= F = 20\ N$
Displacement $= x = 1\ cm = 0.01\ m$
(a) Force Constant $= k = ?$
(b) Mass of Object $= m = 4\ kg$
Time period of Oscillation $= T = ?$

Q.2: A body hanging from a spring is set into motion and the period of oscillation is found to be 0.50 s. After the body has come to rest, it is removed. How much shorter will the spring be when it comes to rest?

<u>Data:</u>
Time period of Vibration= T = 0.5 sec

Displacement = x =?



Solution:

$$T = 2\pi\sqrt{\frac{m}{k}} \text{ ------ (i)}$$

Also, $W = kx$

$$mg = kx$$

$$\frac{m}{k} = \frac{x}{g}$$

Putting in equation (i)

$$T = 2\pi\sqrt{\frac{x}{g}}$$

S.O.B.S

$$T^2 = 4\pi^2\left(\frac{x}{g}\right)$$

$$(0.5)^2 = 4(3.14)^2\left(\frac{x}{g}\right)$$

$$\boxed{x = \frac{(0.5)^2 \times 9.8}{4 \times 3.14^2} = 0.06\ m}$$

Result: The spring will be 0.06 m shorter when the body is removed.

Q.3: A pipe has a length of 2.46 m. (a) Determine the frequencies of the fundamental mode and the first two overtones if the pipe is open at both ends. Take υ = 344 m/s as the speed of sound in air. (b) What are the frequencies determined in (a) if the pipe is closed at one end? (c) For the case of open pipe, how many harmonics are present in the normal human being hearing range (20 to 20000 Hz)?

Data:

Length of Pipe $= L = 2.46\ m$

Speed of Sound $= v = 344\ m/s$

(a) For Open Pipe:

Frequecy of Fundamental Mode $= f_1 = ?$

Frequecy of 2nd Harmonic $= f_2 = ?$

Frequecy of 3rd Harmonic $= f_3 = ?$

(b) For Closed Pipe:

Frequecy of Fundamental Mode $= f_1 = ?$

Frequecy of 2nd Harmonic $= f_2 = ?$

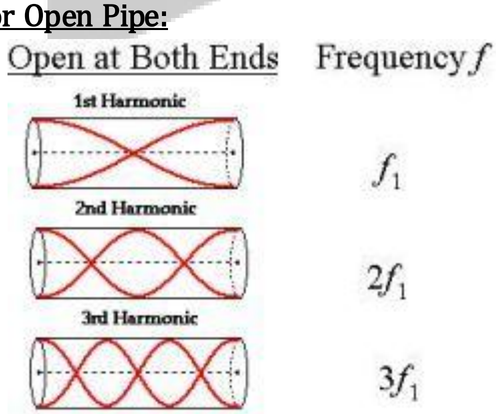
Frequecy of 3rd Harmonic $= f_3 = ?$

(c) Frequency of nth Harmonic $= f_n = 20000\ Hz$

No. of Harmonics $= n = ?$

Solution:

(a) For Open Pipe:



The fundamental Frequency is given by

$$f_1 = \frac{v}{2L} = \frac{344}{2 \times 2.46}$$

$$\boxed{f_1 = 70Hz}$$

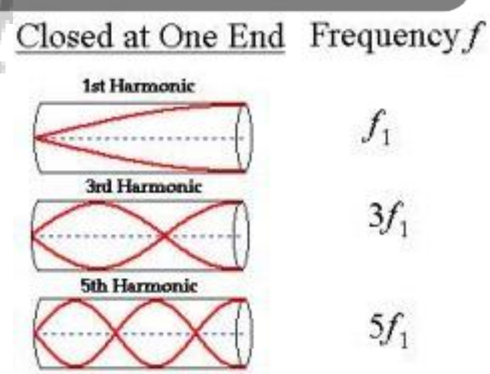
and $f_n = nf_1$

then

$$\boxed{f_2 = 2f_1 = 2 \times 70 = 140\ Hz}$$

and $\boxed{f_3 = 3f_1 = 3 \times 70 = 210\ Hz}$

(b) For Closed Pipe:



The fundamental Frequency is given by

$$f_1 = \frac{v}{4L} = \frac{344}{4 \times 2.46}$$

$$\boxed{f_1 = 35Hz}$$

and $f_n = nf_1$

then

$$f_2 = 3f_1 = 3 \times 35 = 105\ Hz$$

and $$f_3 = 5f_1 = 5 \times 35 = 175\ Hz$$

(c) For Open Pipe:

$$f_n = nf_1$$

then $$n = \frac{f_n}{f_1} = \frac{20000}{70}$$

$$n = 285\ harmonics$$

Result: (a)For Open pipe the frequencies are 70 Hz,140 Hz and 210 Hz.(b) For Closed pipe the frequencies are 35 Hz,105 Hz and 175 Hz (c) 285 harmonics are present in the normal human being hearing range.

Q.4: A standing wave is established in a 120 cm long string fixed at both ends. The string vibrates in four segments when driven at 120 Hz (a) Determine the wavelength (b) What is the fundamental frequency?

Data:

Length of String = l=120 cm = 1.2 m

No. of loops = n = 4

Frequency of fourth harmonic = f_4 =120 Hz

Fundamental Frequency = f_1 = ?

Wavelength = λ = ?

Solution:

The fundamental Frequency is given by

$$f_n = n\frac{v}{2l}$$

Or $$f_n = nf_1$$

$$f_4 = 4f_1$$

$$f_1 = \frac{f_4}{4} = \frac{120}{4} = 30\ Hz$$

The wavelength of stationary wave is given by

$$L = \frac{n\lambda}{2}$$

$$L = \frac{(4)\lambda}{2}$$

$$L = 2\lambda$$

Or $$\lambda = \frac{L}{2} = \frac{1.2}{2} = 0.6\ m$$

Result: The fundamental frequency is 30 Hz and its wavelength is 0.6 m.

Q.5: Calculate the speed of sound in air at atmospheric pressure p = 1.01x10^5 N/m^2 , taking γ = 1.40 and ρ = 1.2kg/m^3 .

Data:

Speed of sound in air = v =?

Atmospheric pressure = $P = 1.01 \times 10^5\ Pa$

Ratio of molar specific heats = $\gamma = 1.40$

Density of air = $\rho = 1.2\ kg/m^3$

Solution:

The speed of sound in air is given by

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

$$v = \sqrt{\frac{1.40 \times 1.01 \times 10^5}{1.2}}$$

$$v = 343.26\ m/s$$

Result: The speed of sound in air at S.T.P. is 343 m/s.

Q.6: A sound wave propagating in air has a frequency of 4000 Hz. Calculate the percent change in wave length when the wave front, initially in a region where T = 27^o C, enters a region where the air temperature decreases to 10^o C.

Data:

Frequency of Sound = f = 500Hz

Initial temperature = T_1 = 27 ^{0}C + 273 = 300 K

Final temperature = T_2 = 10 ^{0}C +273= 283 K

Percent Fractional Change in wavelength = $\frac{\Delta\lambda}{\lambda_1}\%$ =?

Solution:

As we know that

$$v \propto \sqrt{T}$$

$$v = k\sqrt{T}$$

So, $$v_1 = k\sqrt{T_1} \text{ ---- (i)}$$

And $$v_2 = k\sqrt{T_2} \text{ ---- (ii)}$$

Dividing eq (ii) by eq (i)

$$\frac{v_2}{v_1} = \frac{k\sqrt{T_2}}{k\sqrt{T_1}}$$

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$$

$$\frac{v_2}{v_1} = \sqrt{\frac{283}{300}}$$

$$\frac{v_2}{v_1} = 0.971$$

$$\because v = f\lambda$$

$$\therefore \frac{f\lambda_2}{f\lambda_1} = 0.971$$

$$\boxed{\lambda_2 = 0.971\lambda_1}$$

Now, $\Delta\lambda = \lambda_1 - \lambda_2$

$$\Delta\lambda = \lambda_1 - 0.971\lambda_1$$

$$\Delta\lambda = 0.029\lambda_1$$

$$\frac{\Delta\lambda}{\lambda_1} = 0.029$$

Or $\boxed{\frac{\Delta\lambda}{\lambda_1}\% = 2.9\%}$

Result: The percent fractional change in wavelength will be 2.9%.

Q.7: The frequency of the second harmonic of an open pipe (open at both ends) is equal to the frequency of the second harmonic of a closed pipe (open at one end). (a) Find the ratio of the length of the closed pipe to the length of the open pipe.(b) If the fundamental frequency of the open pipe is 256 Hz, what is the length of pipe? (Use υ = 340 m/s).

Data:

(a) Ratio of Lengths of Pipe $= \frac{L_{Closed}}{L_{Open}} = ?$

Speed of Sound $= v = 340\ m/s$

(b) Fundamental Frequency of open Pipe $= f_{1(Open)} = 256\ Hz$

Length of Open Pipe $= L_{Open} = ?$

Length of Closed Pipe $= L_{Close} = ?$

Solution:

(a) According to the given condition

$$f_{2(Open)} = f_{2(Closed)}$$

$$\frac{v}{L_{Open}} = \frac{3v}{4L_{Closed}}$$

$$\frac{1}{L_{Open}} = \frac{3}{4L_{Closed}}$$

$$\boxed{\frac{L_{Closed}}{L_{Open}} = \frac{3}{4}}$$

(b) **For Open Pipe:**

$$f_1 = \frac{v}{2L}$$

then

$$f_{1(open)} = \frac{v}{2L_{Open}}$$

$$L_{Open} = \frac{v}{2\times f_{1(open)}}$$

$$L_{Open} = \frac{340}{2\times 256}$$

$$\boxed{L_{Open} = 0.66\ m}$$

For Closed Pipe:

As we know that

$$\frac{L_{Closed}}{L_{Open}} = \frac{3}{4}$$

then

$$\frac{L_{Closed}}{0.66} = \frac{3}{4}$$

$$L_{Closed} = \frac{3}{4}\times 0.66$$

$$\boxed{L_{Closed} = 0.49\ m}$$

Result: The ratio of the length of the closed pipe to the length of the open pipe. is 3/4 and the length of open pipe is 0.66 m and closed pipe is 0.49 m.

Q.8: A 256 Hz tuning fork produces four beats per second when sounded with another fork of unknown frequency. What are two possible values for the unknown frequency?

Data:

Frequency of 1st Tuning Fork $= f_1 = 256\ Hz$

Beats Frequency $= f_b = 4\ beats/\ sec$

Frequency of 2nd Tuning Fork $= f_2 = ?$

Frequency of 2nd Tuning Fork $= f_2' = ?$

Solution:

For 1st Possible value:

$$f_b = f_1 - f_2$$

$$4 = 256 - f_2$$

$$f_2 = 256 - 4$$

$$\boxed{f_2 = 252\ Hz}$$

For 2nd Possible value:

$$f_b = f_2' - f_1$$

$$4 = f_2' - 256$$

$$f_2' = 256 + 4$$

$\boxed{f_2' = 260\ Hz}$

Result: The two possible values for unkown frequency are 252 Hz and 260Hz.

Q.9: An ambulance travels down a highway at a speed of 75 mi/h. Its siren emits sound at a frequency of 400 Hz . What frequency will heard by a person in a car traveling at 55 mi/h in the opposite direction as the car approaches the ambulance and as the car moves away from the ambulance.

Data:

Speed of Ambulance $= v_s = 75\ mi/h$

Actual Frequency $= f = 400\ Hz$

Speed of Car $= v_L = 55\ mi/h$

(a) Apparent frequncy $= f' = ?$

(b) Apparent frequncy $= f' = ?$

Speed of Sound $= v = 750\ mi/h$

Solution:

(a)When Source and listener move towards each other

$$f' = \left(\frac{V+V_L}{V-V_S}\right) f$$

$$f' = \left(\frac{750+55}{750-75}\right) \times 400$$

$$f' = \left(\frac{805}{675}\right) \times 400$$

$$\boxed{f' = 477\ Hz}$$

(b)When Source and listener move away from each other

$$f' = \left(\frac{V-V_L}{V+V_S}\right) f$$

$$f' = \left(\frac{750-55}{750+75}\right) \times 400$$

$$f' = \left(\frac{695}{825}\right) \times 400$$

$$\boxed{f' = 337\ Hz}$$

Result: When car and ambulance move towards each other the apparent frequency will be 477 Hz and when car and ambulance move away from each other the apparent frequency will be 337 Hz

Q.10: A car has siren sounding a 2 kHz tone. What frequency will be detected as stationary observer as the car approaches him at 80 km/h? Speed of sound = 1200 km/h.

Data:

Frequency of sound $= f = 2\ KHz = 2000\ Hz$

Apparent of frequency $= f' = ?$

Speed of source $= V_s = 80\ km/h$

Speed of Sound $= V = 1200\ km/h$

Solution:

When Source moves towards stationary listener

$$f' = \left(\frac{V}{V-V_S}\right) f$$

$$f' = \left(\frac{1200}{1200-80}\right) \times 2000$$

$$f' = (1.07) \times 2000$$

$$\boxed{f' = 2143\ Hz}$$

Result: The apparent frequency heard by the listener is 2143 Hz

PAST PAPER NUMERICALS

2022

x) Two cars approaching each other from opposite directions with same speed. The horn of one is blowing with the frequency of 3000 Hz and is heard by the people in the other car with the frequency of 3400 Hz. Find the speed of both cars, if speed of sound in air is 340 m/s .

Data:

Speed of Car $= v_s = v_L = v = ?$

Actual Frequency $= f = 3000\ Hz$

Apparent Frequency $= f = 3400\ Hz$

Speed of Sound $= v' = 340\ \ m/s$

Solution:

(a)When Source and listener move towards each other

$$f' = \left(\frac{v'+V_L}{v'-V_S}\right) f$$

$$3400 = \left(\frac{v'+v}{v'-v}\right) 3000$$

$$\frac{3400}{3000} = \left(\frac{340+v}{340-v}\right)$$

$$1.13(340 - v) = 340 + v$$

$$384.2 - 1.13v = 340 + v$$

$$384.2 - 340 = v + 1.13v$$

$$44.2 = 2.13v$$

$$\boxed{v = \frac{44.2}{2.13} = 20.7\ m/s}$$

Result: The speed of both cars is 20.7 m/s

2019

Q.2 (vii) A mass at the end of spring oscillates with a period of 0.4 sec. Find the acceleration when the displacement is 6 cm.

Data:

Time Period= T = 0.4 sec

Displacement = x = 6 cm =0.06 m

Acceleration= a =?

Solution:

$$T = 2\pi\sqrt{\frac{m}{k}} \text{------ (i)}$$

Also, $F = kx$

$$ma = kx$$

$$\frac{m}{k} = \frac{x}{a}$$

Putting in equation (i)

$$T = 2\pi\sqrt{\frac{x}{a}}$$

S.O.B.S

$$T^2 = 4\pi^2(\frac{x}{a})$$

$$(0.5)^2 = 4(3.14)^2(\frac{0.06}{a})$$

$$\boxed{a = 14.8\ m/s^2}$$

Result: The acceleration of body is 14.8 m/s^2.

Q.2 (xiv) A string 2m long and mass 0.004 kg, is stretched horizontally by passing one end over a pulley and attaching a 1 kg mass to it. Find the speed of the transverse waves on the string and frequency of the second harmonic.

Data:

Length= l =2 m

Mass of string = m= 0.004 kg

Mass attached to string =M = 1 kg

Velocity of transverse wave = v=?

Frequency of Second harmonic = f_2 =?

Solution:

$$v = \sqrt{\frac{Mg}{(m/l)}}$$

$$v = \sqrt{\frac{1\times 9.8}{(0.004/2)}}$$

$$\boxed{v = 70\ m/s}$$

$$f_n = n\frac{v}{2l}$$

$$f_2 = 2\frac{70}{2(2)}$$

$$\boxed{f_2 = 35\ Hz}$$

Result: Velocity of transverse wave is 70 m/s and Frequency of Second harmonic 35 Hz.

2018

Q.2(vi) A 15 kg block is suspended by a spring of spring constant 5 x 10^3 N/m. Calculate the frequency of vibration of the block displaced from its equilibrium position when it is released.

Data:

Mass of block= m = 15 kg

Spring constant = K = 5 x 10^3 N/m

Frequency of Vibration = f = ?

Solution:

The time period of spring mass system is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2(3.14)\sqrt{\frac{15}{5 \times 10^3}}$$

$$\boxed{T = 0.344\ s}$$

Now,

$$f = \frac{1}{T} = \frac{1}{0.344}$$

$$\boxed{f = 2.9\ Hz}$$

Result: The frequency of vibration of the block is 2.9 Hz.

Q.2(x) A standing wave is established in a 135 cm long string fixed at both ends. The string vibrates in four loops when driven at 130 Hz. Determine the fundamental frequency.

Data:

Length of String = l=135 cm = 1.35 m

No. of loops = n = 4

Frequency of fourth harmonic = f_4 =130 Hz

Fundamental Frequency = f_1 = ?

Solution:

The fundamental Frequency is given by

$$f_n = n\frac{v}{2l}$$

Or

$$f_n = nf_1$$

$$f_4 = 4f_1$$

$$\boxed{f_1 = \frac{f_4}{4} = \frac{130}{4} = 32.5\ Hz}$$

Result: The fundamental frequency is 32.5 Hz.

2017

Q.2(xv) A 100 cm long string vibrates into four loops at 50 Hz. The linear density of the string is 4 x 10^{-4} gm/cm. Calculate the tension in the string.

Data:

Length of string = l= 100 cm = 1m

No. of Loops = n = 4

Frequency of fourth harmonic = f_4 = 50 Hz

Mass per unit length of string = μ = 4 x 10^{-4} gm/cm

μ = 4 x 10^{-5} kg/m

Tension in the string= T = ?

