ME English Center

New Physics Book

Class 11th Notes

(According to new book)

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Unit # 1: Physics and Measurements

Worked Example 1.1

Show that the equation for impulse $Ft = m\,V_f - m\,V_i = m\,(V_f - V_i) = m\,\Delta V$ is dimensionally correct.

Solution:

Step 1: write above equation in dimensional form we have.

$[M][L][T^{-2}][T] = [M][L][T]^{-1} + [M][L][T]^{-1}$

Step 2:

Therefore $[M][L][T]^{-1} = [M][L][T]^{-1}$ and the equation is correct, both sides having the dimensions of momentum.

Worked Example 1.2

If the radius of sphere is measured a 9 cm with an error of 0.02 cm. Find the approximate error in calculating its volume.

Solution:

Step 1:

$R = 9$ cm and $\Delta R = 0.02$ cm

Volume of sphere $= \frac{4}{3}\pi r^3$ By differentiating both sides, we get

$\therefore \frac{\Delta V}{V} = 3\,\frac{\Delta R}{R}$

$\frac{\Delta V}{V} = 3 \times \frac{\Delta R}{R} = 3 \times \left(\frac{0.02}{9}\right) = 0.0067$

So, the error in volume calculation is approximately 0.67%.

Step 2:

$$V = \frac{4}{3}\pi R^3$$

$$V = \frac{4}{3} \times \pi \times (9)^3 = 3053.628\ cm^3$$

The absolute error in volume is approximately 3053.628 x 0.0067 = 20.46 cm^3.
Therefore, the approximate error in calculating the volume of the sphere is 20.46 cm^3.

Worked Example 1.3

If radius of a sphere is measured as 7.5 cm with error of 0.03 cm, find the approximate error in calculating its volume.

Solution:

Step 1:

Let R be the radius and V be the volume of the sphere, then

$V = \frac{4}{3}\pi R^3$ Differentiating both sides, we get

Let ΔR be the error in R and the corresponding error in V is ΔV, then

$$\frac{\Delta V}{V} = 3 \times \frac{\Delta R}{R} = 3 \times \left(\frac{0.03}{7.5}\right) = 0.012$$

Step 2:

If R is given 7.5 cm and ΔR is 0.03 cm

$V = \frac{4}{3}\pi\, 7.5^3 = 1767.15\ cm^3$

The absolute error in volume is approximately $1767.15 \times 0.012 = 21.2058\ cm^3$.

Therefore, the approximate error in calculating the volume of a sphere is $21.2058\ cm^3$.

Worked Example 1.4

Consider the length of cube is given as 5.75±0.3 cm and you want to find absolute uncertainty in volume.

Solution:

Step 1:

$VOLUME = L^3 = (5.75)^3 = 190\ cm^3$

Step 2:

Percentage uncertainty $= 3 \times \left(\frac{0.3}{5.75}\right) \times 100 = 15.65\%$

Absolute uncertainty in volume $= 190 \pm 15.65\ cm^3$

Section (A): Multiple Choice Questions (MCQs)

1. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are:

a. 5, 1, 2　　b. 5, 1, 5

c. 5,5,2　　d. 4, 4,2

2. Which among the following is the supplementary unit:

a. Mass
b. Time
c. Solid angle
d. Luminosity

3. The unit of solid angle is

a. Second
b. Steradian
c. Kilogram
d. Candela

4. The quantity having the same unit in all system of unit is:

a. mass
b. time
c. length
d. temperature

5. Random errors can be eliminated by:

a. taking number of observations and their mean.
b. measuring the quantity with more than one instrument
c. eliminating the cause
d. careful observations

6. Systemic error can be:

a. either positive or negative
b. negative only
c. positive only
d. zero error

7. [MLT^{-2}] is dimensional formula of

a. strain
b. displacement
c. force
d. pressure

8. Which of the following pair has the same dimension?

a. moment of inertia and torque
b. impulse and momentum
c. surface tension and force
d. specific heat and latent heat

9. Dependent variable is:

a. cause
b. cause and effect
c. effect
d. reason

10. Dimensions of kinetic energy is the same as that of:

a. Acceleration b. Work

c. Velocity d. Force

Ans:

1. a	2. c	3. b	4. b	5. a
6. a	7. c	8. b	9. c	10. b

CRQ's:

1. Give an example of (I) a physical quantity which has a unit but no dimensions, (II) a physical quantity which has neither unit not dimensions, (III) a constant which has a unit, (IV) a constant which has no unit.

Ans)

I. Angle

II. Relative density

III. Gravitational constant

IV. Avagadro's number

2. When rounding the product or quotient of two measurements, is it necessary to consider significant digit?

Ans) Yes, when rounding the product or quotient of two measurements, it is necessary to consider significant digits.

3. Drive the equation for period of oscillations of a mass suspended on a vertical spring by dimensional analysis. i.e. $T = 2\pi\sqrt{\frac{m}{k}}$

Ans) Consider the oscillations of a mass suspended on a vertical spring by dimensional analysis. We assume that the period of the mass suspended on a vertical spring [T] depends on following quantities:

1. Mass (*m*): [M]
2. Spring constant (*k*): $[M][T]^{-2}$
3. Acceleration due to gravity (*g*): $[L][T]^{-2}$

Therefore, the equation can be written as.

$$T = m^x \cdot k^y \cdot g^z$$

Where x, y and z are unknown powers.

$$[T] = [M]^x \cdot [M]^y[T]^{-2y} \cdot [L]^z[T]^{-2z}$$

Equating the indices for M, L and T on both sides of the equation, we get:

M: $0 = x + y$

L: $0 = z$

T: $1 = -2y - 2z$

Therefore

$x = 1/2$, $y = -1/2$ and $z = 0$

The original equation therefore becomes

$$T = m^{\frac{1}{2}} . k^{-\frac{1}{2}} . g^0$$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

4. Find the dimensions of the following.

- **a) Work**
- **b) Energy**
- **c) Power**
- **d) Momentum**

a) $W = F \cdot d$

1. Force(F): $[M] \cdot [L]/[T]^2$
2. Displacement(d): $[L]$

Work = $\frac{[M].[L]}{[T]^2} . [L]$

Work = $[M] \cdot [L]^2 \cdot [T]^{-2}$

b) In SI units, the dimensions of energy ([E]) are the same as the dimensions of work:

Energy = $[M] \cdot [L]^2 \cdot [T]^{-2}$

c) $P = \frac{W}{t}$

1. Work(W): $[M] \cdot [L]^2 \cdot [T]^{-2}$
2. Time(t): $[T]$

Power = $\frac{[M].[L]^2.[T]^{-2}}{[T]}$

Power = $[M]\cdot[L]^2\cdot[T]^{-3}$

d) $p = m\cdot v$

1. Mass(m): [M]
2. Velocity(v): $[L]\cdot[T]^{-1}$

Momentum = $[M]\cdot[L]\cdot[T]^{-1}$

5. You measure the radius of a wheel to be 4.16 cm. If you multiply by 2 to get diameter, should you write the result as 8 cm or as 8.32 cm? Justify your answer.

Ans) According to the rules of significant figures, the result of a calculation should have the same number of significant figures as the least precise measurement used in the calculation. In this case, the least precise measurement is the radius with three significant figures (4.16 cm). When you multiply by 2 to get the diameter, the result should be rounded to three significant figures as well. So the result should be 8.32 cm.