Solution:

As we know that

$$f_n = n\frac{v}{2l}$$

Since

$$v = \sqrt{\frac{T}{\mu}}$$

Therefore

$$f_n = n\frac{\sqrt{\frac{T}{\mu}}}{2l}$$

$$50 = 4\frac{\sqrt{\frac{T}{4\times10^{-5}}}}{2(1)}$$

Or

$$25 = \sqrt{\frac{T}{4\times10^{-5}}}$$

S.O.B.S.

$$625 = T/4 \times 10^{-5}$$

Or

$$T = 625 \times 4 \times 10^{-5}$$

$$\boxed{T = 0.025\ N}$$

Result: The tension in the string is 0.025 N.

2016

Q.2 (vii) Textbook Numerical 10

Q.2 (xiii) A string 2 m long of mass 0.004 kg, is stretched horizontally by passing one end over a frictionless pulley and a mass of 1 kg is suspended, Find the speed of transverse waves on the string.

Data:

Length=l =2 m

Mass of string = m= 0.004 kg

Mass attached to string =M = 1 kg

Velocity of transverse wave = v=?

Solution:

$$v = \sqrt{\frac{Mg}{(m/l)}}$$

$$v = \sqrt{\frac{1\times 9.8}{(0.004/2)}}$$

$$\boxed{v = 70\ m/s}$$

Result: Velocity of transverse wave is 70 m/s.

2015

Q.2 iv) A car emitted a note of frequency 490 Hz, if the car approaching towards a stationary listener at speed of 55 km/h, what frequency will be detected by the listener. Take speed of sound as 334 m/s.

Data:

Actual frequency of sound = f =490 Hz

Speed of car = Vs = 55 km/h = 55x1000/3600

Vs = 15.27 m/s

Apparent frequency of sound = f' =?

Speed of Sound = V = 334 m/s

Solution:

When Source moves towards stationary source

$$f' = \left(\frac{V}{V-Vs}\right) f$$

$$f' = \left(\frac{334}{334-15.27}\right) \times 490$$

$$\boxed{f' = 513.4\ Hz}$$

Result: The frequency heard by the listener is 513.4 Hz

Q.2 xiv) **Textbook Numerical 2**

2014

Q.2 (vi) **Same as 2019 Q.2 (xiv)**

2013

Q.2 (iii) A sound wave of frequency 500 Hz in air enters from a region of temperature 25 degrees C to a region of temperature 5 degrees C. Calculate the percent fractional change in wavelength.

Data:

Frequency of Sound = f = 500Hz

Initial temperature = T_1 = 25 ^{0}C + 273 = 298 K

Final temperature = T_2 = 5 ^{0}C +273= 278 K

Percent Fractional Change in wavelength = $\frac{\Delta\lambda}{\lambda_1}\%$ =?

Solution:

As we know that

$$v \propto \sqrt{T}$$

$$v = k\sqrt{T}$$

So, $v_1 = k\sqrt{T_1}$ ---- (i)

And $v_2 = k\sqrt{T_2}$ ---- (ii)

Dividing eq (ii) by eq (i)

$$\frac{v_2}{v_1} = \frac{k\sqrt{T_2}}{k\sqrt{T_1}}$$

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$$

$$\frac{v_2}{v_1} = \sqrt{\frac{278}{298}}$$

$$\frac{v_2}{v_1} = 0.965$$

$$\because v = f\lambda$$

$$\therefore \frac{f\lambda_2}{f\lambda_1} = 0.965$$

$$\boxed{\lambda_2 = 0.965\lambda_1}$$

Now, $\Delta\lambda = \lambda_1 - \lambda_2$

$$\Delta\lambda = \lambda_1 - 0.965\lambda_1$$

$$\Delta\lambda = 0.035\lambda_1$$

$$\frac{\Delta\lambda}{\lambda_1} = 0.035$$

Or $\boxed{\frac{\Delta\lambda}{\lambda_1}\% = 3.5\ \%}$

Result: The percent fractional change in wavelength will be 3.5%.

Q.2 (x) Textbook Numerical 2

2012

Q.2 (vii) Find the velocity of sound in a gas when two waves, of wavelengths 0.8m and 0.81, respectively, produce 4 beats per seconds.

<u>Data:</u>

Velocity of sound in gas = v =?

Wavelength of 1st wave = λ_1=0.8 m

Wavelength of 2nd wave = λ_2=0.81 m

Beat Frequency = f_b = 4 beats/sec

<u>Solution:</u>

The beat frequency is given by

$$f_b = |f_1 - f_2|$$

$$4 = \left|\frac{v}{\lambda_1} - \frac{v}{\lambda_2}\right|$$

$$4 = v\left|\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right|$$

$$4 = v\left|\frac{1}{0.8} - \frac{1}{0.81}\right|$$

$$4 = v|1.25 - 1.234|$$

$$4 = v|0.016|$$

$$v = \frac{4}{0.016}$$

$$\boxed{v = 250\ m/s}$$

<u>Result:</u> The speed of sound in gas is 250 m/s.

Q.2 (xiii) A string, lm long and of mass 0.004 kg, is stretched with a force. Calculate the force if the speed of the wave in the string is 140m/sec.

<u>Data:</u>

Length=l =1 m

Mass of string = m= 0.004 kg

Force Applies to string =T =?

Velocity of transverse wave = v= 140 m/s

<u>Solution:</u>

$$v = \sqrt{\frac{T}{(m/l)}}$$

$$140 = \sqrt{\frac{T}{(0.004/1)}}$$

S.O.B.S

$$(140)^2 = \left(\sqrt{\frac{T}{(0.004)}}\right)^2$$

$$19600 = \frac{T}{(0.004)}$$

$$\boxed{T = 19600 \times 0.004 = 78.4\ N}$$

<u>Result:</u> Force applied on the string is 78.4 N

2011

Q. 2(xiii) A note of frequency of 500 Hz is being emitted by an ambulance moving towards a listener at rest. If the listener detects a frequency of 526 Hz, calculate the speed of the ambulance. (speed of sound is 340 m/s at that moment)

<u>Data:</u>

$Frequency\ of\ sound = f = 500\ Hz$

$Apparent\ of\ frequency = f' = 526\ Hz$

$Speed\ of\ source = V_s = ?$

$Speed\ of\ Sound = V = 340\ m/s$

<u>Solution:</u>

When Source moves towards stationary listener

$$f' = \left(\frac{V}{V - V_S}\right) f$$

$$526 = \left(\frac{340}{340 - V_s}\right) \times 500$$

$$\frac{526}{500} = \left(\frac{340}{340 - V_s}\right)$$

$$1.052 = \left(\frac{340}{340 - V_s}\right)$$

$$340 - V_s = \frac{340}{1.052}$$

$$340 - V_s = 323.1$$

$$V_s = 340 - 323.1$$

$$\boxed{V_s = 16.8\ m/s}$$

Result: The speed of ambulance is 16.8 m/s.

2010

Q.2 (xi) **Same as 2019 Q.2 (xiv)**

Q.2 (xii) A simple pendulum completes 4 vibrations in 8 seconds on the surface of the earth. Find the time period on the surface of the moon where the acceleration due to gravity is one-sixth that of the earth.

Data:

No. of vibrations = N = 4

Time = t = 8 sec

Time period of Pendulum = T_m =?

Value of "g" on Moon = $g_m = \frac{1}{6}g$

Solution:

$$Time\ Period = \frac{Total\ \ time}{No.of\ Vibrations}$$

$$\boxed{T = \frac{t}{N} = \frac{8}{4} = 2\ sec}$$

The time period of pendulum on the surface of moon is given by

$$T_m = 2\pi\sqrt{\frac{L}{g_m}} \text{----(i)}$$

The time period of pendulum on the surface of Earth is given by

$$T = 2\pi\sqrt{\frac{L}{g}} \text{----(ii)}$$

Dividing eq(ii) by eq(i)

$$\frac{T_m}{T} = 2\pi\sqrt{\frac{L}{g_m}} \div 2\pi\sqrt{\frac{L}{g}}$$

$$\frac{T_m}{T} = 2\pi\sqrt{\frac{L}{g_m}} \times \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

$$\frac{T_m}{T} = \sqrt{\frac{g}{g_m}}$$

$$\frac{T_m}{2} = \sqrt{\frac{g}{\frac{1}{6}g}}$$

$$T_m = 2\times\sqrt{6}$$

$$\boxed{T_m = 4.89\ sec}$$

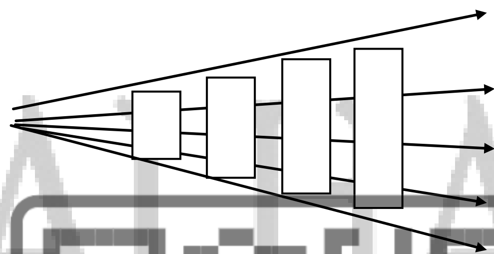
Result: The time period of simple pendulum at the surface of moon is 4.89 sec.

THEORY NOTES

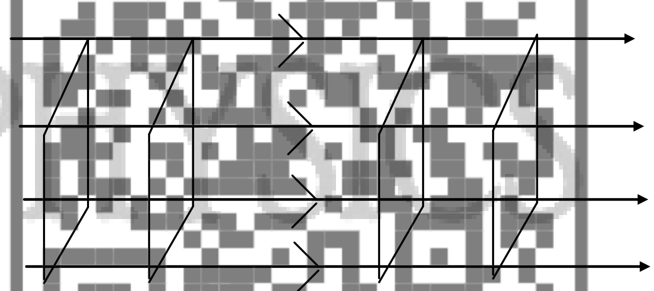
WAVE FRONT AND HUYGEN'S PRINCIPLE

WAVE FRONT:

Whenever a wave passes through a certain medium, its particles execute simple harmonic motion. The focus of at the points in the medium having same phase of vibrations is called a wave front.



In case of a point source of light in a certain homogenous medium the wave front will be concentric spheres with center as the source S. such a wave front is known as spherical a Wave Front.



At a very large distance from source, a small portion of a spherical wave front is nearly a plane, Such a portion of the wave front is called a plane wave front.
In case of light, a Ray is known as the direction in which a wave propagates and it is always normal to the wave front. Thus a please wave front represents a parallel pencil of rays.

HUYGEN'S PRINCIPLE:

Huygen's principle consists of two parts:-

(i) The first part states that every point on a wave front can be considered as a source of secondary spherical wave-lets.

(ii) The secondary wavelets travel with the speed equal to the source of wavelets.

In figure AB represents the position of a spherical front at a particular instant. To get the position of the new wave front after t second we take some points on the wave front AB, according to the first principle. If the wave travels a distance vt in time t sec, then draw the secondary wavelets with radius v t. By the second part of the principle draw a plane CD tangential to these wavelets, then CD will be the new wave front after time t.

INTERFERENCE OF WAVES:

When two waves superpose one another they either enhance the effect of one another or they reduce their effect at that point. This phenomenon is called interference of waves.

CONSTRUCTIVE INTERFERENCE:-

When two waves meet such that the crest of one wave coincides with the crest of other wave, and the trough of one wave coincides with the trough other wave, the resulting crests and troughs are enhanced. Such interference is called constructive interference. For constructive interference the path difference between them must be integral multiple of λ i.e.

$$\boxed{Path\ difference = m\lambda}$$

where $m = 0; \pm 1; \pm 2; \pm 3, \ldots$

DESTRUCTIVE INTERFERENCE:

When two waves meet such that the crest of one wave coincides with the trough of the other wave and the trough of one wave coincides with the crest of the other wave, the resulting crests and troughs are reduced. Such interference is called destructive interference. For destructive interference the path difference between them must be odd multiple of $\lambda/2$ i.e.

$$\boxed{Path\ difference = (m + \tfrac{1}{2})\lambda}$$

Where $m = 0; \pm 1; \pm 2; \pm 3 \ldots$

CONDITIONS OF INTERFERENCE OF LIGHT:

For interference of light the following conditions must be observed.

1. The sources of light must be monochromatic and coherent.
2. The slits must be narrow of the order of wavelength of light.
3. The slits must be separated by a small distance.

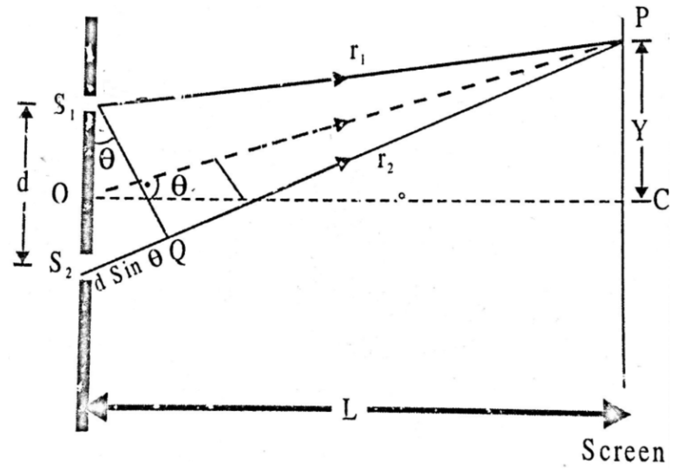
YOUNG'S DOUBLE SLIT EXPERIMENT

The fringes obtained by Young's double slit experiment are the result of interference of light waves. The conditions for interference are:

(i) Monochromatic light must be used.

(ii) The slits must be narrow of the order of wavelength of light and two sources of light must be coherent.

(iii) The two sources must be very close to each other is the wavelength of light is very small, otherwise the bright and dark pattern in front of the source would be two fine to see and interference pattern will be visible.



Light from a monochromatic source falls on the slits S and illuminate the two slits S_1 and S_2 separated by a small distance d. Since the light diverging from the slit S_1 has exactly the same frequency as the light diverging from S_2 act as two close coherent sources interference takes places at the screen placed at a distance L form the double slits.

If O be the central point on the screen then the path difference between S_1O and S_2O will be zero and the constructive interference will take place at O.

Waves traveling from the slit S_2 to any point P on the screen travel a distance $S_2P = r_2$ and from the slit S_1 travel a distance $S_1P = r_1$ the intensity of light at P will be the result of the superposition of the waves coming from S_1 and S_2. The path difference

Between the waves from S1 and S_2 is given by Path difference = $PS_2 - PS_1$

= $(r_2 - r_1)$

= $d \sin\theta$

IN CONSTRUCTIVE INTERFERENCE:

If the path difference is an integral multiple of λ, Constructive interference will take place at P. Hence for constructive interference.

$d \sin\theta = m\lambda$ (Maxima)

Where λ is the wave length of light used and m is the order of the fringe that is

$m = 0, +1, +2, +3, +4, \ldots$

The central bright fringe at $\theta = 0$ ($m = 0$) is called zero order maximum.

IN DESTRUCTIVE INTERFERENCE:

If the path difference is an odd multiple of λ, the waves arriving at P will be out of phase and hence Destructive interference will take place at P. for destructive interference.

$d \sin\theta = (m + ½)\lambda$ (Minimum)

$m = 0, +1, +2, +3, +4, \ldots$

The position of bright and dark fringes is measured from the central bright fringe. Since the distance L is very large from the slits as compared to the distance between the slits, hence the angle θ is very small. Thus for small angles.

$\sin\theta \cong \tan\theta = Y/L$ (From the triangleΔOQP)

Now,

Path diff. = d Y/L

POSITION OF BRIGHT FRINGES:

For bright fringes the path difference is given by

$$d \ \sin\theta = m\,\lambda$$

or $$d\,Y/L = m\,\lambda$$

or $$\boxed{Y = \frac{m\,\lambda L}{d}}$$

=> $$Y = 0\,, \frac{\lambda L}{d}, 2\frac{\lambda L}{d}, 3\,\frac{\lambda L}{d}, 4\,\frac{\lambda L}{d}, \ldots$$

These are the positions of bright fringes

POSITION OF DARK FRINGES:

For dark fringes the path difference is given by

$$d \ \sin\theta = (m+1/2\,)\,\lambda$$

or $$d\,Y/L = (m+1/2)\,\lambda$$

or $$\boxed{Y = \frac{(m+\frac{1}{2})\,\lambda L}{d}}$$

=> $$Y = \frac{\lambda L}{2d}, 3\,\frac{\lambda L}{2d}, 5\frac{\lambda L}{2d}, \ldots$$

These are the positions of dark fringes

FRINGE SPACING:

The distance between two dark or two bright fringes is called Fringe spacing. The fringe spacing Δx is given by.

The position of 1st bright fringe =0

The position of 2nd bright fringe $= \frac{\lambda L}{d}$

$$\Delta x = \frac{\lambda L}{d} - \ 0$$

or

$$\boxed{\Delta x \ = \frac{\lambda L}{d}}$$

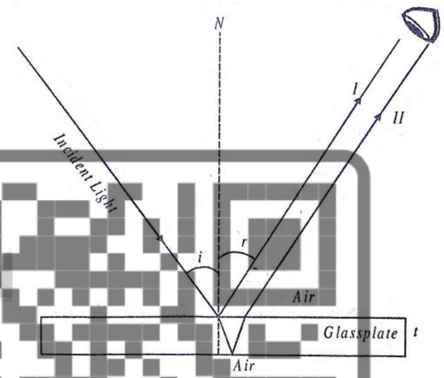
If L, d and Δx are known the value of the wavelength λ can be calculated. The fringe spacing depends upon the wavelength of light used as L and d are constant for a given experiment.

INTERFERENCE IN THIN FILM

Constructive and destructive interference of light waves is also the reason why thin films, such as soap bubbles, show colorful patterns. This is known as thin-film interference, because it is the interference of light waves reflecting off the top surface of a film with the waves reflecting from the bottom surface. To obtain a nice colored pattern, the thickness of the film has to be on the order of the wavelength of light.

Reflection of light waves, when light waves (transverse waves) strike at the boundary of a denser medium, the crest returns as a trough and the trough returns as a crest. There is a change in its phase by a or 180^o or a path difference of ½ λ. On the contrary when it strikes at the boundary of rarer medium, the crest returns as a crest and trough returns as a trough. There is no change in its phase. The wave length of light in a medium is changed by a factor. I/n if λ be the wavelength of light in air and λ_n be its wavelength in a medium of refractive index n, then.

$$\lambda_n = \frac{\lambda}{n}$$



When light falls on a film of a transparent medium of thickness then a part of the light is reflected from the upper surface of the film and the change in the path difference of ½ λt akes place (phase is reversed) Some of the light is refracted into the medium of the film, which is reflected from the lower surface of the film. Since it is reflected from the rare medium, hence there is no change is its phase (ray 1 and 2)If the light rays are normal incident on the film, then the path difference between ray 1 and 2 is "2t". where t is the thickness of the film.

Path difference = 2t

FOR CONSTRUCTIVE INTERFERENCE :

Due to the phase change of 180^0 the conditions for constructive and destructive interference are reversed. Therefore, for constructive interference

$$Path\ difference = (m + \tfrac{1}{2})\lambda_n$$

or $$2t = (m + \tfrac{1}{2})\lambda_n$$

or $$2t = (m + \tfrac{1}{2})\frac{\lambda}{n}$$

or $$\boxed{2tn = (m + \tfrac{1}{2})\lambda}$$

FOR DESTRUCTIVE INTERFERENCE :

Due to the phase change of 180^0 the conditions for constructive and destructive interference are reversed. Therefore, for destructive interference

$$Path\ difference = m\ \lambda_n$$

or $$2t = m\lambda_n$$

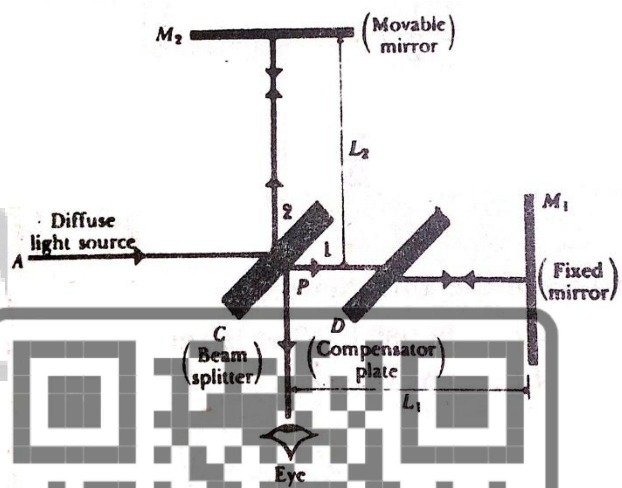
or $$2t = m\,\frac{\lambda}{n}$$

or $\boxed{2tn = m\lambda}$

MICHELSON'S INTERFEROMETER

INTRODUCTION:

Michelson's interferometer was invented by and American physicist A.A Michelson.In case of a Michelson's interferometer we use an extended source of monochromatic light in comparison with Young's double slit experiment where we use two narrow slits.Figure shows the principle diagram of Michelson's interferometer.



PROCEDURE:

Light from an extended source strikes the glass plate C the right side of which has a thin coating of silver. Part of this light is reflected from the silvered surface at P to the mirror M_2 and back through the silvered surface to the observer's eye. The remainder of the light passes through the silvered surface and through the compensator plate D and is reflected from the mirror M_2. It then returns through D and I reflected from the silvered surface of C to the observer eye. Both the plates C and D are of early same thickness so that the rays 1 and 2 through the same thickness of glass. The plate C is called beam splitters.

If the distances L_1 and L_2 are exactly equal and the mirrors M_1 and M_2 are not perpendicular then a wedge film will be obtained under this conditions the virtual image of M_1 and mirror M_2 behave as the two surface of a wedge shaped film. The interference fringes are obtained in the same way as in case of thin wedge shaped film.

CALCULATION OF WAVELENGTH :

Let the extended source be monochromatic of wavelength λ and the mirror M_2 is moved through $\lambda/4$, the path difference is changed by $\lambda/2$, and a dark fringe will appear in place of bright fringe. If now the mirror M_2 is moved through $\lambda/2$, the path difference is changed by λ and a dark fringe will replace a bright fringe. If the fringes are seen, through a telescope, and 'm' bright fringes pass through the cross wire when the mirror M_2 is moved through X then

$$x = m\frac{\lambda}{2}$$

Or $\boxed{\lambda = \frac{2x}{m}}$

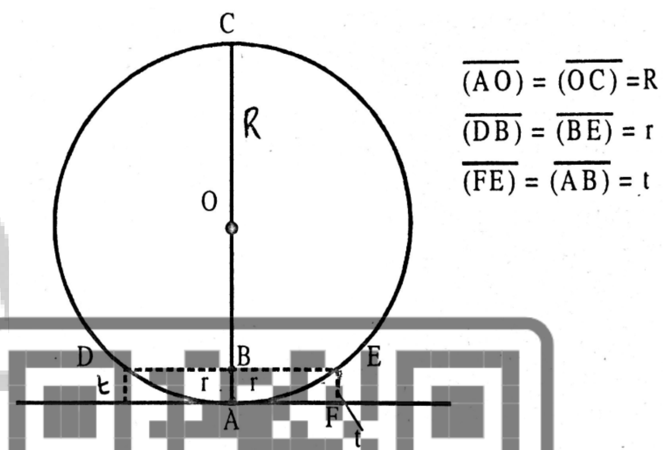
If m is large then X is also large and can be measured with good precision and hence a precise value of λ can be obtained.

NEWTON'S RINGS

DEFINITION:-

The principle of Newton's rings is based on the interference of wedge shaped film. A plano-convex lens is placed on a plane and smooth glass plate. An air film is enclosed between the lens and the glass plate. The thickness of the air film is zero at the point of contact 'A' and increase as we move outward.

Light is allowed to fall on the glass plate which is inclined at an angle of 45^0. The light is reflected from lower surface of the glass, plate and falls normally on the lens. After passing through the lens the light is reflected from the upper and lower surface of the air film. The light reflected from the upper surface of the air film, does not suffer while light reflected from the lower surface of the air film (upper surface of the glass) suffers a phase change of 180o (if the thickness of the air film is such that the path difference between ray No. 1 and Ray No.2 is an integral multiple of λ, destructive interference will take place at those points. If the thickness of the air film is such that the path difference between ray No.1 and Ray No.2 is an integral multiple of λ destructive interference will take place at those points. If the thickness of the film is such that the path difference between ray No.1 and Ray No.2 is an integral multiple of λ, destructive interference will take place at those points. If the thickness of the film is such that the path difference between ray No.1 and Ray No.2 is odd multiple of λ/2 constructive interference will take place at those point of contact, hence the interference fringes will be circular, (dark and bright circles) when seen through the reflected light, the center will be dark. These circles are called Newton's rings.



According to the geometrical theorem ,the product of intercepts of intersecting chord is equal to the product of section of diameter then,

$$\overline{BE} \times \overline{DB} = \overline{AB} \times \overline{BC}$$

$$r \times r = t(2R - t)$$

$$r^2 = 2Rt - t^2$$

As t is small then t^2 will be so small so that it can be neglected, then

$$r^2 = 2Rt$$

=> $$t = \frac{r^2}{2R} \text{ --------(i)}$$

FOR CONSTRUCTIVE INTERFERENCE:(BRIGHT RING)

The condition for constructive interference in thin film interference is,

$$2tn = (m + \frac{1}{2})\lambda$$

putting the value of "t" from eq(i),we get,

$$\cancel{2}\left(\frac{r^2}{\cancel{2}R}\right)n = \left(m + \frac{1}{2}\right)\lambda$$

$$r^2 = \left(m + \frac{1}{2}\right)\lambda R \qquad \text{(For air n=1)}$$

or $$r = \sqrt{\left(m + \frac{1}{2}\right)\lambda R}$$

For "Nth " bright ring

$$\boxed{r_N = \sqrt{\left(N - \frac{1}{2}\right)\lambda R}}$$

FOR DESTRUCTIVE INTERFERENCE:(DARK RING)

The condition for destructive interference in thin film interference is,

$$2tn = m\lambda$$

putting the value of "t" from eq(i),we get,

$$\cancel{2}\left(\frac{r^2}{\cancel{2}R}\right)n = m\lambda$$

$$r^2 = m\lambda R \qquad \text{(For air n=1)}$$

or $$\boxed{r = \sqrt{m\lambda R}}$$

In case of dark ring , for 1st dark ring m=1 because for m=0 , r=0.

DIFFRACTION OF LIGHT

According to geometrical optics when light falls on an opaque body it casts it shadow of the same shape, and

1. No light enters into the region of shadow and
2. Outside the shadow, the screen in uniformly illuminated

However, it is found when light falls on sharp edges and narrow slits, the light bends into the shadow showing that light can bend inside the geometrical shadows. "The bending of waves around the corners of obstacles, sharp edges or narrow slits is called diffraction".

There are two types of Diffraction.

i. Fresnel's Diffraction ii. Fraunhofer Diffraction

FRESNEL DIFFRACTION:

If the source of light and screen are at finite distance from the aperture, the diffr action obtained is called Fresnel diffraction.

FRAUNHOFER DIFFRACTION:

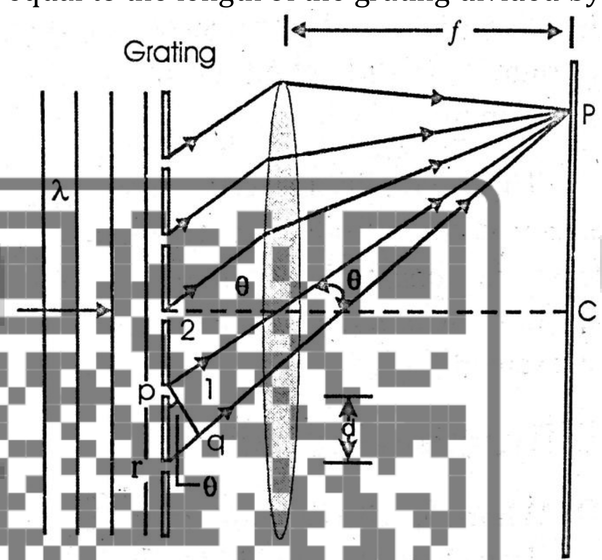
If the source of light and screen are at infinite distance from the aperture, the rays reaching the slit will be parallel to one another, and then the diffraction is called Fraunhofer diffraction.

Fraunhofer diffraction can be obtained in the laboratory by putting a convex lens in between source and slit at a distance of its focal length from the screen to focus them on the screen.

DIFFRACTION GRATING

DEFINITION:

If instead of a single slit there are large number of slits parallel to one another with equal width, such an arrangement is called diffraction grating.A diffraction grating consists of a glass piece with a number of parallel opaque lines marked on it there is transparent portion which acts as slit. The distance between two lines on a grating is called grating element it is equal to the length of the grating divided by the number of lines on it, the grating element 'd' is given by



$$d = \frac{length\ of\ grating}{number\ of\ opaque\ lies}$$

In the figure if 'n' width of the opaque line and 'b' is width of the opening then grating element is given by.

$$d = (a + b)$$

EXPLANATION:

A diffraction grating is shown in the figure. A parallel beam of monochromatic light is normal incident upon it, which sends out waves from each slit. A convex lens is placed in the path of diffracted waves which bring them together at a point on the screen in a certain direction waves of particular wavelength which are in phase reinforce each other and focus at a point in chase of white light.

If the parallel rays of light after diffraction differ in path difference by λ, they will interfere constructively at P.

For constructive interference the path difference between any two adjacent rays must be zero or an integral multiple of λ.

Path difference = m λ

$(a + b) \sin\theta = m\lambda$

or $d \sin\theta = m\lambda$

The central maximum has a path difference equal to zero, hence it is called zero order.

The first order maximum occurs when …………… $d \sin\theta = 1\lambda$

The second order maximum occurs when ……… $d \sin\theta = 2\lambda$

The third order maximum occurs when…………… $d \sin\theta = 3\lambda$

The m^{th} order diffraction occurs when

$$\boxed{d \sin\theta \; = \; m\,\lambda}$$

Where m called the order of spectrum and has values

$m = 0, \pm, \pm 1, \pm 2, \pm 3, ..$

DIFFRACTION OF X-RAYS

X rays have very Short wavelength less than that of visible and ultraviolet light. Therefore diffraction of X - rays cannot be observed with the help of common grating as X-rays pass through the slits of grating. However it is possible to obtain it rays diffraction by making use of crystals such as rock salt in which the atoms are uniformly spaced planes and separated by a distance of the order of 2 -5 A°. Therefore the diffraction of X - rays takes place when they incident on the crystals as shown in fig."

Suppose parallel Lattice planes having spacing 'd' between each other. It is clear from figure that 2[nd] ray covers more distance as compared to 1[st] ray and the path difference between two reflected rays is,

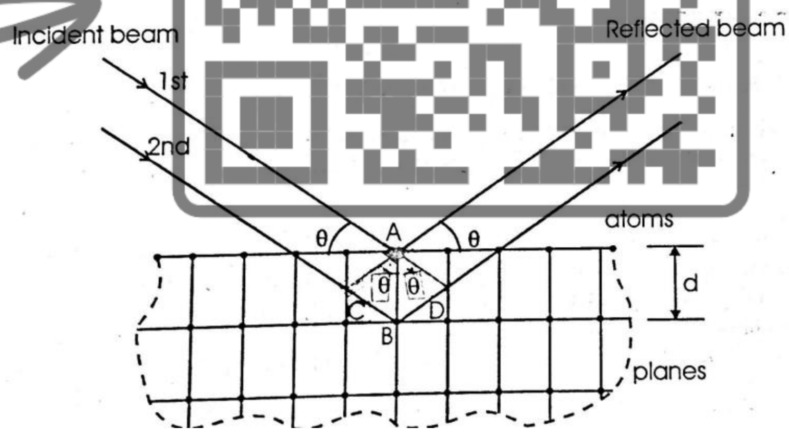
Path diff = BC + BD - (i)

In right angled Δ BAC,

$$Sin\ \theta = \frac{BC}{AB}$$

$$Sin\ \theta = \frac{BC}{d}$$

$$\overline{BC} = dSin\ \theta$$



Similarly,

$$\overline{BD} = dSin\ \theta$$

Putting values in eq(i),we get

$$Path\ Diff = dSin\ \theta + dSin\ \theta$$

$$Path\ Diff = 2dSin\ \theta \text{ ---------(ii)}$$

Now, For constructive interference,

$$Path\ Diff = m\lambda \text{ ---------------(iii)}$$

Comparing eq(ii) and eq(iii)

$$\boxed{dSin\ \theta = m\lambda}$$

This is known is Braw's Law.Now if 'd' the interplanar distance of crystal is known where 'm' and "θ" are experimentally measured then wave length of x ray light used can be calculated with the help of this equation.

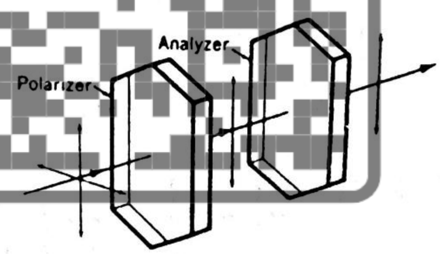
POLARIZATION OF LIGHT

The interference and diffraction phenomena verify the wave nature of light but these phenomena do not provide any information about the type of light waves.

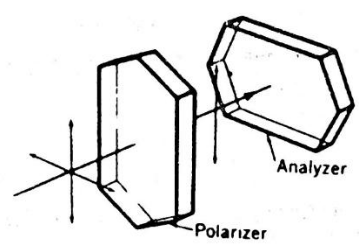
It is polarization phenomenon which tells us that light waves are transverse in nature.Actually, there is a periodic fluctuation in electric and magnetic fields along the propagation of light waves. These fields vary at right angles to the direction of the propagation of the light wave. so light wave is a transverse wave and this makes it possible to produce and detect polarized light.

Unpolarized light means ordinary light whose waves have electric and magnetic vibrations in all directions in a plane perpendicular to the direction of propagation of light. Polarized light means light waves having vibrations in certain directions 'y' perpendicular to the direction of propagation of light. Tourmaline crystals are often used to polarize the light and to analyse the polarized light. A tourmaline crystal has its crystallographic axis parallel to its face and it transmits" only those vibrations of light waves which are parallel to the axis of crystal.

When unpolarized light is incident on two tourmaline crystals placed with their crystallographic axes parallel,the light beam is transmitted. If however, one of the crystal is rotated with respect to other the emergent beam becomes dimmer and ultimately light is totally cut off when the axes of the two crystals become perpendicular to each other. On further rotation, the light reappears and becomes brighter ,when the axes of crystals again parallel as shown in fig.
When a beam of light passes through crystal one component of the vibration is absorbed and the other component is transmitted.



Consequently the emerging beam differs from incident light in the sense that all the vibrations are in one direction. Such a beam is said to be plane polarized. When it falls on a second crystal, vibrations can only pass if they are parallel to the transmission direction of the crystal i.e. crystallographic axis of the crystal.It means that polarization of light is due to selective absorption by tourmaline of all light waves vibrating in one particular plane. The first crystal is known as polarizer
where as the second crystal is known as analyzer.



APPLICATIONS OF POLARIZATION OF LIGHT:

(i) The simplest application of polaroid is curtain less windows. An outer polarizing disc is fixed in position and an inner disc may be rotated to adjust the amount of light admitted.

(ii) In photography it is often desirable to enhance the effect of sky and clouds. Since light from the blue sky is partially polarized by scattering, suitable orientation polarization discs in front of the camera lens will serve as a sky filter.

(iii) Control of headlight glass in night driving is possible if each car has light polarizing viewer.

M.C.Qs.