6. If $y = a + bt + ct^2$ where y is in meters and t in seconds, what is the unit of c?

Ans) In the equation $y = a + bt + ct^2$, where y is in meters and t is in seconds, let's consider the units of each term in the equation:

- a is a constant, and since y is in meters, a must also be in meters.
- bt involves multiplication of *b* (a coefficient) and t (in seconds). The unit of b will be in meters per second (m/s) because it's multiplied by time (t).
- ct^2 involves multiplication of c (a coefficient) and t^2 (in seconds squared). The unit of c should be such that the resulting unit, when multiplied by time squared (t2), gives meters. **Therefore, the unit of c should be in meters per second squared (m/s^2)**.

7. Differentiate between accuracy and precision.

Ans)

Accuracy	Precision
Accuracy is referred to the level of agreement between the actual measurement and the absolute measurement.	Precision suggests the level of variation that happens in the values of several measurements of the same factor.
It represents how closely the results agree with the standard value.	Represents how closely results agree with one another.
Single-factor or measurement is required.	Multiple measurements are needed to comment about precision.
Occasionally, a measurement may happen to be accurate by chance, while consistent accuracy and precision are required for a measurement to be reliable	Results can be precise without being accurate.

8. Define dependent and independent variables.

Ans) Dependent variables: The value of a dependent variable depends on the values of one or more independent variables.

Independent variables: The values of independent variables are not influenced by other variables.

9. Differentiate systematic error and random error.

Aspect	Systematic Error	Random Error
Definition	Consistent and predictable	Inconsistent and unpredictable
Nature	Bias towards a certain value	No specific bias
Causes	Faulty equipment, calibration issues, procedural errors	Fluctuations in measurement conditions, environmental factors
Effects	Shifts results consistently	Scatters results randomly
Correction	Can be identified and corrected once identified	Can't be fully eliminated

10. Enlist the limitations of dimensional analysis?

Ans) Limitations of Dimensionality:

Some limitations of dimensionality are:

- It doesn't give information about the dimensional constant.
- The formula containing trigonometric function, exponential functions, logarithmic function, etc. cannot be derived.
- It gives no information about whether a physical quantity is a scalar or vector.

11. Describe least count of Vernier and screw gauge micrometer.

Ans)

1. **Vernier Caliper:** The least count of a Vernier caliper is the difference between one smallest division on the main scale and one smallest division on the Vernier scale that aligns with a main scale division.

The formula to calculate the least count of a Vernier caliper is given by:

$$\text{Least Count} = \frac{\text{Smallest division on the main scale}}{\text{Total number of divisions on the Vernier scale}}$$

For example, if the main scale has divisions of 1 mm each and the Vernier scale has 10 divisions that cover the same length as 9 main scale divisions, then the least count of the Vernier caliper would be:

$$\text{Least Count} = \frac{1\ \text{mm}}{10}$$

2. **Screw Gauge (Micrometer):**

The least count of a screw gauge is determined by the pitch of the screw and the number of divisions on the circular scale.

The formula to calculate the least count of a screw gauge is given by:

$$\text{Least Count} = \frac{\text{Pitch of the screw}}{\text{Total number of divisions on the circular scale}}$$

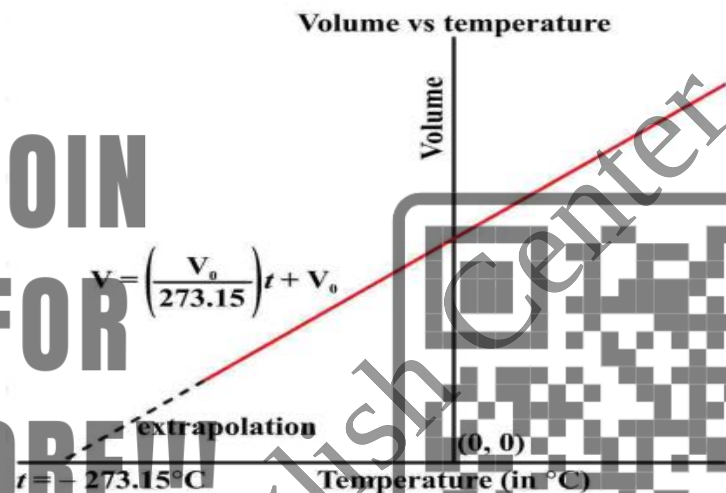
For example, if the pitch of the screw is 0.5 mm and the circular scale has 100 divisions, then the least count of the screw gauge would be:

$$\text{Least count} = \frac{5\ mm}{100} = 0.005\ \text{mm}$$

12. Describe extrapolation methods.

Ans) Extrapolation is a statistical technique that involves using observed data to estimate values beyond the range of the data that was collected. In other words, it is the process of making predictions or estimates about future or unseen data based on the trends or patterns in the existing data.

For example, in case volume temperature graph as shown in figure below, If we extrapolate the line till it intercepts the temperature axis, in result we reach zero kelvin temperature.



ERQ's:

1. Discuss graphs and its various types by supported an example?

Ans) Graphs: Graphs are visuals that show relationships between; intended to display the data in a way that is easy to understand and remember

Types: There are two types of graph.

1. Linear graph
2. Non-linear graph:

Linear graph: Observe on the graph as shown in figure below, x-axis is showing time and Y axis is showing position. It is observed that position is linearly increasing in positive direction with the time. So, the graph is linear.

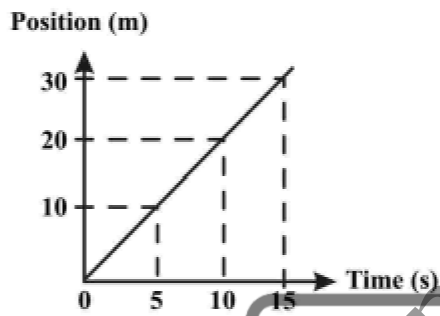
To determine the slope and intercept of the graph in figure shown below, compare it with the equation of straight line y= mx +c

Where m is the slope, c is the point of intercept. Now slope can be determined as

$$\text{Slope} = m = \frac{\Delta y}{\Delta x}$$

$$= \frac{30}{15} = 2ms^{-1}$$

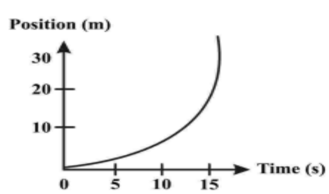
As the intercept of the line is at origin so the intercept will be zero (c = 0). Note that slope of a straight line always constant.



linear graph

Non-linear graph: In contrast to the previous example, let's graph the position of an object with a constant, non-zero acceleration starting from rest at the origin as shown in figure below. The primary difference between this curve and those on the previous graph is that this line actually curves. The relation between position and time is quadratic when the acceleration is constant and therefore this curve exhibits a non-linear relationship.

Slope is a characteristic exclusive to straight lines. This emphasizes that there is no single, constant velocity in such cases. The velocity of an object under these circumstances must be undergoing change, indicating acceleration. In the case of a curved position-time graph, since the slope varies at each point along the curve, it is not possible to determine a uniform velocity by calculating the slope alone.



Non linear graph

2. Elaborate rules of significant figures. State the rules for determining the number of significant figures in a given measurement.

Ans) Rule 1: All Non-Zero Digits are Significant: Every non-zero digit in a number is considered significant.

For example: In the number 123.45, all digits (1, 2, 3, 4, 5) are significant.

Rule 2: All Zeros between Significant Digits are Significant: Zeros that appear between non-zero digits are considered significant.

For example: In the number 101.203, all digits (1, 0, 1, 2, 0, 3) are significant.

Rule 3: Leading Zeros are Not Significant: Zeros that appear to the left of the first non-zero digit are not considered significant. They merely indicate the scale of the number.

For example: In the number 0.00345, only the digits 3, 4, and 5 are significant.

Rule 4: Trailing Zeros in a Decimal Number are Significant: Zeros that appear to the right of the last non-zero digit in a decimal number are considered significant.

For example: In the number 56.700, all digits (5, 6, 7, 0, 0) are significant.

Rule 5: Trailing Zeros in Whole Numbers are Not Always Significant: Trailing zeros in a whole number (a number without a decimal point) are generally not considered significant unless they are explicitly indicated.

For example: In the number 700, only the digit 7 is significant.

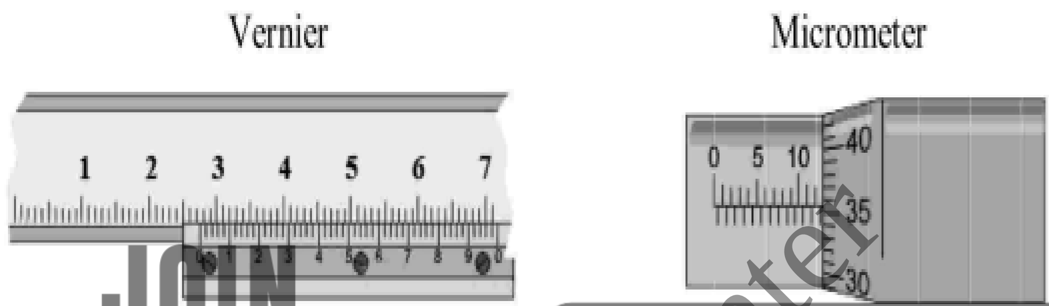
3. Here are the main uses of dimensional analysis:

Ans)

1. **Checking Equations for Consistency:** Dimensional analysis helps verify the correctness and consistency of equations. If the dimensions on both sides of an equation are not the same, it indicates an error in the equation. By comparing dimensions, scientists and engineers can catch mistakes in their calculations and ensure the validity of their equations.
2. **Predicting Relationships:** Dimensional analysis can be used to predict how different physical quantities are related to each other. If you have a set of variables and their dimensions, you can determine how they must be combined to create a meaningful equation. This is particularly useful when deriving equations for physical phenomena where experimentation or theoretical derivation is challenging.

3. **Unit Conversion:** Dimensional analysis aids in converting units from one system to another. By using conversion factors with appropriate dimensions, you can convert measurements from, for instance, the metric system to the imperial system or vice versa.

4. A vernier and micrometer are shown as under. Observe their readings and write correctly.



Ans)

Vernier:

Main scale reading = 27 mm

Vernier scale reading = 5 x 0.02 = 0.1 mm

Main scale reading + Vernier scale reading = 27.1 mm

Micrometer:

Main scale reading = 12.5 mm

Circular scale reading = 36 x 0.01 = 0.36 mm

Main scale reading + Circular scale reading = 12.86 mm

Numerical:

1. What is the percent uncertainty in the measurement 3.67 ± 0.25 mm?

Data:

Measurement = 3.67 ± 0.25 mm

Percent uncertainty = ?

Solution:

Percent Uncertainty = $\frac{\text{Absolute Uncertainty}}{\text{Measured Value}}$ x 100

Percent Uncertainty = $\frac{0.25}{3.67}$ x 100

Percent Uncertainty = 6.8 %

2. What is the area, and its approximate uncertainty, of a circle with radius 3.7 x 10^4 cm.

Data:

A = ?

Uncertainty = ΔA = ?

r = 3.7 x 10^4 cm.

Solution: First calculating the area.

$A = \pi . r^2$

$A = \pi(3.7 \times 10^4)^2$

$A = 4.3 \times 10^9\ cm^2$

Now, let's calculate the uncertainty in the area. The uncertainty in the radius is given as 0.05×10^4 cm (it is not provided in the question by mistake)

To find the uncertainty in the area, we'll use the concept of differentials:

$\Delta A = 2\pi r \Delta r$

$\Delta A = 2\pi\ (3.7 \times 10^4)\ (0.05 \times 10^4)$

$\Delta A = 0.11 \times 10^9\ cm^2$

3. An aero plane travels at 850 km/h. How long does it take to travel 1.00 km?

Data:

v = 850 km/h = $\frac{850 \times 1000}{3600}$ = 236.111 m/s

s = 1.00 km = 1 x 1000 = 1000 m

Solution:

$v = \frac{s}{t}$

$t = \frac{s}{v}$

$t = \frac{1000}{236.111}$

$t = 4.23529$ s

Since the given distance value (1.00 km) has three significant figures, the calculated time should also be expressed with three significant figures.

$t = 4.2353$

$t = 4.235$

t = 4.24 s

4. A rectangular holding tank 25.0 m in length and 15.0 m in width is used to store water for short period of time in an industrial plant. If 2980m^3 water is pumped into the tank. What is the depth of the water?

Data:

L = 25.0 m

W = 15.0 m

V = 2980m^3

H = ?

Solution:

V = L x W x H

$H = \frac{V}{L \times W}$

$H = \frac{2980}{25.0 \times 15.0} = 7.9466$ m

We are going to round off it up to three decimal places.

H = 7.947 m

H = 7.95 m

5. Find the volume of rectangular underground water tank has storage facility of 1.9 m by 1.2 m by 0.8 m.

Data:

L = 1.9 m

W = 1.2 m

H = 0.8 m

Solution:

V = L x W x H

V = 1.9 x 1.2 x 0.8

V = 1.824 m^3

We need to use the lowest number of significant figures among the given dimensions, which is 1 significant figure. Therefore, we should round the volume to 1 significant figure.

V= 2 m^3

6. Two students derive following equations in which x refers to distance traveled, v the speed, a the acceleration, and t the time and the subscript (o) means a quantity at time t=0

a) **$x = v t^2 + 2at$**
b) **$x = v_o t + 2at^2$**

Which of these could possibly be correct according to dimensional check?

a) $[L] = [L].[T]^{-1} .[T]^2 + 2 [L].[T]^{-2} .[T]$

$[L] = [L].[T] + 2 [L].[T]^{-1}$

$[L] = [L].[T] + 2 [L].[T]^{-1}$

Since L.H.S ≠ R.H.S, it is incorrect according to dimension.

b) $[L] = [L].[T]^{-1} .[T] + 2 [L].[T]^{-2} .[T]^2$

[L] = 3[L]

After omitting constant

[L] = [L]

Since L.H.S = R.H.S, it is correct according to dimension.

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7. One hectare is defined as 10^4 m². One acre is 4 x 10^4 f t^2. How many acres are in one hectare? (Hint: 1 m = 3.28 ft).