1. The wave theory of light was proposed by:
(a) Galileo (b) Huygens
(c) Kepler (d) Newton

2. Yellow light of a single wavelength can't be:
(a) Reflected (b) Refracted
(c) Dispersed (d) Polarized

3. The characteristic property of light wave which does not vary with the medium is:
(a) Frequency (b) Amplitude
(c) Velocity (d) Wavelength

4. The waveform of waves will be spherical when the rays of light are:
(a) parallel (b) perpendicular
(c) monochromatic (d) not parallel

5. Color of light is determined by its.
(a) Frequency (b) Amplitude
(c) Speed (d) Wavelength

6. When a transverse wave travelling through a rare medium is reflected from a dense medium, then phase change produced in it will be equal to:
(a) 0° (b) 90° (c) 180° (d) 360°

7. The locus of all points in the same phase of vibration is:
(a) Wave front (b) Interference
(c) Diffraction (d) Polarization

8. A thin layer of oil on the surface of water looks colored due to:
(a) Polarization of light.
(b) different elements presenting the oil
(c) Interference of light
(d) The transmission of light

9. When Newton's rings are observed by reflected light , the centre of rings appear dark due to
(a) phase reversal only
(b) path difference zero only
(c) intensity of light being maximum
(d) both phase reversal and path difference being zero.

11. The experimental evidence of transverse nature of light is:
(a) diffraction (b) interference
(c) polarization (d) dispersion

12. In Newton's rings seen throughout reflected light:
(a) The central spot is dark
(b) The central spot is bright
(c) The last spot is dark
(d) The last spot is bright

13. The number of lines per cm of a diffraction

grating is 4000. Its grating element is:
(a) 2.5×10^{-4} cm (b) 2.5×10^{-6} cm
(c) 4×10^{2} cm (d) 4×10^{5} cm

14. The phenomenon of interference come out because wave obey:
(a) The impulse moment theorem
(b) The 1st law of thermodynamics
(c) The inverse square law
(d) The principle of superposition

15. Which of the following is used to plane polarize light?
(a) A sheet with small opening
(b) A thick glass sheet
(c) A plano-convex lens (d) A paper sheet

16. The conditions of interference in thin film are reversed due to:
(a) Diffraction (b) Phase coherence
(c) Refraction (d) Phase reversal

17. This equation represents Bragg's Law:
(a) $m\lambda = 2d \sin\Theta$ (b) $m\lambda = d \sin\Theta$
(c) $2m\lambda = d \sin\Theta$ (d) $2m\lambda = 3d \sin\Theta$

18. Which of the following phenomenon produce the colors in soap bubble?
(a) Interference (b) Polarization
(c) Diffraction (d) Dispersion

19. The path difference in destructive interference must be:
(a) $d = 0, 2\lambda, 3\lambda$ (b) $d = \lambda/2, 3\lambda/2, 5\lambda/2$
(c) $d = 0, \lambda/6, 3\lambda/6, 5\lambda/6$ (d) $d = 0, 3\lambda/4, 5\lambda/4$

20. Monochromatic yellow light is unable to show:
(a) reflection (b) refraction
(c) dispersion (d) interference

21. According to Maxwell theory, light travels in the form of:
(a) transverse wave (b) longitudinal wave
(c) mechanical wave (d) electromagnetic wave

22. One condition for interference is that the two sources should be coherent and:
(a) Close together (b) at a far off distance
(c) Opposite to each other (d) Coinciding

23. In Young's double slit experiment; the condition for the constructive interference is that the path diff. must be:
(a) an odd multiple of the half wavelength
(b) an odd multiple of the whole wavelength
(c) an integral multiple of the wavelength
(d) an even number of the wavelength

24. Width of the interference fringes in young's double slit experiment increase with increase in:
(a) Slit separation (b) Wave length
(c) order of the fringes (d) frequency of the source

25. The property which enables waves to bend around the edge of an opening or obstacle in its path is called:
(a) Dispersion (b) Diffraction
(c) Super position (d) Interference

26. Which of the following are types of diffraction?
(a) Interfering and non interfering
(b) Transparent – semi transparent
(c) Fresnel - Fraun hoffer diffraction
(d) Johnson- element diffraction

27. A plate with a number of tiny holes or thin lines is called ____.
a) Sheet b) Sieve

c) Grating d) Collimator

28. A lens which converges a parallel beam of light is called _____.
a) Concave b) Convex
c) Plane lens d) Compound lens

29. The wave theory of light cannot explain:
a) polarization b) Photoelectric effect
c) Interference d) Diffraction

30. Electromagnetic wave consist of oscillatory electric field and magnetic field. Both fields are:
(a) Parallel to each other
(b) Parallel to the direction of propagation
(c) Perpendicular to each other
(d) None of these

PAST PAPER M.C.Qs.

2022

4.Bragg's law is:
mλ = 2d sinΘ *mλ = d sinΘ *2mλ = d sinΘ *2mλ = 3d sinΘ

11.Huygen's Principle is used to determine:
*speed of light ***position of wave front*** *polarization *refractive index

15.Wave front near the point source is:
* Plane **_* Spherical_** * Conical * Cylindrical

17.The bending of light around sharp obstacle is called:
* Interference ***Diffraction*** *Polarization * Refraction

2021

(xxiii) The equation represents Bragg's law:
mλ = 2d sinΘ *mλ = d sinΘ *2mλ = d sinΘ *2mλ = 3d sinΘ

(xxiv) In Young's double slits experiment, the condition for the constructive interference is that the path difference must be
*An odd multiple of the half wavelength
*An odd multiple of the quarter wavelength
An integral multiple of the wavelength
*An even multiple of one third the wavelength

(xxx) The fringe spacing in Young's double slit experiment is:
* $d\lambda$ / L ***λL/d*** * Ld/λ *Lλd

xxxiv) The motion of number of lines per cm of a diffraction grating are 4000, its grating element is:

***2.5×10^{-4} cm** *2.5×10^{-6} cm *4×10^{-2} cm *4×10^{5} cm

2019

3.The waveform of waves will be spherical when the rays of light are:

* parallel *perpendicular *monochromatic ***not parallel**

11. When a transverse wave travelling through a rare medium is reflected from a dense medium, then phase change produced in it will be equal to:

*0° *90° ***180°** *360°

2018

9. When Newton's rings are observed by reflected light , the centre of rings appear dark due to *phase reversal only *path difference zero only
* intensity of light being maximum *** both phase reversal and path difference being zero.**

11. The experimental evidence of transverse nature of light is:

*diffraction *interference *** polarization** * dispersion

2017

12. The number of lines per cm of a diffraction grating is 4000. Its grating element is:

***2.5×10^{-4} cm** *2.5×10^{-6} cm *4×10^{2} cm *4×10^{5} cm

2016

6. The conditions of interference in thin film are reversed due to:

*Diffraction *Phase coherence *Refraction ***Phase reversal**

8. This equation represents Bragg's Law:

***$m\lambda = 2d \sin\theta$** *$m\lambda = d \sin\theta$ *$2m\lambda = d \sin\theta$ *$2m\lambda = 3d \sin\theta$

2015

9. Newton's rings illustrate the phenomenon of:

*Polarization *diffraction *** interference** * dispersion

12. In thin film interference, the positions of constructive and destructive interference are interchanged due to:

*Phase coherence ***Phase reversal** *Diffraction *Interference

15. Polarization of light due to tourmaline crystals takes place because of:

*Reflection *** Absorption** * Refraction * Collision

2014

8. Diffraction of light is a special type of:

* reflection *refraction ***interference** *polarization

14. In Young's double slit experiment, the fringe spacing is:

*$d\lambda/L$ ***$L\lambda/d$** *Ld/λ * λLd

2013

11. Monochromatic yellow light is unable to show:

*reflection *refraction ***dispersion** *interference

2012

7. According to Maxwell theory, light travels in the form of:

*transverse wave * longitudinal wave
* mechanical wave *** electromagnetic wave**

10. Huygens's principle is used to:

*determine the speed of light *** locate the wave front**
* expressed polarization * find the refractive index

2011

2. The wave theory cannot explain:

*polarization *** photo electric effect**
* interference * diffraction

8. Electromagnetic waves consist of oscillating electric and magnetic fields, both are:

*parallel to each other ***perpendicular to each other**
*non parallel to each other *none of these

2010

8. In Young's double slit experiment; the condition for the constructive interference is that the path diff. must be:

*an odd multiple of the half wavelength
* an odd multiple of the whole wavelength
*** an integral multiple of the wavelength**
* an even number of the wavelength

TEXTBOOK NUMERICALS

Q.1: How many fringes will pass a reference point if the mirror of a Michelson's interferometer is moved by 0.08 mm. The wavelength of light used is 5800 A° .

Data:

No. of Fringes= m = ?

Distance moved = Δx = 0.08 mm=8 x 10^{-5} m

Wavelength = λ= 5800 $A^0 = 5.8 \times 10^{-7} m$

Solution:

$$\Delta x = \frac{m\lambda}{2}$$

$$m = \frac{2\Delta x}{\lambda}$$

$$m = \frac{2\times 8 \times 10^{-5}}{5.8\times10^{-7}}$$

$$\boxed{\lambda = 276}$$

Result: 276 fringes will pass the reference point.

Q.2: In a double slit experiment the separation of the slits is 1.9 mm and the fringe spacing is 0.31mm at a distance of 1 metre from the slits. Find the wavelength of light?

Data:

Slits separation = d =1.9 mm= 1.9 x 10^{-3} m

Fringe Spacing = Δx = 0.31 mm=0.31 10^{-3} m

Distance of screen from slits =L = 1 m

Wavelength of Light = λ = ?

Solution:

The Fringe spacing is given by

$$\Delta x = \frac{\lambda L}{d}$$

$$0.31\times\ 10^{-3}\ = \frac{\lambda\,(1)}{1.9\text{ x }10^{-3}}$$

$$\lambda = 0.31\times\ 10^{-3}\times 1.9\text{ x }10^{-3}$$

$$\boxed{\lambda = 5.89\times10^{-7}m}$$

Result: The wavelength of light used is $5.89\times10^{-7}m$

Q.3: Interference fringes were produced by two slits 0.25 mm apart on a screen 150 mm from the slits. If eight fringes occupy 2.62 mm. What is the wavelength of the light producing the fringes?

Data:

Slits separation = d =0.25 mm = 2.5 x 10^{-4}m

Fringe Spacing = $\Delta x = \Delta x/8$ =

=2.62 10^{-3} / 8 = 3.27 x 10^{-4}m

Distance of screen from slits =L = 150 mm =0.15 m

Wavelength of Light = λ = ?

Solution:

The Fringe spacing is given by

$$\Delta x = \frac{\lambda L}{d}$$

$$\lambda = \frac{\Delta x\times d}{L}$$

$$\lambda = \frac{3.27\ \times10^{-4}\times2.5\times10^{-4}}{0.15}$$

$$\boxed{\lambda = 5.45\times10^{-7}m}$$

Result: The wavelength of light used is $5.45\times10^{-7}m$

Q.4: Green light of a wavelength 5400 A° is diffracted by grating having 2000 line/cm. (a) Compute the angular deviation of the third order image. (b) Is a 10th order image possible?

Data:

Order = m = 3

Wavelength = $\lambda = 5400\ A = 5.4\times10^{-7}m$

Angle = θ = ?

No. of lines per cm = n =2000 lines/cm

Solution:

$$grating\ element = \frac{length\ of\ grating}{No.of\ Lines}$$

$$d = \frac{1}{n}$$

$$\boxed{d = \frac{1}{2000} = 5\times10^{-4}cm = 5\times10^{-6}m}$$

(a)

$$m\lambda = dSin\theta$$

$$(3)(5.4\times10^{-7}) = 5\times10^{-6}Sin\theta$$

$$Sin\theta\ = \frac{(3)(5.4\times10^{-7})}{5\times10^{-6}} = 0.324$$

$$\theta = Sin^{-1}(0.324)$$

$$\boxed{\theta = 18.9^0}$$

(b) For 10th order image:

$$m\lambda = dSin\theta$$

$$(10)(5.4\times10^{-7}) = 5\times10^{-6}Sin\theta$$

$$Sin\theta\ = \frac{(10)(5.4\times10^{-7})}{5\times10^{-6}} = 1.08$$

$$\theta = Sin^{-1}(1.08)$$

$$\boxed{\theta = Not\ Possible}$$

Result: The angular deviation of 3rd order image is 18.9^0 and 10th order is not possible.

Q.5: Light of a wavelength 6x10^{-7} m falls normally on a diffraction grating with 400 lines per mm. At what angle to the normal are the 1st , 2nd and 3rd order spectra produced?

Data:

Order = m = 1,2,3

Wavelength = $\lambda = 6\times10^{-7}m$

Angle = θ = ?

No. of lines per cm = n =400 lines/mm

Solution:

$$grating\ element = \frac{length\ of\ grating}{No.of\ Lines}$$

$$d = \frac{1}{n}$$

$$\boxed{d = \frac{1}{400} = 2.5 \times 10^{-3} mm = 2.5 \times 10^{-6} m}$$

(a) When m = 1

$$m\lambda = dSin\theta$$

$$(1)(6 \times 10^{-7}) = 2.5 \times 10^{-6}\ Sin\theta$$

$$Sin\theta = \frac{(1)(6\times10^{-7})}{2.5\times10^{-6}} = 0.24$$

$$\theta = Sin^{-1}(0.24)$$

$$\boxed{\theta = 13.8^0}$$

(b) When m = 2

$$m\lambda = dSin\theta$$

$$(2)(6 \times 10^{-7}) = 2.5 \times 10^{-6}\ Sin\theta$$

$$Sin\theta = \frac{(2)(6\times10^{-7})}{2.5\times10^{-6}} = 0.48$$

$$\theta = Sin^{-1}(0.48)$$

$$\boxed{\theta = 28.6^0}$$

(b) When m = 3

$$m\lambda = dSin\theta$$

$$(3)(6 \times 10^{-7}) = 2.5 \times 10^{-6}\ Sin\theta$$

$$Sin\theta = \frac{(3)(6\times10^{-7})}{2.5\times10^{-6}} = 0.72$$

$$\theta = Sin^{-1}(0.72)$$

$$\boxed{\theta = 46^0}$$

Result: The angular deviation of 1st order image is 13.8^0 , 2nd order is 28.6^0 and for 3rd order is 46^0.

Q.6: If a diffraction grating produced a 1st order spectrum of light of wavelength 6x10^{-7} m at an angle of 20° from the normal. What is its spacing and also calculate the number of lines per mm?

Data:

Order = m = 1

Wavelength = λ = 6 x 10^{-7} m

Angle = θ = 20°

Grating Element = d = ?

No. of lines per mm = n = ?

Solution:

$$m\lambda = dSin\theta$$

$$d = \frac{m\lambda}{Sin\theta}$$

$$d = \frac{(1)(6 \times 10^{-7})}{Sin\ 20^0}$$

$$\boxed{d = 1.75 \times 10^{-6} m}$$

Or

$$d = 1.75 \times 10^{-3}\ mm$$

$$No. of\ Lines = \frac{length\ of\ grating}{grating\ element}$$

$$n = \frac{1}{d}$$

$$\boxed{n = \frac{1}{1.75 \times 10^{-3}} = 571 \frac{lines}{mm}}$$

Result: The grating element is $1.75 \times 10^{-3}\ mm$ and no. of lines are $571 \frac{lines}{mm}$

Q.7: Newton's rings are formed between a lens and a flat glass surface of wavelength 5.88x10^{-7} m. If the light passes through the gab at 30° to the vertical and the fifth dark ring is of diameter 9 mm. What is the radius of the curvature of the lens?

Data:

No. of Ring = N = 5

Radius of ring = r = 9 mm = 9 x10^{-3}m

(Correction)

Wavelength of light = $\lambda = 5.89 \times 10^{-7} m$

Angle with the vertical = $\theta = 30°$

Radius of curvature = R = ?

Solution:

The radius of Nth dark ring is given by

$$r = \sqrt{\frac{N\lambda R}{\cos\ \theta}}$$

.

$$9 \times 10^{-3} = \sqrt{\frac{5\times5.89\times10^{-7}\times R}{\cos\ 30°}}$$

S.O.B.S

$$(9 \times 10^{-3})^2 = \frac{5\times5.89\times10^{-7}\times R}{0.866}$$

$$R = \frac{(9\times10^{-3})^2\times0.866}{5\times5.89\times10^{-7}}$$

$$\boxed{R = 23.8\ m}$$

Result: The radius of curvature of the lens used is 23.8 m

Q.8: How far apart are the diffracting planes in a NaCl crystal for which X-rays of wavelength 1.54 A° make a glancing angle of 15° -54' in the 1st order?

Data:

Order of maximum= m = 1

Glancing angle $=\theta = 15$ degrees 54 minutes

$$=\theta = 15 + \frac{54}{60} = 6 + 0.9$$

$$\theta = 15.9°$$

Wavelength of x rays= $\lambda = 1.54\ A° = 1.54 \times 10^{-10} m$

Distance between the atomic planes = d = ?

Solution:

According to Bragg's Law

$$m\lambda = 2dsin\theta$$

$$d = \frac{m\lambda}{2\sin\theta}$$

$$d = \frac{1\times1.54\times10^{-10}}{2\sin 15.9°}$$

$$\boxed{d = 2.81 \times 10^{-10} m}$$

Result: The between the atomic planes is $2.81 \times 10^{-10} m$.

Q.9: A parallel beam of X-rays is diffracted by rocksalt crystal the 1st order maximum being obtained when the glancing angle of incidence is 6 degree and 5 minutes. The distance between the atomic planes of the crystal is 2.8x10^{-10} m. Calculate the wavelength of the radiation.