Data:

Hectare = 10^4 m²

Acre = 4 x 10^4 f t^2

Acres in one hectare = ?

1 m = 3.28 ft

Solution:

Area in square feet= $(10^4\ m^2) \times (3.28 ft/m)^2$

Area in square feet= $(10^4\ m^2) \times (10.76\ ft^2/m^2)$

Area in square feet = 107600 ft^2

Number of acres = $\frac{\text{Area in square feet}}{4\times104\ \text{ft}^2/\text{acre}}$

Number of acres = $\frac{107600\ \text{ft}^2}{4\times104\ \text{ft}^2/\text{acre}}$

Number of acres = 2.69

8. A watch factory claims that its watches gain or lose not more than 10 seconds in a year. How accurate is this watch, express as percentage?

Data:

Acceptable range = 10 s

Percentage Accuracy = ?

Solution:

Percentage Accuracy = $\frac{\text{Acceptable Range}}{\text{Total Seconds in a Year}} \times 100$

Percentage Accuracy = $\frac{10}{365 \text{ x } 24 \text{ x } 60 \text{ x } 60} \times 100$

Percentage Accuracy = 3.17 x 10^{-5}

9. The diameter of Moon is 3480 km. What is the volume of the Moon? How many Moons would be needed to create a volume equal to that of Earth?

Data:

d = 3480 km

V =?

No. of Moons = ?

Solution:

First we calculate the volume of moon.

$r = d/2 = 3480 / 2 = 1740 \text{ km} = 1740 \times 10^3 \text{ m}$

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi (1740 \times 10^3)^3$

$V = 2.2 \times 10^{19} \text{ m}^3$

Now, we calculate the no. of moons:

$\text{No. of Moons} = \frac{\text{Volume of Earth}}{\text{Volume of Moon}}$

Volume of Earth = $1.08321 \times 10^{12} \text{ km}^3 = 1.08321 \times 10^{21} \text{ m}^3$

$\text{No. of Moons} = \frac{1.08321 \times 10^{21}}{2.2 \times 10^{19}}$

No. of Moons = 49.24

Unit 2: Kinematics

Worked Example 2.1

Find $\vec{a}.\vec{b}$ when $\vec{a} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $\vec{b} = 3\hat{\imath} - 4\hat{\jmath} - 2\hat{k}$

Step 1:

$\vec{a} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$

$\vec{b} = 3\hat{\imath} - 4\hat{\jmath} - 2\hat{k}$

$\vec{a}.\vec{b} = (\hat{\imath} - 2\hat{\jmath} + \hat{k}).(3\hat{\imath} - 4\hat{\jmath} - 2\hat{k}) = 3 + 8 - 2$

Step 2: $\vec{a}.\vec{b} = 9$

$\therefore\ \hat{\imath}.\hat{\jmath} = \hat{k}.\hat{\jmath} = \hat{\imath}.\hat{k} = 0$

$\therefore\ \hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$

Worked Example 2.2

A turtle starts at the origin and moves with the speed of v_0=10 cm/s in the direction of 25° to the horizontal.

(a) Find the coordinates of a turtle 10 seconds later.

(b) How far did the turtle walk in 10 seconds?

(a)

(a)

Step 1: you can solve the equations independently for the horizontal (x) and vertical (y) components of motion and then combine them!

$$\vec{v}_0 = \vec{v}_x + \vec{v}_y$$

- X components:

$$v_{0x} = v_0 \cos 25^\circ = 9.06 \text{ cm/s} \qquad \Delta x = v_{0x} t = 90.6 \text{ cm}$$

- Y components:

$$v_{0y} = v_0 \sin 25^\circ = 4.23 \text{ cm/s} \qquad \Delta y = v_{0y} t = 42.3 \text{ cm}$$

Step 2:

- Distance from the origin:

$$d = \sqrt{\Delta x^2 + \Delta y^2} = 100.0 \text{ cm}$$

Worked Example 2.3

A Car with an initial speed of 1 m/sec was in motion for 10 minutes, and then it came to a stop, the velocity right before it stopped was 5 m/sec. What was the constant acceleration of the car?

Solution:

Step 1: Write the known quantities and point out quantities to be found.

$Initial\ Velocity = 1\ m/sec$

$Final\ Velocity = 5\ m/sec$

Time for which the car was in motion = 10 mint

Acceleration = ?

Step 2: Write the formula and rearrange if necessary

Using First equation of motion,

$v = u + at$

Step 3: Put the value in formula and calculate

$5 = 1 + a \times (10 \times 60)$

$a \times 600 = 4$

$a = 4/600$

$a = 0.0066\ m/sec^2$

Worked Example 2.4

A cycle covered 2 km in 8 minutes and the initial velocity of the cycle was 1 m/sec. Find the acceleration that the cycle had in its motion.

Solution:

Step 1: Write the known quantities and point out quantities to be found.

Displacement covered = 2km

Total Time Taken = 8minutes = 8×60 = 480 seconds.

Initial Velocity = 1 m/sec

Using Second equation of motion to find the acceleration of the cycle,

Step 2: Write the formula and rearrange if necessary

Second Equation of motion, $S = ut + 1/2(at^2)$

Step 3: Put the value in formula and calculate

$2000 = 1 \times 480 + 1/2(a \times 480^2)$

$2000 = 480 + 115200a$

$1520 = 115200a$

$a = 0.0139 m/sec^2$

Worked Example 2.5

A body is projected with a velocity of $20ms^{-1}$ at the angle of 50 degree to the horizontal plane. Find the time of flight of the projectile.

Solution:

Step 1: Write the known quantities and point out quantities to be found.

Initial Velocity $V_o = 20ms^{-1}$

angle $\theta = 50°$

$g = 9.8ms^{-2}$

Step 2: Write the formula and rearrange if necessary

Formula for time of flight is,

$$T = \frac{2V_o \sin\theta}{g}$$

Step 3: Put the value in formula and calculate

$T = 2 \times 20 \times \sin 50° / 9.8$

$T = 2 \times 20 \times 0.766 / 9.8$

$T = 30.64 / 9.8$

$\mathbf{T = 3.126\ sec}$

Therefore time of flight is 3.126 second.

Section (A): Multiple Choice Questions (MCQs)

1. To get a resultant displacement of 10m, two displacement vectors of magnitude 6m and 8m should be combined:

(a) Parallel (b) Antiparallel

(c) At an angle of 45^{o} (d) Perpendicular to each other

2. The velocity of a particle at an instant is 10 m/s and after 5sec the velocity of particle is 20m/s. The velocity 3 sec before in m/s is:

(a) 8 (b) 4

(c) 6 (d) 7

3. A ball is thrown upwards with a velocity of 100 m/s. It will reach the ground after

(a) 10 sec (b) 20 sec

(c) 5 sec (d) 40 sec

4. Two projectiles are fired from the same point with the same speed at angles of projection 60° and 30° respectively. Which one of the following is true?

(a) The range will be same
(b) Their maximum height will be same
(c) Their landing velocity will be same
(d) Their time of flight will be same

5. The ratio of numerical values of average velocity and average speed of a body is always:

(a) Unity
(b) Unity or less
(c) Unity or more
(d) Less than unity

6. If the average velocities of a body become equal to the instantaneous velocity, body is said to be moving with:

(a) Uniform acceleration
(b) Uniform velocity
(c) Variable velocity
(d) Variable acceleration

7. At the top of a trajectory of a projectile, the acceleration is:

(a) maximum
(b) minimum
(c) zero
(d) g

8. At what angle the range of projectile becomes equal to the height of projectile?

(a) 65^o
(b) 45^o
(c) 76^o
(d) 30^o

9. The angle at which dot product becomes equal to the cross product is:

(a) 65^o
(b) 45^o
(c) 76^o
(d) 30^o

10. If the dot product of two non-zero vectors vanishes, the vectors will be:

(a) in the same direction
(b) opposite direction to each other
(c) perpendicular to each other
(d) zero

KEY:

1. d	2. b	3. b	4. c	5. b
6. b	7. d	8. c	9. b	10. c

CRQS:

1. Is the range equivalent to the horizontal distance in the projectile motion, or do they have distinct meaning? Explain.

Ans) The range and the horizontal distance in projectile motion are related concepts, but they are not exactly the same and have distinct meanings.

Horizontal Distance: This refers to the total distance covered horizontally by a projectile from its initial point to its final point. It is the straight-line distance measured along the horizontal axis, parallel to the ground. Horizontal distance takes into account the entire path of the projectile, including both its vertical and horizontal components of motion.

Range: The range specifically refers to the horizontal distance covered by a projectile along its trajectory before it hits the ground again (assuming the projectile returns to the same height from which it was launched). In other words, the range is the horizontal distance between the launch point and the point of impact on the ground. It does not include the vertical distance traveled by the projectile.

2. If air resistance is taken into account in case of projectile motion, then what parameters of projectile are influenced?

Ans) Here are the parameters that are influenced by air resistance in projectile motion:

1. **Trajectory Shape:** The trajectory of the projectile will deviate from a perfect parabolic shape due to air resistance. Instead of a smooth curve, the trajectory will be more flattened out.
2. **Range:** The range of the projectile will be affected. Air resistance will cause the projectile to slow down over time, reducing its horizontal velocity and thus its range.
3. **Time of Flight:** The time the projectile takes to travel from its initial point to the impact point will increase due to air resistance. This is because the object's horizontal velocity decreases as it travels, causing the overall time of flight to be longer compared to a scenario without air resistance.
4. **Maximum Height:** The maximum height reached by the projectile will also be influenced. The vertical component of velocity decreases due to air resistance, so the projectile won't reach as high a maximum height as it would in the absence of air resistance.
5. **Impact Velocity:** The velocity of impact at the ground will be lower due to air resistance. This means the projectile will hit the ground with less speed than it would without air resistance.
6. **Launch Angle:** The optimal launch angle for maximizing the range (which is 45 degrees in the absence of air resistance) may change slightly when air resistance is taken into account. This is because the balance between the

horizontal and vertical components of motion is altered by the slowing effect of air resistance.

7. **Surface Area:** Objects with larger surface areas facing the direction of motion experience greater air resistance.

3. What are some real-life examples of projectile motion? Can motion of an aero plane be considered as projectile motion? Explain

Ans) Here are some real-life examples of projectile motion:

1. **Throwing a Ball:** When you throw a ball, it follows a parabolic trajectory as it rises and falls under the influence of gravity. The ball's horizontal motion is uniform, while its vertical motion is affected by gravity.
2. **Kicking a Soccer Ball:** When a soccer player kicks the ball, it follows a curved path through the air before landing on the ground.
3. **Diving and Cliff Jumping:** When a person dives or jumps off a cliff into the water, they follow a curved path before entering the water.
4. **Projectile Weapons:** Weapons like cannons, catapults, and artillery shells rely on projectile motion. These objects are launched at certain angles to hit distant targets.

The motion of an airplane cannot be considered as pure projectile motion, but it does share some similarities with projectile motion while also involving significant differences due to the principles of aerodynamics, thrust, and control.

Projectile motion is the motion of an object that is launched into the air and moves under the influence of gravity alone, without any additional forces acting on it after the launch. In the case of an airplane, it is subjected to various forces and controls that make its motion more complex than simple projectile motion.

4. Where do we use the concept of speed and velocity in our daily life? Can you give some examples?

Ans) Here are some examples of how we use these concepts in our daily life:

1. **Driving a Car:** When you drive a car, you monitor your speed to adhere to speed limits and ensure safe driving. Speed is the rate of motion, and it helps you control how fast you're traveling.
2. **Running or Jogging:** People use speed to describe how fast they are running or jogging. For instance, you might say you're running at a speed of 8 miles per hour.
3. **Traveling:** Whether you're flying on an airplane or taking a train, speed is crucial for estimating travel time and planning your journey.
4. **Mobile Apps and GPS:** Navigation apps use speed and velocity information to estimate arrival times and provide real-time directions.

5. **Shipping and Logistics:** Shipping companies use speed and velocity to estimate delivery times and optimize routes for efficient delivery.

5. Can an object have an initial velocity of zero and still experience uniformly accelerated motion?

Ans) Yes, an object can have an initial velocity of zero and still experience uniformly accelerated motion.

For example, consider an object that is released from rest and allowed to fall freely under the influence of gravity. At the moment it's released, its initial velocity is zero. However, due to the constant acceleration caused by gravity, the object's velocity increases at a constant rate as it falls.

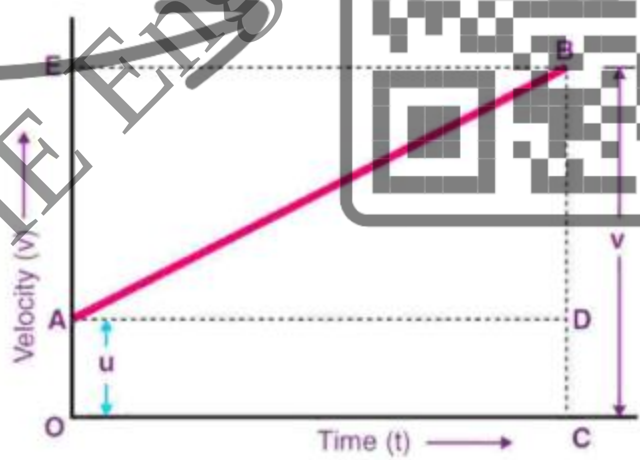
ERQS:

1. What are three equations of uniformly accelerated motion and how they are derived?

Ans)

Derivation of First Equation of Motion by Graphical Method: The first equation of motion can be derived using a velocity-time graph for a moving object with an initial velocity of u, final velocity v, and acceleration a.

In the figure below.



The velocity of the body changes from A to B in time t at a uniform rate.

BC is the final velocity and OC is the total time t. A perpendicular is drawn from B to OC, a parallel line is drawn from A to D. and another perpendicular is drawn from B to OE (represented by dotted lines).

Following details are obtained from the graph above.