Data:

Order of maximum= m = 1

Glancing angle $=\theta = 6$ degrees 5 minutes

$$=\theta = 6 + \frac{5}{60} = 6 + 0.083$$

$$\theta = 6.083^0$$

Distance between the atomic planes = d=2.8x10^{-10} m

Wavelength of x rays= $\lambda = ?$

Solution:

According to Bragg's Law

$$m\lambda = 2dsin\theta$$

$$\lambda = \frac{2dsin\theta}{m}$$

$$\lambda = \frac{2\times2.81\times10^{-10}\times\sin(6.083)}{(1)}$$

$$\boxed{\lambda = 5.93 \times 10^{-10} m}$$

Result: The wavelength of x rays will be $5.93 A°$.

PAST PAPER NUMERICALS

2021

Q.2 (ii) Textbook problem 8

2019

Q.2(v) 271 fringes passed through a reference point when a movable mirror of Michelson interferometer is moved by 0.08mm.Find the wavelength used in A^0.

Data:

No. of Fringes= m = 271

Distance moved = $\triangle x$ = 0.08 mm=8 x 10^{-5} m

Wavelength = λ=?

Solution:

$$\triangle x = \frac{m\lambda}{2}$$

$$\lambda = \frac{2\triangle x}{m}$$

$$\lambda = \frac{2\times8 \text{ x } 10^{-5}}{271}$$

$$\lambda = 5.9 \times 10^{-7} m$$

$$\boxed{\lambda = 5900\ A^0}$$

Result: The wavelength used is 5900 A^0.

Q.2(xii) A diffraction grating produces 3rd order spectrum of light of wavelength 7000 A° at an angle of 30° from the normal. What is the grating element? Calculate the number of lines per cm.

Data:

Order = m = 3

Wavelength = λ = 7000 A° = 7 x 10^{-7} m

Angle = θ = 30°

Grating Element = d = ?

No. of lines per cm = n = ?

Solution:

$$m\lambda = dSin\theta$$

$$d = \frac{m\lambda}{Sin\theta}$$

$$d = \frac{(3)(7 \times 10^{-7})}{Sin\ 30^0}$$

$$\boxed{d = 4.2 \times 10^{-6} m}$$

Or

$$d = 4.2 \times 10^{-4}\ cm$$

$$No.of\ Lines = \frac{length\ of\ grating}{grating\ element}$$

$$n = \frac{1}{d}$$

$$\boxed{n = \frac{1}{4.2 \times 10^{-4}} = 2381 \frac{lines}{cm}}$$

Result: The grating element is $4.2 \times 10^{-4}\ cm$ and no. of lines are 2381 $\frac{lines}{cm}$

2018

No Numerical

2017

2(vi) If the radius of 5th dark ring Newton's ring is 3mm when light of wavelength 5.89 x 10^{-7}m is used, what will be the radius of curvature of the lower surface of the lens used?

Data:

No. of Ring = N = 5

Radius of ring = r = 3 mm = 3 x10^{-3}m

Wavelength of light = $\lambda = 5.89 \times 10^{-7} m$

Radius of curvature = R = ?

Solution:

The radius of Nth dark ring is given by

$$r = \sqrt{N\lambda R}$$

$$3 \times 10^{-3} = \sqrt{5 \times 5.89 \times 10^{-7} \times R}$$

S.O.B.S

$$(3 \times 10^{-3})^2 = 5 \times 5.89 \times 10^{-7} \times R$$

$$R = \frac{(3 \times 10^{-3})^2}{5 \times 5.89 \times 10^{-7}}$$

$$\boxed{R = 3.05\ m}$$

Result: The radius of curvature of the lens used is 3.05 m.

2(xi) If a diffraction grating produced a 3rd order spectrum of light of wavelength 7x10^{-7} m at an angle of 30° from the normal. What is its spacing and also calculate the number of lines per mm?

Data:

Order = m = 3

Wavelength = λ = 7 x 10^{-7} m

Angle = θ = 30°

Grating Element = d = ?

No. of lines per cm = n = ?

Solution:

$$m\lambda = dSin\theta$$

$$d = \frac{m\lambda}{Sin\theta}$$

$$d = \frac{(3)(7 \times 10^{-7})}{Sin\ 30^0}$$

$$\boxed{d = 4.2 \times 10^{-6} m}$$

Or

$$d = 4.2 \times 10^{-3}\ mm$$

$$No.of\ Lines = \frac{length\ of\ grating}{grating\ element}$$

$$n = \frac{1}{d}$$

$$\boxed{n = \frac{1}{4.2 \times 10^{-3}} = 238 \frac{lines}{mm}}$$

Result: The grating element is $4.2 \times 10^{-3}\ mm$ and no. of lines are 238 $\frac{lines}{mm}$

2016

Q.2 (xv) Textbook Numerical 2

2015

Q.2 (xv) Textbook Numerical 9

2014

Q.2 (vii) Textbook Numerical 4

2013

No Numerical

2012

Q.2(v) In a double slit experiment, eight fringes occupy 2.62mm on a screen 145 mm away from the slits. The wave length of light is 545nm. Find the slit separation.

Data:

Slits separation = d =1.9 mm= ?

Fringe Spacing = $\Delta x = \Delta x/8$ =

=2.62 10^{-3} / 8 = 3.27 x 10^{-4}m

Distance of screen from slits = L = 145 mm

=0.15 m

Wavelength of Light = λ = 545 nm =545 x 10^{-9}m

Solution:

The Fringe spacing is given by

$$\Delta x = \frac{\lambda L}{d}$$

$$d = \frac{\lambda L}{\Delta x}$$

$$d = \frac{545 \times 10^{-9} \times 0.15}{3.27 \times 10^{-4}}$$

$$\boxed{d = 2.5 \times 10^{-4} m}$$

Result: The slit separation is $2.5 \times 10^{-4} m$

2011

Q.2 (vii) Textbook Numerical 6

2010

Q.2 (xiv) If the radius of the 14th bright Newton's ring is 1 mm and the radius of curvature of the lens is 125 mm, calculate the wavelength of the light.

Data:

No. of Ring = N = 14

Radius of ring = r = 1 mm = 1 x10^{-3}m

Wavelength of light = λ =?

Radius of curvature = R = 125 mm = 0.125 m

Solution:

The radius of Nth dark ring is given by

$$r = \sqrt{N\lambda R}$$

$$1 \times 10^{-3} = \sqrt{14 \times \lambda \times 0.125}$$

S.O.B.S

$$(1 \times 10^{-3})^2 = 14 \times \lambda \times 0.125$$

$$\lambda = \frac{(1 \times 10^{-3})^2}{14 \times 0.125}$$

$$\boxed{\lambda = 571 \times 10^{-9}\ m}$$

Result: The light of wavelength used is 571 nm.

THEORY NOTES

GEOMETRICAL OPTICS:-

The study of nature and behavior of light is called optics which is a branch of physics., where the wave nature of radiation need not be considered, but situation can be discussed in terms of rays, such study is traditionally called Geometrical optics.

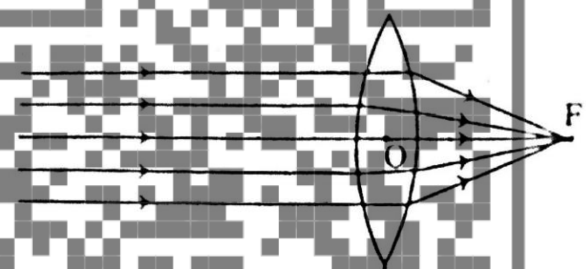
LENSES

A lens is a piece of transparent material that can focus a transmitted beam of light. This is usually bounded by two spherical surfaces. or a spherical and a plane surface. Basically lenses fall into two categories.

1. Converging or Convex lenses
2. Diverging or Concave lenses.

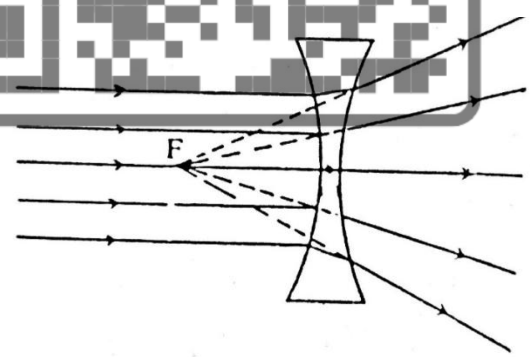
1. CONVERGING OR CONVEX LENSES:

A convex lens is thicker in the middle and thinner at the edges light rays towards its optical axis. (the line through its centre of curvature) ,so that a beam of parallel rays converges at a point F.For example in bright sunlight , a convex lens can produce bright spot of light intense enough to ignite paper.



2. DIVERGING OR CONCAVE LENSES:

A concave lens is thinner in the middle and thicker at the edges light rays towards its optical axis. (the line through its centre of curvature) ,so that a beam of parallel rays diverges in different direction.



PRINCIPAL FOCUS OR THE FOCAL POINT:

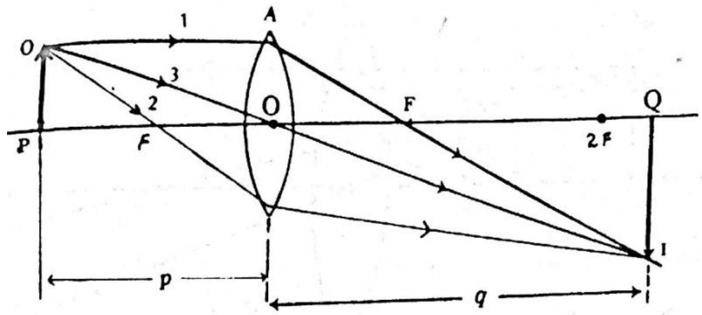
The point F to which the rays are brought to focus is called the principal focus or the focal point.

FOCAL LENGTH:

The distance between the optical centre of the lens and its principal focus is called focal length. For Concave lens it is taken negative and for convex lens it is positive.

PROPERTIES OF RAYS:

(1) A ray comes parallel to principal axis after serving refraction passes through focus.
(2) A ray comes through focus after refraction becomes parallel to principal axis.
(3) A ray strikes at center of lens refract without any deviation.



THIN LENS FORMULA

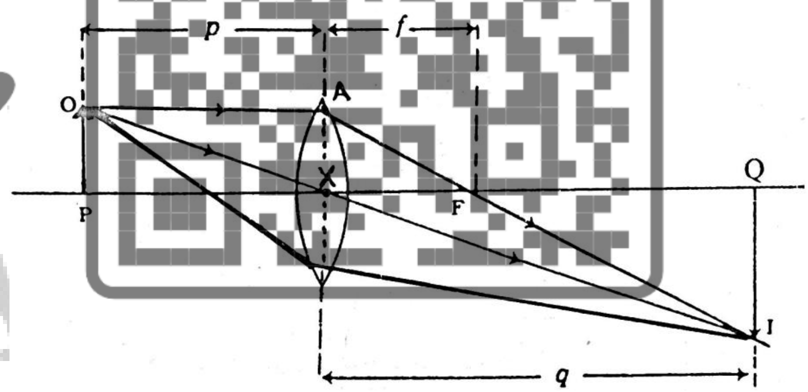
Let us consider an object whose real and inverted image is formed by a thin convex lens as shown in figure.

As shown in the figure the right angled triangle OPX and IQX are similar, therefore we can write as,

$$\frac{OP}{IQ} = \frac{PX}{QX} = \frac{p}{q}$$

Again right angled triangles AXF and IQF are also similar, Therefore,

$$\frac{AX}{IQ} = \frac{XF}{QF} = \frac{f}{q-f}$$



Since AX=OP, Therefore,

$$\frac{OP}{IQ} = \frac{XF}{QF}$$

or

$$\frac{p}{q} = \frac{f}{q-f}$$

$$p(q-f) = qf$$

$$pq - pf = qf$$

Dividing by "pqf" on both sides we get,

$$\frac{pq}{pqf} - \frac{pf}{pqf} = \frac{qf}{pqf}$$

$$\frac{1}{f} - \frac{1}{q} = \frac{1}{p}$$

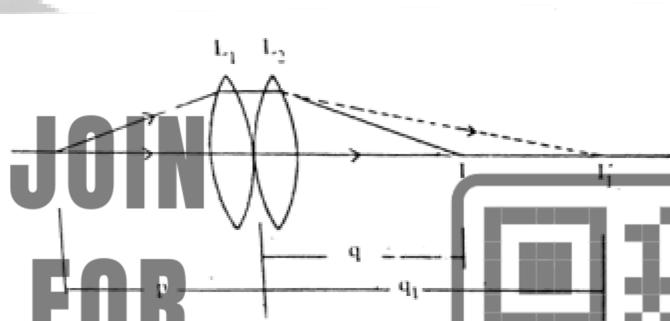
or

$$\boxed{\frac{1}{f} = \frac{1}{p} + \frac{1}{q}}$$ This is known as lens equation of lens formula.

COMBINATION OF LENSES

Lens are also used in combination as desired this combination may be of some type lens are in different lens continuation may be in styles. i.e. (1) lenses are separated sufficiently or (2) may be in contact or closer. Whenever parallel rays are desired to make again parallel the lenses are places such that their separation is sum of their focal lengths are shown above i.e. focus of both lenses lies at a some point.

Consider two lenses of focal length f_1 and f_2 placed in contact. First lens form the image at I_1 of object O at a distance q_1 the lens equation for first be.



$$\frac{1}{f_1} = \frac{1}{p} + \frac{1}{q_1} \text{-------(i)}$$

This image now serves as the virtual object for the second lens of focal length f_2. It forms the image at I of object O at a distance q ,For second lens formula is

$$\frac{1}{f_2} = -\frac{1}{q_1} + \frac{1}{q} \text{ -------(ii)}$$

By adding equation (1) and (2)

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{p} + \frac{1}{q_1} - \frac{1}{q_1} + \frac{1}{q}$$

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

or $$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}}$$

This close combination behaves as a single lens whose focal length is given by the above relation.The power of combination is given by,

$$\boxed{P = P_1 + P_2} \qquad \because P = \frac{1}{f}$$

MAGNIFYING GLASS(SIMPLE MICROSCOPE)

Sometimes tiny objects are desired view in fine detail of which brought closer to eye for getting greater visual angle but object becomes within least distance of distinct vision consequently being closer could not viewed distinctly . For removing this difficulty converging lens (convex) is used in between object and eye to view virtual image at least distance 25cm at greater visual angle as shown in figure such used convex lens is called magnifying glass.

ANGULAR MAGNIFICATION:

Let us consider a small object OP which is placed at a distance p within the focal length of magnifying glass “L” such that its erect and virtual magnified image IQ is formed at least distance of distinct vision “d” as shown in figure. The magnifying power of magnifying glass is given by

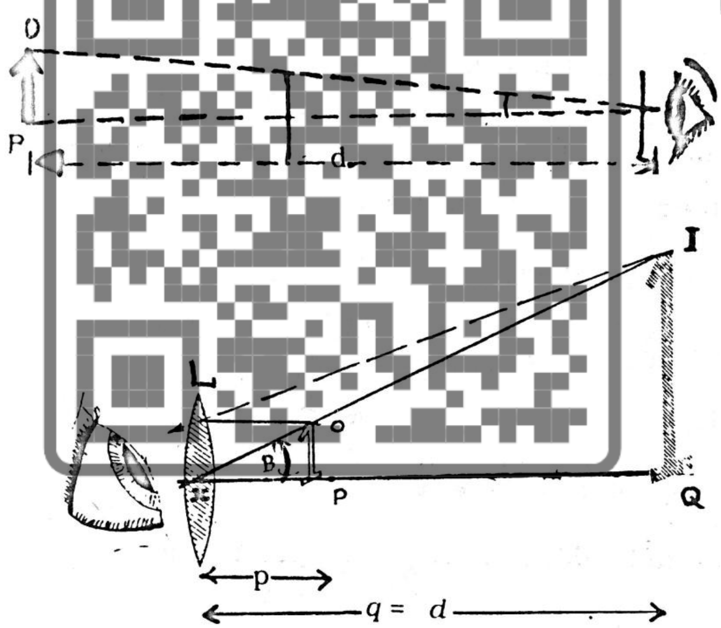
$$M = \frac{\beta}{\alpha} \text{------------(i)}$$

where α is the visual angle subtended by object when placed at least distance of distinct vision. β is the visual angle subtended by the image seen through the magnifying glass.

$\tan \propto = \frac{OP}{d}$, since $\propto$ is small

$\therefore \tan \propto = \alpha$

$\therefore \alpha = \frac{OP}{d}$



In right angled triangle OPX, we have

$\tan \beta = \beta$

$\therefore \beta = \frac{OP}{p}$

putting values of α and β in eq(i), we get

$$M = \left(\frac{OP}{p}\right) / \left(\frac{OP}{d}\right)$$

or $$M = \left(\frac{OP}{p}\right) / \left(\frac{d}{OP}\right)$$

$$\boxed{M = \frac{d}{p}} \text{--------(ii)}$$

From the lens formula ,we have

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

For Magnifying Glass we have p=+p , q=-d and f= +f

$$\therefore \frac{1}{f} = \frac{1}{p} - \frac{1}{d}$$

Multiply by “d” on both sides, we get

$$\frac{d}{f} = \frac{d}{p} - \frac{d}{d}$$
$$\frac{d}{f} = \frac{d}{p} - 1$$

From eq(ii), we get

$$\frac{d}{f} = M - 1$$

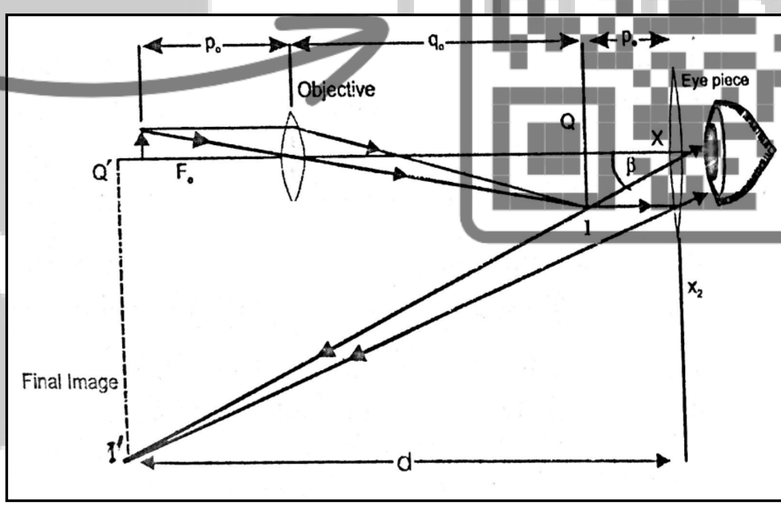
$\boxed{M = 1 + \frac{d}{f}}$ This gives magnifying power of magnifying glass.