The initial velocity of the body u = OA

The final velocity of the body, v = BC

From the graph, we know that

BC = BD + DC

Therefore, v = BD + DC

v = BD + OA (since DC = OA)

Finally,

v = BD + u (since OA = u) …….. (i)

Now, since the slope of a velocity-time graph is equal to acceleration a,

So,

a = slope of line AB

a = BD / AD

Since AD = AC = t the above equation becomes:

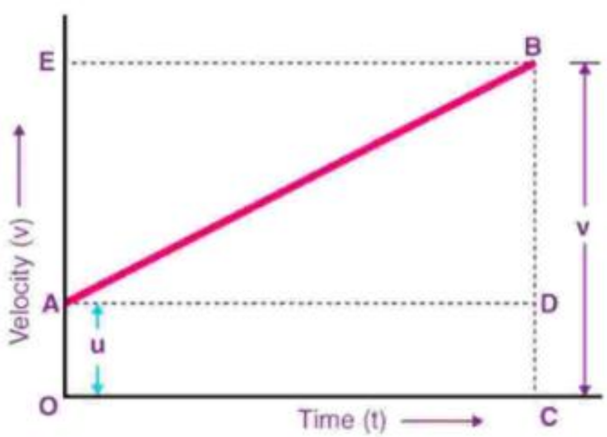
BD = at ……… (ii)

Now, combining Equation (i) & (ii) the following is obtained:

v = u + at

Derivation of Second Equation of Motion by Graphical Method:

From the figure shown, we can say that



Distance travelled (s) = Area of figure OABC

= Area of triangle ABD + Area of rectangle OADC

s = (1/2 x AD x BD) + (OA x OC)

Since **BD = EA**, the above equation becomes

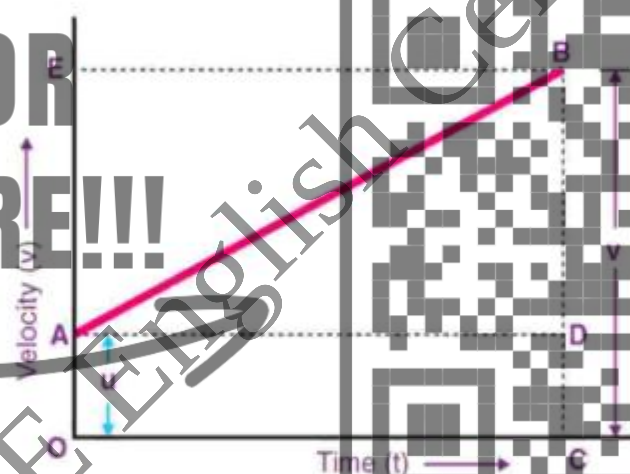
As EA = at, the equation becomes because V = at; EA represents the velocity (V)

s=1/2 x at x t + ut

by rearranging, the equation becomes

$s = ut + \frac{1}{2} at^2$

Derivation of Third Equation of Motion by Graphical Method: From the figure shown,



We can say that the total distance travelled, s is given by the Area of trapezium OABC.

Hence,

S = ½ (Sum of Parallel Sides) x Height

S = ½ (OA+CB) x OC

Since, OA= u, CB = v, and OC = t

The above equation becomes

S = ½ (u+v) x t

Now, since t = (v-u)/a

The above equation can be written as:

$$S = \frac{1}{2}\,((u+v) \times (v-u))/a$$

Rearranging the equation, we get

$$S = \frac{1}{2}\,(v+u) \times (v-u)/a$$

$$S = (v^2-u^2)/2a$$

Third equation of motion is obtained by solving the above equation:

$$\mathbf{v^2 = u^2 + 2aS}$$

2. How does the horizontal component of velocity in projectile motion behaves?

Ans) Here's how it behaves:

1. **Constant Magnitude:** The horizontal component of velocity remains constant throughout the entire motion. This means that the speed of the object in the horizontal direction does not change as long as there are no horizontal forces (like air resistance) acting on it.
2. **Unaffected by Gravity:** Gravity only affects the vertical motion of the projectile, not its horizontal motion. This is due to the fact that gravity acts vertically downwards, and there are no horizontal forces causing acceleration in that direction.
3. **Uniform Motion:** Uniform means no change with distance. Because there are no forces causing acceleration in the horizontal direction, the projectile's horizontal velocity remains the same, resulting in uniform horizontal motion.
4. **Independence of Vertical Motion:** The horizontal and vertical motions of a projectile are independent of each other. This means that the vertical motion, affected by gravity, does not impact the horizontal motion, and vice versa. The two motions occur simultaneously but are not directly linked.
5. **Horizontal Distance:** The constant horizontal velocity contributes to the projectile's horizontal distance, or range. The greater the initial horizontal velocity, the greater the horizontal distance covered by the projectile.

3. What is the significance of the area under a velocity-time graph in the context of accelerated motion?

Ans) Here are the key points regarding the significance of the area under a velocity-time graph in the context of accelerated motion:

1. **Constant Acceleration:** When an object undergoes constant acceleration, the velocity-time graph is a straight line. The area under the line within a specific time interval corresponds to the displacement of the object during that time interval.

2. **Sign of area:** The algebraic sign of the area indicates the direction of displacement. If the area is positive, the displacement is positive (motion in the positive direction). If the area is negative, the displacement is negative (motion in the negative direction).
3. **Uniformly Accelerated Motion Equations:** This relationship between area and displacement aligns with the equations of motion for uniformly accelerated motion. For instance, for an object experiencing constant acceleration a during the time interval t, the displacement S is given by:

   $s = ut + ½ at^2$

   Here, the first term ut corresponds to the area of the rectangle under the velocity-time graph, and the second term at^2 corresponds to the area of the triangle.

4. How do the initial launch angle and velocity affect projectile motion?

Ans) Here's how these factors affect projectile motion:

1. Initial Launch Angle:

- **Range and Trajectory Shape:** Different launch angles result in different trajectory shapes. A launch angle of 45 degrees results in the maximum range (horizontal distance) for a given initial velocity, as it provides an optimal balance between horizontal and vertical motion. Launch angles greater or smaller than 45 degrees will result in shorter ranges.

- **Maximum Height:** The launch angle also affects the maximum height reached by the projectile. An angle close to 90 degrees (almost vertical) results in a high maximum height but a shorter range. Conversely, shallow angles like 0 degrees (horizontal launch) result in no vertical motion and no maximum height.

- **Symmetry:** For a given initial speed, launch angles that are complementary (add up to 90 degrees) result in the same range. For example, launching at 30 degrees and launching at 60 degrees will yield the same range.

2. Initial Velocity:

- **Range:** The initial velocity directly affects the range covered by the projectile. A higher initial velocity leads to a greater range, assuming the launch angle remains the same.

- **Maximum Height:** While the launch angle determines the shape of the trajectory, the initial velocity influences the maximum height reached. A higher initial velocity will result in a higher maximum height, assuming the launch angle remains constant.

- **Time of Flight:** A higher initial velocity generally results in a longer time of flight since the projectile covers a greater horizontal distance before hitting the ground.

- **Impact of Air Resistance:** In the presence of air resistance, a higher initial velocity may result in a more significant impact on the trajectory due to increased air resistance affecting the projectile's motion.

5. Explain the concept of independence between horizontal and vertical motion in projectile motion.

Ans) The concept of independence between horizontal and vertical motion in projectile motion is a fundamental principle that states that the motion of a projectile in the horizontal direction and its motion in the vertical direction are completely separate from each other. In other words, the object's motion along one axis does not affect or influence its motion along the other axis. This concept simplifies the analysis of projectile motion by allowing us to treat the two motions as separate components.

Here's a detailed explanation of this concept:

1. **Gravity's Effect on Vertical Motion:** In projectile motion, the only force acting on the object is gravity, which accelerates the object downward in the vertical direction. The vertical motion of the projectile is influenced by this constant acceleration due to gravity. As the object moves upward, its vertical velocity decreases until it reaches its peak height, where its vertical velocity becomes zero. Then, as the object descends, its vertical velocity increases in the downward direction.
2. **Horizontal Motion:** In the absence of horizontal forces (assuming no air resistance), the horizontal component of the projectile's velocity remains constant throughout its motion. There are no forces acting horizontally to change the object's horizontal velocity. This means that the object's horizontal motion is essentially uniform motion, where the horizontal velocity doesn't change.
3. **Independence:** The key insight is that these two motions—the vertical and horizontal—are entirely independent of each other. The motion in one direction doesn't affect the motion in the other direction. The vertical motion depends only on the gravitational acceleration, while the horizontal motion remains unaffected by gravity.
4. **Simultaneous Motion:** Even though the vertical and horizontal motions are independent, they occur simultaneously. As the object moves horizontally, its vertical motion is also taking place. This simultaneous motion creates the characteristic curved trajectory of a projectile.

Numerical:

1. A helicopter is ascending at the rate of 12 m/s. At a height of 80 m above the ground, a package is dropped. How long does the package take to reach the ground?

Solution:

Consider upward motion from B to C.

Initial velocity of package at B = v_i = 12 m/sec

Final velocity of package at C = v_f = 0

Time from B to C = t_1 =?

a= -g = -9.8m/s²

Using the equation of motion

$V_f = V_i + at$

$0 = 12 + (-9.8)\ t_1$

$9.8\ t_1 = 12$

$t_1 = \frac{12}{9.8}$

t_1 = 1.2 sec

To calculate distance from B to C

$S_1 = v_i t + \frac{1}{2} gt^2$

$S_1 = (12)\ (1.2) + \frac{1}{2}\ (9.8)\ (1.2)^2$

S_1 = 14.4 - 7.06

S_1 = 7.34 m

Consider downward motion from C to A

$S = 80 + S_1$

S=80 + 7.34 = 87.34m

$v_i = 0$

g = 9.8m/sec^2

$t_2 = ?$

using equation of motion

$S = v_i t + \frac{1}{2} g t^2$

$87.34 = 0 \times t_2 + \frac{1}{2} (9.8) t_2^2$

$87.34 = 4.9\ t_2^2$

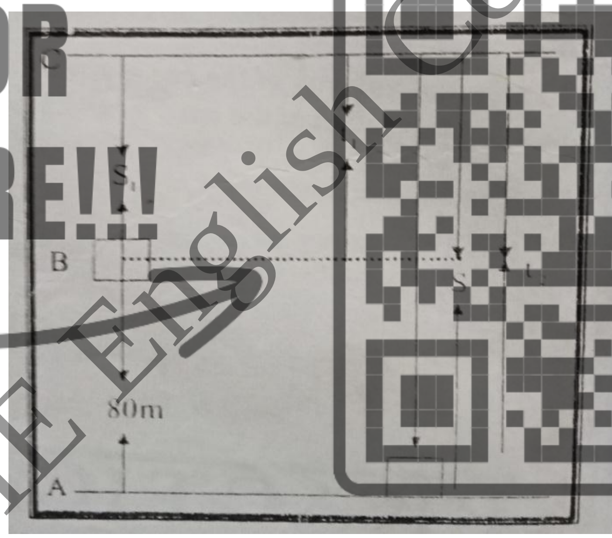
$t_2 = 4.22$ sec

Hence total time the package take to reach the ground is

$t = t_1 + t_2$

$t = 1.2 + 4.22$

t = 5.42 sec



2. Two tug boats are towing a ship each exerts a force of 6000 N, and the angle between two ropes is 60°. Calculate the resultant force on the ship?

Data:

$T_1 = T_2 = 6000$ N

$\theta = 60°$

F = ?

Solution:

$T_{1x} = 6000 \times \sin(30) = 3000$ N

$T_{1y} = 6000 \times \cos(30) = 5196.15$ N

$T_{2x} = 6000 \times \sin(30) = 3000$N

$T_{2y} = 6000 \times \cos(30) = 5196.15$ N

$F_x = 3000 - 3000 = 0$ (since they are antiparallel)

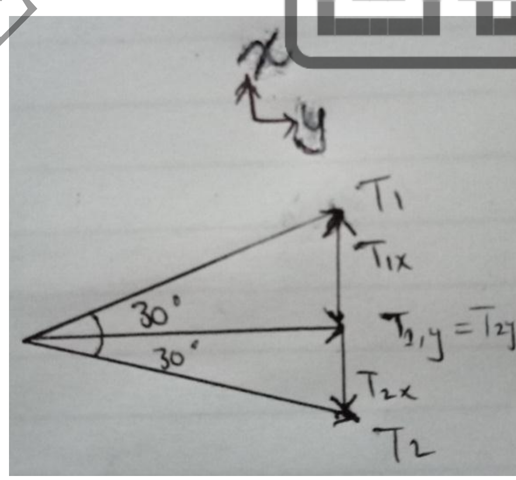
$F_y = = 5196.15$ N + = 5196.15 N = 10392.3 N

Resultant Force

$F = \sqrt{{F_x}^2 + {F_y}^2}$

$F = \sqrt{0^2 + 10392.3^2}$

F = 10392.3 N



3. A car starts from rest and moves with a constant acceleration. During the 5^{th} second of its motion, it covers a distance of 36 meters. Calculate:

a) **acceleration of the car**
b) **the total distance covered by the car during this time**

Solution:

Distance covered by the car in 5 second = S_5

Initial velocity of the car = $v_i = 0$

Time = t = 5 sec

a = ?

Using equation of motion

$S = v_i t + \frac{1}{2} at^2$

$S_5 = 0 \times 5 + \frac{1}{2}(a)(5)^2$

$S_5 = 12.5\ a \longrightarrow (1)$

Distance covered by the car in 4 second = S_4

Initial velocity of the car = $v_i = 0$

Time = t = 4 sec

a = ?

Using equation of motion

$S = v_i t + \frac{1}{2} at^2$

$S_4 = 0 \times 4 + \frac{1}{2}(a)(4)^2$

$S_4 = 8\ a \longrightarrow (2)$

Now the distance covered by the minibus in 5^{th} second.

$= S_5 - S_4 = 36$

$12.5a - 8a = 36$

$\mathbf{a = 8\ m/s^2}$

Putting the value of a in equation. (1)

$S_5 = 12.5\ (8) = 100\ m$

4. Show that the range of projectile at complementary angles is same with examples?

Solution:

Two angles are complementary if

$\alpha + \beta = 90^o$

Let $\alpha = 30^o$ and $\beta = 60^o$

$R = \frac{v_o^2 \sin 2\alpha}{g}$

$R = \frac{v_o^2 \sin 2(30)}{g}$

$R = \frac{\sqrt{3}}{2} \frac{v_o^2}{g}$

$R' = \frac{v_o^2 \sin 2\beta}{g}$

$R' = \frac{v_o^2 \sin 2(60)}{g}$

$R' = \frac{\sqrt{3}}{2} \frac{v_o^2}{g}$

Therefore, $R = R'$

5. At what angle the range of projectile becomes equal to the height of projectile?

Solution:

$H = R$

$\frac{v_o^2\ (\sin\theta)^2}{2g} = \frac{v_o^2\ \sin 2\theta}{g}$

$\frac{v_o^2\ (\sin\theta)^2}{2g} = \frac{v_o^2\ 2\sin\theta\cos\theta}{g}$

$\sin\theta = 4\cos\theta$

$\tan\theta = 4$

$\boldsymbol{\theta = 76^o}$

6. A mortar shell is fired at a ground level target 500 m distance with an initial velocity of 90 m/s. What is the launch angle?