COMPOUND MICROSCOPE

Microscope is used extensively in biological researches and in sum researches physic by microscope small objects as bacteria and other carriers of disease are magnifications sufficiently for detail examination.

CONSTRUCTION:

It consists of two converging lenses. One called objective of shorter focal length 'f_o' and other eye piece of relatively longer length 'f_e' ($f_o > f_e$) object is placed very close to objective lens just beyond the focus in order to get magnified real image which is then viewed through eye piece used as magnificent glass as shown in figure.



ANGULAR MAGNIFICATION:

The magnification power of microscope could as calculated. The magnification of objective is

$$M_o = \frac{q_o}{p_o}$$

Where q_o image distance from objective and p_o object distance from objective. Magnification of eye piece uses as magnifying glass is

$$M_e = 1 + \frac{d}{f_e}$$

Where d = least distance of distinct Vision = 25cm

Net magnification of combination in microscope is

$$M = M_o \times M_e$$

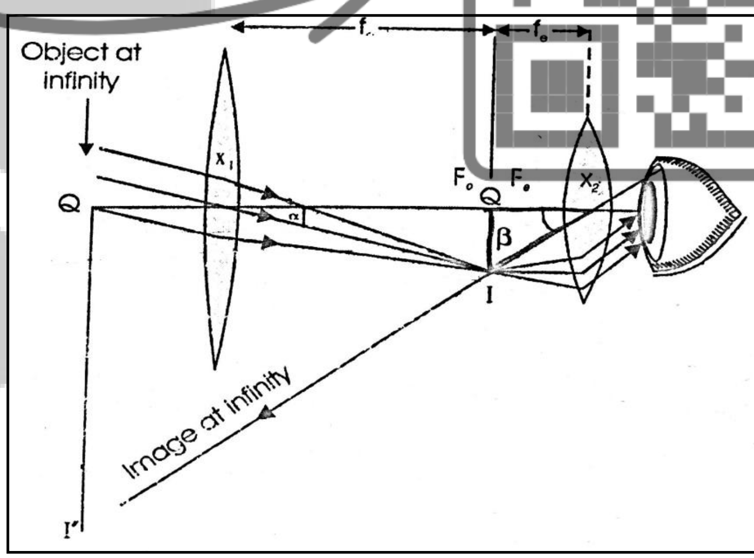
Putting values of M_o *and* M_e ,we get

$\boxed{M = \frac{q_o}{p_o}\left(1 + \frac{d}{f_e}\right)}$ This gives magnifying power of compound microscope.

ASTRONOMICAL TELESCOPE

CONSTRUCTION:

The telescope is used to view the distant objects like stars, planets the image form by telescope is not greater one but near to eye. Telescope is consists of two converging lens, one objective of longer focal length 'f_o' and eye-piece of some shorter focal length 'f_e' ($f_o > f_e$) the image formation shown in figure,final image is virtual and inverted. Hence use to view heavenly object having spherical shape. That is why called Astronomical telescope.



ANGULAR MAGNIFICATION:

The magnifying power of telescope is given by

$M = \frac{\beta}{\alpha}$ ------------(i)

where α is the visual angle subtended by object and image respectively.

in right angled triangle IX_1Q

$\tan \propto = \frac{IQ}{QX_1}$, since $\propto$ is small

$\therefore \tan \propto = \alpha$

$\therefore \alpha = \frac{IQ}{QX_1}$

or $\alpha = \frac{IQ}{f_o}$

in right angled triangle IX_2Q

$\tan \beta = \frac{IQ}{QX_2}$, since $\propto$ is small

$\therefore \tan \beta = \beta$

$\therefore \beta = \frac{IQ}{QX_2}$

or $\beta = \frac{IQ}{f_e}$

putting values of α and β in eq(i), we get

$$M = \left(\frac{IQ}{f_e}\right) / \left(\frac{IQ}{f_o}\right)$$

or $$M = \left(\frac{\cancel{IQ}}{f_e}\right) / \left(\frac{f_o}{\cancel{IQ}}\right)$$

$\boxed{M = \frac{f_o}{f_e}}$ This gives magnifying power of astronomical telescope.

and $\boxed{L = f_o + f_e}$,is the length of astronomical telescope.

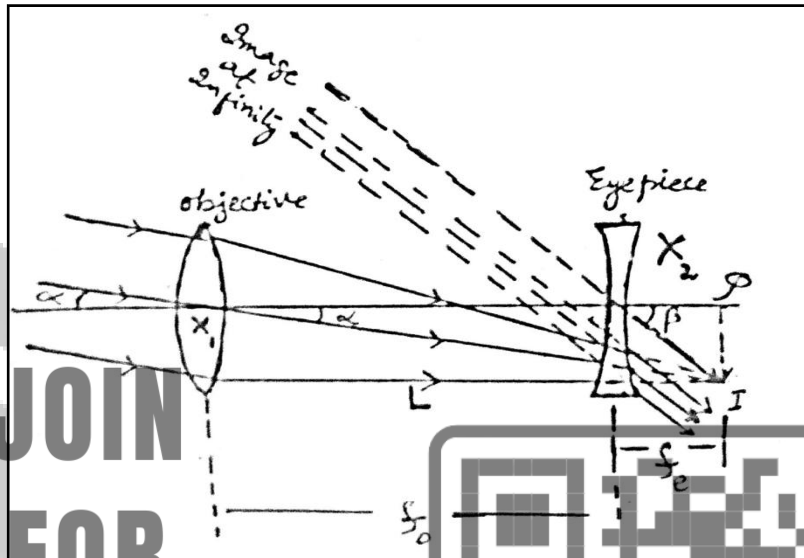
TERRESTRIAL TELESCOPE:

As astronomical telescope forms inverted final images of heavenly bodies like moon and stars which are acceptable. But when terrestrial objects are to be viewed, it is necessary to have an erect final image. The erection of image can be made by introducing a third lens between objective and eye piece of telescope. This modified telescope is known as terrestrial telescope whose magnifying power is just equal to the magnification of astronomical telescope but it just gives erect image.

GALILEAN TELESCOPE

Galilean telescope is a convenient reduced length terrestrial telescope developed by Galileo. He replaced two convex lenses erecting and eye-piece by single concave lens which does two jobs erection and magnification simultaneously as shown in diagram.

Magnification of Galilean telescope is computed maximum magnification i.e. when parallel rays, coming from object at infinite are again made parallel by concave lens for getting such focusing of telescope, focus of both converging and lenses lies at a single point a shown below diagram.



ANGULAR MAGNIFICATION:

The magnifying power of telescope is given by

$$M = \frac{\beta}{\alpha} \text{ ------------(i)}$$

where α is the visual angle subtended by object and image respectively.

in right angled triangle IX_1Q

$\tan \propto = \frac{IQ}{QX_1}$, since $\propto$ is small

$\therefore \tan \propto = \alpha$

$\therefore \alpha = \frac{IQ}{QX_1}$

or $\alpha = \frac{IQ}{f_o}$

in right angled triangle IX_2Q

$\tan \beta = \frac{IQ}{QX_2}$, since $\propto$ is small

$\therefore \tan \beta = \beta$

$\therefore \beta = \frac{IQ}{QX_2}$

or $\beta = \frac{IQ}{f_e}$

putting values of α and β in eq(i), we get

$$M = \left(\frac{IQ}{f_e}\right) / \left(\frac{IQ}{f_o}\right)$$

or
$$M = \left(\frac{IQ}{f_e}\right) / \left(\frac{f_o}{IQ}\right)$$

$$\boxed{M = \frac{f_o}{f_e}}$$ This gives magnifying power of astronomical telescope.

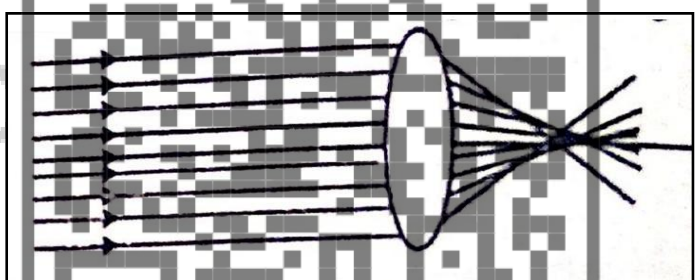
and $\boxed{L = f_o + f_e}$,is the length of astronomical telescope.

DEFECTS IN LENSES OR LENS ABERRATION

The defect in the image formation by the lenses are due to two common reasons.
1. Spherical Aberration
2. Chromatic Aberration

1.SPHERICAL ABERRATION:

When rays of light parallel to the principal axis of a lens pass through zones near its edges, these are brought to focus nearer the lens than those rays which pass through the region nearer to its center. Therefore, the focus is not a sharp point but is contained in a region called 'focal region' This produces blurred images. This problem is knows as spherical aberration as shown in fig.



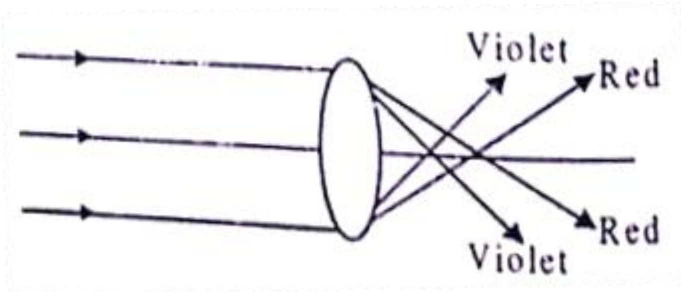
This defect can be minimized by following ways:

i. By using only the central portion of the lens. This can be achieved by using a stop on the lens which makes effective aperture of the lens small.
ii. By making opposite surfaces of the lens of different curvatures.
iii.By combining a strongly convergent lens of producing little spherical aberration with a weaker diverging lens producing equal amount of aberration in the opposite direction.

CHROMATIC ABERRATION:

The fact that different wavelengths of light refracted by lens focus at different points give rise to chromatic aberration as shown in fig. Actually a thick lens may be regarded as made up of two prism placed one above the other and due to dispersion of the refracting medium of the lens different wave length of light focus at different points. Hence image formed by lens consists of small linear spectrum. Chromatic aberration for a diverging lens is opposite to that for a converging lens.
This defect can be reduced to a large extent by combining Converging lens of crown glass with a

diverging lens of flint glass. These lenses are so chosen that dispersion of one is equal and opposite to that of the other. Lenses which are free from dispersion are called “achromatic lenses”



M.C.Qs.

1. Power of a lens is equal to
(a) Focal length in meters
(b) Reciprocal of focal length
(c) Double of focal length
(d) Half of focal length

2. A terrestrial telescope can be made by adding an erecting lens to a:
(a) Prism spectroscope (b) Reflecting telescope
(c) Field telescope (d) Astronomical telescope

3. In an astronomical telescope objective is a:
(a) Concave lens of large focal length
(b) Convex lens of large focal length
(c) Concave lens of small focal length.
(d) Convex lens of small focal length.

4. The focal length of magnifying glass is equal to the least distance of the distinct vision. Its angular magnification will be :
(a) 6 (b) 5 (c) 2 (d) 1

5. A lens of linear magnification 2 is placed between as object and a fixed screen. If size of image is 6cm, the size of the object will be:
(a) 0.5cm (b) 2 cm
(c) 3 cm (d) 18 cm

6. The length of a simple astronomical telescope is:
(a) The difference of the focal length of two lenses.
(b) The sum of the focal length of two lenses.
(c) Half the sum of the focal length
(d) Equal to the focal length of the objective lens

7. The magnifying power of a lens of focal length 1/2m is:
(a) 1 dioptre (b) 2 dioptres
(c) 50 dioptres (d) 100 dioptres

8. In a convex lens of large aperture fails to converge the light rays incident on it to a single point, it is said to suffer from?
(a) Chromatic aberration (b) Spherical aberration
(c) Both spherical and chromatic (d) Distortion

9. The distance between the principal focus and the optical center is called:

(a) Aperture (b) Focal length
(c) Principal axis (d) Radius of curvature

10. If the length of the telescope is 96cm the focal lengths of its lenses is:
(a) 100cm, -4cm (b) -80cm, -6cm
(c) 90cm, -6cm (d) 90cm, 6cm

11. Two convex lenses of same focal length 'f' are combined together. The focal length of the combines lens is:
(a) 2f (b) f/2
(c) 2 + f (d) 2 – f

12. The magnifying power of a simple microscope:
(a) Increase with increase in focal length
(b) Increase with decrease in focal length
(c) No effect with decrease or increase with the focal length
(d) Equals to distinct vision

13. If an object is placed at principle focus 'F' of a converging lens, the image will be formed:
(a) at F (b) at 2F
(c) at infinity (d) between focus & optical center

14. Least distance of distinct vision of normal and healthy people:
(a) Increases with increase in age
(b) Decreases with increase in age
(c) Neither increases nor decreases
(d) Becomes infinite after 60 years

15. The length of a Galilean telescope when focused for infinity is:
(a) fo/fe (b) fe /fo
(c) fo + fe (d) fo – fe

16. Final image produced by a compound microscope with respect to the object is?
(a) Real and inverted (b) Real and erect
(c) Virtual and erect (d) Virtual and inverted

17. By using adjustable aperture of a lens we can reduce the defect of the lens which is called:
(a) astigmatism (b) spherical aberration
(c) chromatic aberration (d) none of them

18. A lens, which is thicker at the center and thinner at the edges, is called?
(a) Concave lens (b) Convex lens
(c) Plano-convex lens (d) Plano concave lens

19. If magnifying power of simple microscope is 6, the focal length of lens used is:
(a) 6 cm (b) 5 cm (c) 25 cm (d) -5 cm

20. A spectrometer is used to find?
(a) Wavelength flight
(b) Refractive index of the prism
(c) The wavelength of different colors
(d) All of the above

21. A convex and concave lens of focal length 'f' are in contact. The focal length of the combination will be?
(a) Zero (b) f/2 (c) 2f (d) Infinite

22. Magnification of the astronomical telescope is?
(a) fo + fe (b) fo / fe
(c) fe / fo (d) (1 + fo/fe) L/fo

23. The function of collimator in spectrometer is?
(a) To produce parallel beam of light
(b) To filter the light rays
(c) To make them mutually perpendicular
(d) No function

24. Chromatic aberration can be removed by combining:

(a) A convex lens and concave lens of same type of glass.
(b) Two convex lenses of different types of glass
(c) Two concave lenses of different types of glass.
(d) A concave lens of one type of glass and a convex lens of another types of glass

25 .The final image of Astronomical telescope is:
(a) Real erect enlarged (b) Virtual erect enlarged (c) Real inverted enlarged (d) Virtual inverted enlarged

PAST PAPER M.C.Qs.

2022

21.If Mo and Me are magnifying powers of objective and eye piece of a compound microscope respectively, then total magnification will be
*** Mo x Me** * Mo - Me * Mo + Me * Mo/Me

25.In a spectrometer, parallel beam of light is obtained by
* Beam Splitter ***Collimater** * Diffraction grating *Prism

30.2nd condition of equilibrium is:
* $\sum F = 0$ * $\sum P = 0$ * $\sum \boldsymbol{\tau} = \mathbf{0}$ * $\sum a = 0$

2021

(xii) In compound microscope, the image formed by objective is:
*Virtual and magnified ***Real and magnified** *Real and diminished *Virtual and diminished

(xiii) The point in the lens through which the light rays pass without any deviation is called:
*** Optical centre** *Principle axis * Optical axis *Pole

(xvi) An astronomical telescope when focused for infinity with focal length of objective is 60cm and a focal length of eye piece is 3cm, the length of telescope is:
***63cm** *20cm * 57cm * 180cm

(xvii) In convex lens when an object is placed beyond 2F then its image will be formed:
*at 2F on the other side ***between F and 2F on the other side**
* beyond 2F on the other side * at infinity

(xxxix) The magnifying power of Astronomical telescope is:
* fo+fo *fofe *** fo+fe** * f-fe

2019

8. In a spectrometer, the focal length of convex lens is equal to length of its:

*telescope *obstacles ***collimator** *turntable

2018

8. The focal length of magnifying glass is equal to the least distance of the distinct vision. Its angular magnification will be :

* 6 * 5 *** 2** * 1

12. A lens of linear magnification 2 is placed between as object and a fixed screen. If size of image is 6cm, the size of the object will be:

*0.5cm *2 cm *** 3 cm** * 18 cm

2017

13. An astronomical telescope is focused at infinity. The focal length of objective is 0.2 cm and that of the eye piece is 5cm, the length of telescope is:

*2.5 cm *4 cm ***5.2 cm** *25 cm

2016

7.The magnifying power of a lens of focal length 1/2m is:

*1 dioptre ***2 dioptres** *50 dioptres *100 dioptres

9.The distance between the principal focus and the optical center is called:

*Aperture ***Focal length** *Principal axis *Radius of curvature

2015

10. The final image formed by a compound microscope is:

*virtual and diminished *real and diminished

* real and magnified ***virtual and magnified**

16. Two convex lenses of same focal length 'f' are combined together. The focal length of the combines lens is:

*2f ***f/2** * 2 + f * 2 – f

2014

6. If an object is placed at principle focus 'F' of a converging lens, the image will be formed:

*at F * at 2F ***at infinity** * between focus & optical center

10. The length of a Galilean telescope when focused for infinity is:

*fo/fe *fe /fo *fo + fe ***fo – fe**

2013

4. In the terrestrial telescope, the central lens is used to:

***erect the image** *increase magnifying power

*both of these *none of these

12. Power of magnifying glass having focal length 5cm is:

*5 dioptre *10 dioptre ***20 dioptre** *50 dioptre

2012

1. The length of Galilean telescope is equal to:

*f_o/f_e **<u>* f_o- f_e</u>** * f_e- f_o * $f_e + f_o$

4. If magnifying power of simple microscope is 6, the focal length of lens used is:

*6 cm **<u>* 5 cm</u>** * 25 cm * -5 cm

<u>2011</u>

3. The magnifying power of a lens of focal length 25 cm is:

*1/2 * 1 * zero **<u>* 2</u>**

<u>2010</u>

3. By using adjustable aperture of a lens we can reduce the defect of the lens which is called:

*astigmatism **<u>* spherical aberration</u>**
* chromatic aberration * none of them

9. If the power of converging lens is 4 diopter, what is the focal length of the lens:

*20 cm **<u>*25 cm</u>**
*10 cm *50 cm

TEXTBOOK NUMERICALS

Q.1: An object 4 cm tall is placed near the axis of a thin converging lens. If the focal length of the lens is 25 cm, where will the image be formed and what will be the size of the image? Sketch the principal ray diagram.

<u>Data:</u>

Height of Object $= h_o = 4\ cm$

Focal length of lens $= f = 25\ cm$

Image distance $= q = ?$

Height of Image $= h_i = ?$

Object distance $= p = 33.33\ cm$

<u>Solution:</u>

Using Thin Lens Formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{25} = \frac{1}{33.33} + \frac{1}{q}$$

$$\frac{1}{25} - \frac{1}{33.33} = \frac{1}{q}$$

$$0.04 - 0.03 = \frac{1}{q}$$

$$0.01 = \frac{1}{q}$$

$$\boxed{q = 100\ cm}$$

As We know that

$$\frac{h_i}{h_o} = \frac{q}{p}$$

$$\frac{h_i}{4} = \frac{100}{33.33}$$

$$\boxed{h_i = 12\ cm}$$

Result: The image will be formed at 100 cm from lens and the size of the image will be 12 cm.

Q.2: A convex lens has a focal length of 10 cm. Determine the image distances when an object is placed at the following distances from the lens. 50 cm, 20 cm, 15 cm, 10 cm and 5 cm

Data:

Focal length of lens $= f = 10\ cm$

Image distance $= q = ?$

(i) Object distance $= p = 50\ cm$

(ii) Object distance $= p = 20\ cm$

(iii) Object distance $= p = 15\ cm$

(iv) Object distance $= p = 10\ cm$

(v) Object distance $= p = 5\ cm$

Solution:

Using Thin Lens Formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

(i) Object distance $= \boldsymbol{p = 50\ cm}$

$$\frac{1}{10} = \frac{1}{50} + \frac{1}{q}$$

$$\frac{1}{10} - \frac{1}{50} = \frac{1}{q}$$

$$\frac{4}{50} = \frac{1}{q}$$

$$\boxed{q = \frac{50}{4} = 12.5\ cm}$$

(ii) Object distance $= \boldsymbol{p = 20\ cm}$

$$\frac{1}{10} = \frac{1}{20} + \frac{1}{q}$$

$$\frac{1}{10} - \frac{1}{20} = \frac{1}{q}$$

$$\frac{1}{20} = \frac{1}{q}$$

$$\boxed{q = 20\ cm}$$

(iii) Object distance $= \boldsymbol{p = 15\ cm}$

$$\frac{1}{10} = \frac{1}{15} + \frac{1}{q}$$

$$\frac{1}{10} - \frac{1}{15} = \frac{1}{q}$$

$$\frac{5}{150} = \frac{1}{q}$$

$$\boxed{q = 30\ cm}$$

(iv) Object distance $= \boldsymbol{p = 10\ cm}$

$$\frac{1}{10} = \frac{1}{10} + \frac{1}{q}$$

$$\frac{1}{10} - \frac{1}{10} = \frac{1}{q}$$

$$0 = \frac{1}{q}$$

$$\boxed{q = \infty}$$

(v) Object distance $= \boldsymbol{p = 5\ cm}$

$$\frac{1}{10} = \frac{1}{5} + \frac{1}{q}$$

$$\frac{1}{10} - \frac{1}{5} = \frac{1}{q}$$

$$-\frac{1}{10} = \frac{1}{q}$$

$$\boxed{q = -10\ cm}$$

Result: The image will be formed at 12.5 cm , 20 cm ,30 cm,Infinity and -10 cm.

Q.3: Two converging lenses of focal lengths 40 cm and 50 cm are placed in contact. What is the focal length of this lens combination? What is the power of the combination in diopters?

Data:

$focal\ length\ of\ first\ lens = f_1 = 40\ cm$

$focal\ length\ of\ second\ lens = f_2 = 50\ cm$

$focal\ length\ of\ combination = f = ?$

$Power\ of\ combination = P = ?$

Solution:

The power of combination of lenses is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f} = \frac{1}{40} + \frac{1}{50}$$

$$\frac{1}{f} = \frac{9}{200}$$

$$\boxed{f = 22.2\ cm}$$

or $$f = 0.22\ m$$

According to the definition of power of lens

$$P = \frac{1}{f}$$

$$\boxed{P = \frac{1}{0.22} = 4.5\ diopters}$$

Result: The power of combination is 4.5 diopters and focal length is 22.2 cm.

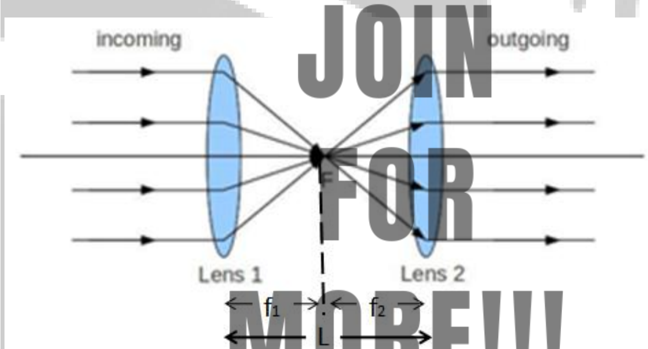
Q.4: A converging lens of focal length 20 cm is placed in front of a converging lens of focal length 4 cm. What is the distance between the lenses if parallel rays entering the first lens leave the second lens as parallel rays?

Data:

Distance between lenses= L = ?

Focal length of 1st lens = f_1 = 20 cm

Focal length of 2nd lens = f_2 = 4 cm



Solution:

According to the figure

$$L = f_1 + f_2$$

$$L = 20 + 4$$

$$\boxed{L = 24\ cm}$$

Result: The distance between lenses is 24 cm

Q.5: A parallel light beam is diverged by a concave lens of focal length -12.5 cm and then made parallel once more by a convex lens of focal length 50 cm. How far are the two lenses apart.

Data:

Distance between lenses= L = ?

Focal length of 1st lens = f_1 = -12.5 cm

Focal length of 2nd lens = f_2 = 50 cm

Solution:

As explained in Q.4

$$L = f_1 + f_2$$

$$L = -12.5 + 50$$

$$\boxed{L = 37.5\ cm}$$

Result: The distance between lenses is 37.5 cm

Q.6: Two lenses are in contact, a converging one of focal length 30 cm and a diverging one of focal length -10cm. What is the focal length and power of the combination?

Data:

$focal\ length\ of\ first\ lens = f_1 = 30\ cm$

$focal\ length\ of\ second\ lens = f_2 = -10\ cm$

$focal\ length\ of\ combination = f = ?$

$Power\ of\ combination = P = ?$

Solution:

The power of combination of lenses is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f} = \frac{1}{30} + \frac{1}{(-10)}$$

$$\frac{1}{f} = \frac{1}{30} - \frac{1}{10}$$

$$\frac{1}{f} = -\frac{2}{30}$$

$$\boxed{f = -15\ cm}$$

or $$f = -0.15\ m$$

According to the definition of power of lens

$$P = \frac{1}{f}$$

Or $$P = \frac{1}{(-0.15)} = -6.67\ diopters$$

Result: The power of combination is -6.67 diopters and focal length is 15 cm.

Q.7: Moon light passes through a converging lens of focal length 19 cm, which is 20.5 cm from a second converging lens of focal length 2 cm. Where is the image of the moon produced by the lens combination?

Focal length of Objective $= f_o = 19\ cm$

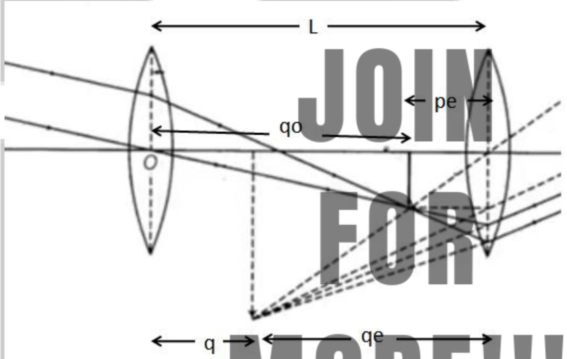
Focal length of Eye Piece $= f_e = 2\ cm$

Distance between Lenses $= L = 20.5\ cm$

Object distance from objective $= p_0 = \infty$

Image distance from Objective $= q = ?$

Image distance from Eye piece $= q_e = ?$



Solution:

For Objective:

$$\frac{1}{19} = \frac{1}{\infty} + \frac{1}{q_0}$$

$$\frac{1}{19} = 0 + \frac{1}{q_0}$$

$$\frac{1}{19} = \frac{1}{q_0}$$

$$q_0 = 19\ cm$$

Now , Distance between lenses

$$L = p_e + q_0$$

$$20.5 = p_e + 19$$

$$p_e = 1.5\ cm$$

For Eye Piece:

$$\frac{1}{f_e} = \frac{1}{p_e} + \frac{1}{q_e}$$

$$\frac{1}{2} = \frac{1}{1.5} + \frac{1}{q_e}$$

$$\frac{1}{2} - \frac{1}{1.5} = \frac{1}{q_e}$$

$$-0.16 = \frac{1}{q_e}$$

$$q_e = \frac{1}{-0.16} = -6\ cm$$

Final Image distance from Objective

$$q = L + q_e$$

$$q = 20.5 - 6$$

$$q = 14.5\ cm$$

Result: The image distance from Eye Piece is 6cm and from objective is 14.5 cm.

Q.8: Find the distance at which an object should be placed in front of a convex lens of focal length 10 cm to obtain an image of double its size?

Data:

Object distance $= p = ?$

Focal length of lens $= f = 10\ cm$

Linear Magnification $= M = 2$

Solution:

Magnification is given by

$$M = \frac{q}{p}$$

$$2 = \frac{q}{p}$$

$$q = 2p$$

For Real Image:

Using Thin Lens Formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{10} = \frac{1}{p} + \frac{1}{2p}$$

$$\frac{1}{10} = \frac{2p + p}{2p^2}$$

$$\frac{1}{10} = \frac{3p}{2p^2}$$

$$\frac{1}{10} = \frac{3}{2p}$$

$$p = 15\ cm$$

For Virtual Image:

Using Thin Lens Formula

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{q}$$

$$\frac{1}{10}=\frac{1}{p}-\frac{1}{2p}$$

$$\frac{1}{10}=\frac{2p-p}{2p^2}$$

$$\frac{1}{10}=\frac{p}{2p^2}$$

$$\frac{1}{10}=\frac{1}{2p}$$

$$\boxed{p=5\ cm}$$

Result: For real image the object should be placed at 15 cm and for virtual image object should be placed at 5 cm.

Q.9: A compound microscope has a 12 mm focal length objective and a 75 mm focal length eye piece, and the two lenses are mounted 200 mm apart. If the final image is 225 mm from the eye piece, what is the magnification produced?

Data:

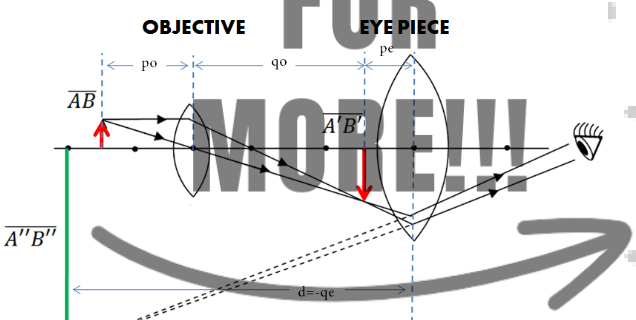
Focal length of Objective $= f_0 = 12\ mm$

Focal length of Eye Piece $= f_e = 75\ mm$

Distance between lenses $= L = 200\ mm$

Magnifying Power $= M = ?$

Image distance from eye piece $= q_e = -225\ mm$



RAY DIAGRAM OF COMPOUND MICROSCOPE

Solution:

For Eye Piece:

$$\frac{1}{f_e}=\frac{1}{p_e}+\frac{1}{q_e}$$

$$\frac{1}{75}=\frac{1}{p_e}+\frac{1}{(-225)}$$

$$\frac{1}{75}+\frac{1}{225}=\frac{1}{p_e}$$

$$\frac{4}{225}=\frac{1}{p_e}$$

$$\boxed{p_e=\frac{225}{4}=56.25\ mm}$$

Now , Distance between lenses

$$L=p_e+q_0$$

$$200=56.25+q_0$$

$$\boxed{q_0=143.75\ mm}$$

For Objective:

$$\frac{1}{f_0}=\frac{1}{p_0}+\frac{1}{q_0}$$

$$\frac{1}{12}=\frac{1}{p_0}+\frac{1}{143.75}$$

$$\frac{1}{12}-\frac{1}{143.75}=\frac{1}{p_0}$$

$$0.0763=\frac{1}{p_0}$$

$$\boxed{p_0=\frac{1}{0.0763}=13\ mm}$$

And Magnification of micro scope is given by

$$M=\left(\frac{q_0}{p_0}\right)\left(1+\frac{d}{f_e}\right)$$

$$M=\left(\frac{143.75}{13}\right)\left(1+\frac{225}{75}\right)$$

$$M=(11.05)(4)$$

$$\boxed{M=44}$$

Result: The magnification of compound microscope is 44.

Q.10: An astronomical telescope of angular magnification 1000 has an objective of 15 m focal length. What is the focal length of the eye piece?

Data:

Angular Magnication $= M = 1000$

Focal Length of Objective $= f_0 = 15\ m$

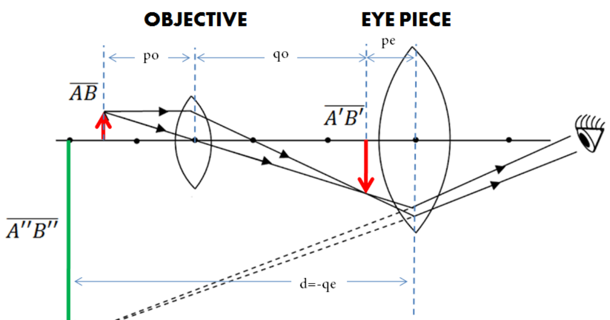
Focal Length of Eye Piece $= f_e = ?$

Solution:

Angular Magnification of Astronomical telescope

$$M=\frac{f_o}{f_e}$$

$$1000=\frac{15}{f_e}$$

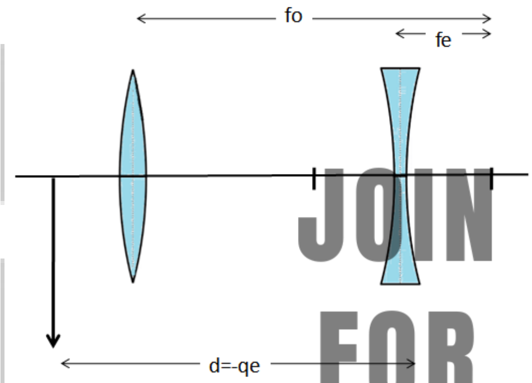


RAY DIAGRAM OF COMPOUND MICROSCOPE

$$f_e = \frac{15}{1000}$$
$$f_e = 0.015\ m$$
or $$f_e = 15\ mm$$

Result: The Focal Length of Eye Piece is 15 mm.

Q.11: A Galilean telescope has an objective of 120 mm focal length and an eye piece of 50 mm focal length. If the image seen by the eye is 300 mm from the eye piece, what is angular magnification?



Data:

Angular Magnication $= M = ?$

Focal Length of Objective $= f_0 = 120\ mm$

Image distance from Eye Piece $= q_e = -300\ mm$

Focal Length of Eye Piece $= f_e = 50\ mm$

Solution:

Angular Magnification of Galelean Telescope is given by

$$M = \frac{f_o}{p_e} \text{ ---(i)}$$

For Eye Piece:

$$\frac{1}{f_e} = \frac{1}{p_e} + \frac{1}{q_e}$$
$$\frac{1}{50} = \frac{1}{p_e} + \frac{1}{(-300)}$$
$$\frac{1}{50} + \frac{1}{300} = \frac{1}{p_e}$$
$$\frac{7}{300} = \frac{1}{p_e}$$
$$\boxed{p_e = \frac{300}{7} = 42.8\ mm}$$

Putting in eq(i)

$$M = \frac{120}{42.8}$$
$$\boxed{M = 2.8}$$

Result: The magnification of telescope is 2.8

Q.12: A compound microscope has an objective with a focal length of 10 mm and a tube 100 mm long. An image is produced 250 mm from the eye piece when the object is 12 mm from the objective. What is the angular magnification?

Data:

Focal length of Objective $= f_0 = 10\ mm$

Distance between lenses $= L = 100\ mm$

Magnifying Power $= M = ?$

Image distance from eye piece $= q_e = -250\ mm$

Object distance from objective $= p_0 = 12\ mm$

Solution:

For Objective:

$$\frac{1}{f_0} = \frac{1}{p_0} + \frac{1}{q_0}$$
$$\frac{1}{10} = \frac{1}{12} + \frac{1}{q_0}$$
$$\frac{1}{10} - \frac{1}{12} = \frac{1}{q_0}$$
$$\frac{1}{60} = \frac{1}{q_0}$$
$$\boxed{q_0 = 60\ mm}$$

Now, Distance between lenses

$$L = p_e + q_0$$
$$100 = p_e + 60$$
$$\boxed{p_e = 40\ mm}$$

For Eye Piece:

$$\frac{1}{f_e} = \frac{1}{p_e} + \frac{1}{q_e}$$
$$\frac{1}{f_e} = \frac{1}{40} + \frac{1}{(-250)}$$
$$\frac{1}{f_e} = \frac{1}{40} - \frac{1}{250}$$
$$\frac{1}{f_e} = \frac{21}{1000}$$

$$\boxed{f_e = \frac{1000}{21} = 47.6\ mm}$$

And Magnification of microscope is given by

$$M = \left(\frac{q_0}{p_0}\right)\left(1 + \frac{d}{f_e}\right)$$

$$M = \left(\frac{60}{12}\right)\left(1 + \frac{250}{47.6}\right)$$

$$M = (5)(6.25)$$

$$\boxed{M = 31}$$

Result: The magnification of compound microscope is 31.

Q.13: A converging lens of 4 dioptres is combined with a diverging lens of -2 dioptres. Find the power and focal length of the combination?

Data:

$Power\ of\ first\ lens = P_1 = 4\ diopters$

$Power\ of\ second\ lens = P_2 = -2\ diopters$

$Power\ of\ combination = P = ?$

$focal\ length\ of\ combination = f = ?$

Solution:

The power of combination of lenses is given by

$$P = P_1 + P_2$$

$$P = 4 + (-2)$$

$$\boxed{P = 2\ diopters}$$

According to the definition of power of lens

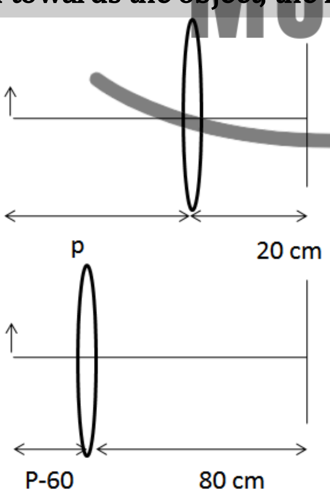
$$P = \frac{1}{f}$$

Or $f = \frac{1}{P} = \frac{1}{2} = 0.5\ m$

$$\boxed{f = 50\ cm}$$

Result: The power of combination is 2 diopters and focal length is 50 cm.

Q.14: A convex lens forms image of an object on a fixed screen 20 cm from the lens. On moving the lens 60 cm towards the object, the image is again formed on the screen. What is the focal length of the lens?



Data:

Initial Image distance $= q = 20cm$

Initial Oject distance $= p$

Final Image distance $= q' = 80cm$

Final Object distance $= p' = p - 60$

Focal length of lens $= f = ?$

Solution:

For 1st Case:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\boxed{\frac{1}{f} = \frac{1}{p} + \frac{1}{20}} \text{ ---- (i)}$$

For 2nd Case:

$$\frac{1}{f} = \frac{1}{p'} + \frac{1}{q'}$$

$$\boxed{\frac{1}{f} = \frac{1}{p-60} + \frac{1}{80}} \text{ ---- (ii)}$$

Comparing eq(i) and eq(ii)

$$\frac{1}{p} + \frac{1}{20} = \frac{1}{p-60} + \frac{1}{80}$$

$$\frac{20+p}{20p} = \frac{80+p-60}{80\,(p-60)}$$

$$\frac{20+p}{20p} = \frac{20+p}{80p-4800}$$

$$\frac{1}{20p} = \frac{1}{80p-4800}$$

$$80p - 4800 = 20p$$

$$60p = 4800$$

$$\boxed{p = 80\ cm}$$

To find focal length putting in eq(i)

$$\frac{1}{f} = \frac{1}{80} + \frac{1}{20}$$

$\frac{1}{f} = \frac{5}{80}$

$$\boxed{f = \frac{80}{5} = 16\ cm}$$

Result: The focal length of the lens is $16\ cm$.

Q.15: Two converging lenses are 25 cm part. Focal length of each is 10 cm. An object is placed in front of one lens at 50 cm. Find the distance between the object and the final image?

Data:

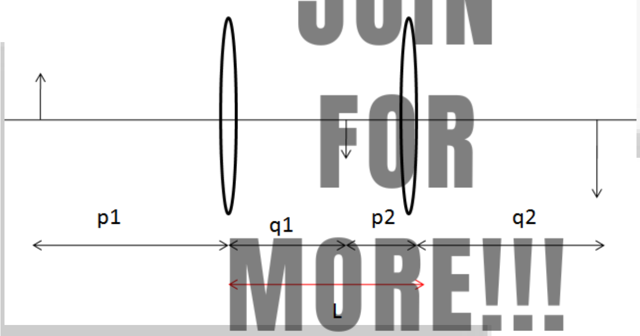
Focal length of 1st lens $= f_1 = 10\ cm$

Focal length of 2nd lens $= f_2 = 10\ cm$

Distance between lenses $= L = 25\ cm$

Distance between the object and the final image $= p_1 + L + q_2 = ?$

Object distance from 1st lens $= p_1 = 50\ cm$



Solution:

For 1st lens:

$\frac{1}{f_1} = \frac{1}{p_1} + \frac{1}{q_1}$

$\frac{1}{10} = \frac{1}{50} + \frac{1}{q_1}$

$\frac{1}{10} - \frac{1}{50} = \frac{1}{q_1}$

$\frac{4}{50} = \frac{1}{q_1}$

$$\boxed{q_1 = 12.5\ cm}$$

Now , Distance between lenses

$L = p_2 + q_1$

$25 = p_2 + 12.5$

$$\boxed{p_2 = 12.5\ cm}$$

For 2nd lens:

$\frac{1}{f_2} = \frac{1}{p_2} + \frac{1}{q_2}$

$\frac{1}{10} = \frac{1}{12.5} + \frac{1}{q_2}$

$\frac{1}{q_2} = \frac{1}{10} - \frac{1}{12.5}$

$\frac{1}{q_2} = \frac{2.5}{125}$

$$\boxed{q_2 = \frac{125}{2.5} = 50\ cm}$$

According to the figure

Distance between the object and the final image $= p_1 + L + q_2$

$p_1 + L + q_2 = 50 + 25 + 50$

$$\boxed{p_1 + L + q_2 = 125\ cm}$$

Result: Distance between the object and the final image is $125\ cm$.

PAST PAPER NUMERICALS

2022

iii) Two converging lenses of focal length 40 cm and 50 cm are placed in contact. What is the focal length of this tens combination?

Data:

$focal\ length\ of\ first\ lens = f_1 = 40\ cm$

$focal\ length\ of\ second\ lens = f_2 = 50\ cm$

$focal\ length\ of\ combination = f = ?$

$Power\ of\ combination = P = ?$

Solution:

The power of combination of lenses is given by

$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

$\frac{1}{f} = \frac{1}{40} + \frac{1}{50}$

$\frac{1}{f} = \frac{9}{200}$

$\boxed{f = 22.2\ cm}$

Q.2 (vi) An astronomical telescope with small amplitude has an objective whose power is 2 diopters. The lens is placed 60cm from the eye piece, when the telescope is adjusted for minimum eye strain. Calculate the angular magnification of the telescope.

Data:

Power of Objective= Po =2 diopters

Distance between lenses = L = 60 cm=0.6 m

Angular magnification = M=?

Solution:

The angular magnification of telescope is given by

$$M = \frac{P_e}{P_o} \text{-------(i)}$$

And the length of telescope is given by

$$L = \frac{1}{P_o} + \frac{1}{P_e}$$

$$0.6 = \frac{1}{2} + \frac{1}{P_e}$$

$$0.6 = 0.5 + \frac{1}{P_e}$$

$$\frac{1}{P_e} = 0.6 - 0.5 = 0.1$$

So, $P_e = \frac{1}{0.1} = 10\ diopter$

Putting values in equation (i)

$$M = \frac{10}{2}$$

$$\boxed{M = 5}$$

Result: The magnification of telescope is 5.

2019

Q.2(ii) An astronomical telescope having angular magnification 5 consists of two thin lenses 24 cm apart. Find the power of its lenses.

Data:

Magnification= M = 5

Length of Telescope= L = 24 m

Power of Objective = P_o=?

Power of Eye Piece = P_e=?

Solution:

$$L = f_0 + f_e$$

$$24 = f_0 + f_e \text{ ----(i)}$$

$$\because\ M = \frac{f_0}{f_e}$$

$$5 = \frac{f_0}{f_e}$$

$$\boxed{f_0 = 5\,fe}$$

Putting in equation (i)

$$24 = 5\,fe + f_e$$

$$24 = 6\,fe$$

$\boxed{fe = 4\ cm}$ Or $\boxed{fe = 0.04\ m}$

Then

$\boxed{f_0 = 5 \times 4 = 20\ cm}$ Or $\boxed{f_0 = 0.2\ m}$

Now,

$$\boxed{P_0 = \frac{1}{fo} = \frac{1}{0.2} = 5\ Dioptre}$$

And

$$\boxed{P_e = \frac{1}{fe} = \frac{1}{0.04} = 25\ Dioptre}$$

Result: The powers of lenses are 5 dioptre and 25 dioptre.

2018

Q.2(vi) An astronomical telescope has a length of 105 cm and its magnification is 6. Determine the power of objective and eye piece?

Data:

Magnification= M = 6

Length of Telescope= L = 105 cm

Power of Objective = P_o=?

Power of Eye Piece = P_e=?

Solution:

$$L = f_0 + f_e$$

$$105 = f_0 + f_e \text{ ----(i)}$$

$\because\ M = \frac{f_0}{f_e}$

$6 = \frac{f_0}{f_e}$

$\boxed{f_0 = 6\,fe}$

Putting in equation (i)

$105 = 6\,fe + f_e$

$105 = 7\,fe$

$\boxed{fe = 15\ cm}$ Or $\boxed{fe = 0.15\ m}$

Then

$\boxed{f_0 = 6 \times 15 = 90\ cm}$ Or $\boxed{f_0 = 0.9\ m}$

Now,

$$\boxed{P_0 = \frac{1}{fo} = \frac{1}{0.15} = 6.66\ Dioptre}$$

And

$$\boxed{P_e = \frac{1}{fe} = \frac{1}{0.9} = 1.11\ Dioptre}$$

Result: The powers of lenses are 6.66 dioptre and1.11 dioptre.

2(xv) A microscope has an objective of 12mm focal length and eye piece of 25 mm focal length. What is the distance between the lenses? What is the magnifying power if the object is in sharp focus when it is 15 mm from the objective?

Data:

Focal length of Objective $= f_0 = 12\ mm$

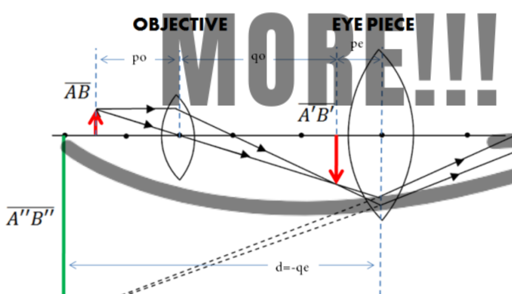
Focal length of Eye Piece $= f_e = 25\ mm$

Distance between lenses $= L = ?$

Magnifying Power $= M = ?$

Object distance from objective $= p_o = 15\ mm$

Image distance from eye piece $= q_e = -250\ mm$



RAY DIAGRAM OF COMPOUND MICROSCOPE

Solution:

For Objective:

$$\frac{1}{f_0} = \frac{1}{p_0} + \frac{1}{q_0}$$

$$\frac{1}{12} = \frac{1}{15} + \frac{1}{q_0}$$

$$\frac{1}{12} - \frac{1}{15} = \frac{1}{q_0}$$

$$\frac{3}{180} = \frac{1}{q_0}$$

$$\boxed{q_0 = \frac{180}{3} = 60\ mm}$$

For Eye Piece:

$$\frac{1}{f_e} = \frac{1}{p_e} + \frac{1}{q_e}$$

$$\frac{1}{25} = \frac{1}{p_e} + \frac{1}{(-250)}$$

$$\frac{1}{25} + \frac{1}{250} = \frac{1}{p_e}$$

$$\frac{11}{250} = \frac{1}{p_e}$$

$$\boxed{p_e = \frac{250}{11} = 22.72\ mm}$$

Now , Distance between lenses

$$L = p_e + q_0$$

$$L = 22.72 + 60$$

$$\boxed{L = 82.72\ mm}$$

And Magnification of micro scope is given by

$$M = \left(\frac{q_0}{p_0}\right)\left(1 + \frac{d}{f_e}\right)$$

$$M = \left(\frac{60}{15}\right)\left(1 + \frac{250}{25}\right)$$

$$M = (4)(11)$$

$$\boxed{M = 44}$$

Result: The distance between lenses is 82.72 mm and the magnification is 44

2017

Q.2(xii) A lens 2 cm focal length is to be used as a magnifying glass. How far from the lens should the object be placed? What is its magnifying power?

Data:

Focal Length = f = 2cm

Image distance = q = -d = -25cm

Object distance = p = ?

Magnifying Power = M = ?

Solution:

As we know that

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{2} = \frac{1}{p} + \frac{1}{(-25)}$$

$$\frac{1}{2} + \frac{1}{25} = \frac{1}{p}$$

$$\frac{27}{50} = \frac{1}{p}$$

Or $\boxed{P = \frac{50}{27} = 1.85\ cm}$

Now, Magnification of Magnifying Glass is given by

$$M = 1 + \frac{d}{f} = 1 + \frac{25}{2}$$

$$\boxed{M = 13.5}$$

Result: The object should be placed at 1.85 cm and the magnification is 13.5.

2016

Q.2 (x) Textbook Numerical 13

2015

No Numerical

2014

Q.2(xv) A watch maker uses a magnifying glass of local length 5cm to see the damaged spring of a watch. If he holds the glass close to the eye what is the best position of the object? What is the linear magnification?

Data:

Focal length of lens = 5 cm

Position of object = p = ?

Linear magnification = M = ?

Solution:

The magnification of Magnifying Glass

$$M = 1 + \frac{d}{f}$$

$$M = 1 + \frac{25}{5}$$

$$\boxed{M = 6}$$

As we know that

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

For best view the minimum distance is 25 cm

$$\frac{1}{5} = \frac{1}{p} + \frac{1}{(-25)}$$

$$\frac{1}{5} = \frac{1}{p} - \frac{1}{25}$$

$$\frac{1}{5} + \frac{1}{25} = \frac{1}{p}$$

$$\frac{1}{p} = \frac{6}{25}$$

$$\boxed{p = 4.16\ cm}$$

Result: The magnification is 6 and object distance should be 4.16 cm.

2013

Q.2 (vii) A magnifying glass of focal length 6 cm is used to see a small specimen. The least distance of distinct vision of the observer is 25 cm. What is the magnifying power of the lens?

Data:

Focal length of Magnifying Glass = f = 6 cm

Least distance of distinct vision = d = 25 cm

Magnifying Power = M = ?

Solution:

The magnification of magnifying glass is

$$M = 1 + \frac{d}{f}$$

$$M = 1 + \frac{25}{6}$$

$$\boxed{M = 5.16}$$

Result: The magnifying power of the lens is 5.16.

2012

Q.2 (xii) Same as 2018 Q.2 (vi)

2011

2(xv) Two converging lenses of focal lengths 30 cm and 60 cm are placed in contact. What is the focal length of this combination? Calculate the power of the combination in dioptres.

Data:

$focal\ length\ of\ first\ lens = f_1 = 30\ cm$

$focal\ length\ of\ second\ lens = f_2 = 60\ cm$

$focal\ length\ of\ combination = f = ?$

$Power\ of\ combination = P = ?$

Solution:

The power of combination of lenses is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f} = \frac{1}{30} + \frac{1}{60}$$

$$\frac{1}{f} = \frac{3}{60}$$

$$\boxed{f = 20\ cm}$$

or $f = 0.2\ m$

According to the definition of power of lens

$$P = \frac{1}{f}$$

Or $\boxed{P = \frac{1}{0.2} = 5\ diopters}$

Result: The power of combination is 5 diopters and focal length is 20 cm.

2010

Q.2 (xiii) A magnifying glass produces an image of magnifying 6. What is the power of the lens? What is the best position of the object if a watch maker holds the same lens close to his eye to see the damaged spring of the watch?

Data:

Power of lens = ?

Position of object = p = ?

Linear magnification = M = 6

Solution:

The magnification of Magnifying Glass

$$M = 1 + \frac{d}{f}$$

$$6 = 1 + \frac{25}{f}$$

$$5 = \frac{25}{f}$$

$$\boxed{f = 5\ cm}$$

Power of lens is given by

$$\boxed{P = \frac{1}{f} = \frac{1}{0.05} = 20\ diopter}$$

As we know that

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

For best view the minimum distance is 25 cm

$$\frac{1}{5} = \frac{1}{p} + \frac{1}{(-25)}$$

$$\frac{1}{5} = \frac{1}{p} - \frac{1}{25}$$

$$\frac{1}{5} + \frac{1}{25} = \frac{1}{p}$$

$$\frac{1}{p} = \frac{6}{25}$$

$$\boxed{p = 4.16\ cm}$$

Result: The magnification is 6 and object distance should be 4.16 cm.