Data:

R = 500 m

v_o = 90 m/s

θ = ?

Solution:

$R = \frac{v_o^2 \sin 2\theta}{g}$

$500 = \frac{90^2 \sin 2\theta}{9.8}$

$\theta = 18.61^o$

If the shell is fired with angle (90 - 18.6) = 71.4° then range will also 500m.

Unit # 3: Dynamics

Worked Example 3.1

A 50 N force applied on a box of mass 8.16 kg to move on the right across a horizontal surface. What is the acceleration of produced in the box.

Solution:

Step 1: Write the known quantities and point out quantities to be found.

$\vec{F} = 50\text{ N}$

$m = 8.16\,kg$

$a = ?$

Step 2: Write the formula and rearrange if necessary

$\vec{F} = ma \;\; or \; a = \frac{\vec{F}}{m}$

Step 3: Put the value in formula and calculate

$a = \frac{\vec{F}}{m}$

$a = \frac{50}{8.16}$

$a = 6.13\text{ ms}^{-2}$

The acceleration of the box is found to be 6.13ms^{-2}.

Worked Example 3.2

A hockey puck with a mass of 0.2 kg is sliding on the ice at a velocity of 10 m/s. It collides with a wall and bounces back with a velocity of -8 m/s. The collision lasts for 0.1 seconds. Calculate the impulse experienced by the hockey puck and the change in its momentum.

Solution:

Step 1: Calculate the initial momentum (p_initial) of the hockey puck before the collision:

p_initial = mass × initial velocity

p_initial = 0.2 kg × 10 m/s = 2 kg·m/s

Step 2: Calculate the final momentum (p_final) of the hockey puck after the collision:

p_final = mass × final velocity

p_final = 0.2 kg × (-8 m/s) = -1.6 kg·m/s

Step 3: Calculate the change in momentum (Δp):

Δp = p_final - p_initial

Δp = (-1.6 kg·m/s) - (2 kg·m/s) = -3.6 kg·m/s

Step 4: Calculate the impulse (J) experienced by the hockey puck during the collision:

J = Δp

J = -3.6 kg·m/s

The impulse experienced by the hockey puck during the collision is -3.6 kg·m/s, and the change in its momentum is also 3.6 kg·m/s.

Worked Example 3.3

Two objects, A and B, are initially at rest on a frictionless surface. Object A has a mass of 0.5 kg, and object B has a mass of 0.8 kg. Object A collides with object B. After the collision, object A moves to the right at a velocity of 4 m/s, and object B moves to the left at a velocity of 2 m/s.

a) Calculate the total momentum before the collision.

b) Calculate the total momentum after the collision.

c) Calculate the kinetic energy before and after the collision.

Step 1: a) Total momentum before the collision:

Step 2: Initial momentum of both objects is zero since they are at rest. Therefore, the total momentum before the collision is 0 kg·m/s.

Step 3: b) Total momentum after the collision:

Total momentum after the collision is the sum of the momenta of objects A and B:

$P_A = m_A \times v_A = 0.5 \text{ kg} \times 4 \text{ m/s} = 2 \text{ kg·m/s}$ (to the right)

$P_B = m_B \times v_B = 0.8 \text{ kg} \times (-2 \text{ m/s}) = -1.6 \text{ kg·m/s}$ (to the left)

Total momentum after the collision: 2 kg·m/s - 1.6 kg·m/s = 0.4 kg·m/s (to the right)

Step 4: c) Kinetic energy before the collision:

Both objects are initially at rest, so the initial kinetic energy is zero.

Step 5: d) Kinetic energy after the collision:

Kinetic energy after the collision for object A:

$K{\cdot}E_A = (1/2) \times m_A \times v_A^2 = (1/2) \times 0.5 \text{ kg} \times (4 \text{ m/s})^2 = 4 \text{ J}$

Kinetic energy after the collision for object B:

$K{\cdot}E_B = (1/2) \times m_B \times v_B^2 = (1/2) \times 0.8 \text{ kg} \times (-2 \text{ m/s})^2 = 1.6 \text{ J}$

Total kinetic energy after the collision: 4 J + 1.6 J = 5.6 J

Section (A): Multiple Choice Questions (MCQs)

1. The rate of change of linear momentum of a body is called a:

a) Linear force b) Angular force

c) Power d) Impulse

2. The term mass refers to the same physical concept as:

a) weight b) inertia

c) force d) acceleration

3. Which one of the following force is also called as self-adjusting force?

a) frictional force b) tension

c) weight d) thrust

4. The laws of motion show the relationship between:

a) velocity and acceleration b) mass and velocity

c) mass and acceleration d) force and acceleration

5. The motion of rocket in the space is according to the law of conservation of

a) Energy b) linear momentum

c) mass d) angular momentum

6. A bomb of mass 12 kg initially at rest explodes into two pieces of masses 4kg and 8kg. The speed of 8kg mass is 6 m/s. The kinetic energy of 4 kg mass is:

a) 32 J b) 48 J

c) 114 J d) 288 J

7. If momentum is increased by 20% then K.E increases by:

a) 44% b) 55%

c) 66% d) 77%

8. The kinetic energy of body of mass 2kg and momentum of 2 Ns is:

a) 1 J b) 2 J

c) 3 J d) 4 J

9. For the same kinetic energy, the momentum is maximum for:

a) an electron b) a proton

c) a deuteron d) alpha particle

10. A 3kg bowling ball experiences a net force of 15N. What will be its acceleration.

a) 35 m/s^2 b) 7 m/s^2

c) 5 m/s^2 d) 35 m/s^2

KEY:

1. d	2. b	3. a	4. d	5. b
6. d	7. a	8. a	9. d	10. c

Section (B): Structured Questions

CRQs:

1. State Newton's second law of motion.

Ans) If a certain unbalanced force acts upon a body, it will accelerate the body in the direction of force. The magnitude of the acceleration is directly proportional to the magnitude of the unbalanced force.

2. How does mass affect an object's acceleration?

Ans) If we increase the mass at a given force the rate of acceleration slows. Therefore, mass is inversely proportional to acceleration.

3. What are the different types of forces that can act on an object?

Ans) A variety of force types were placed into two broad category headings on the basis of whether the force resulted from the contact or non-contact of the two interacting objects.

Contact Forces	Non-contact Forces
Frictional Force	Gravitational Force
Tension Force	Electrical Force
Normal Force	Magnetic Force
Air Resistance Force	
Applied Force	
Spring Force	

4. How do you calculate net force?

Ans) The net force is a term used in a system when there is significant number of forces.

Formula of Net Force:

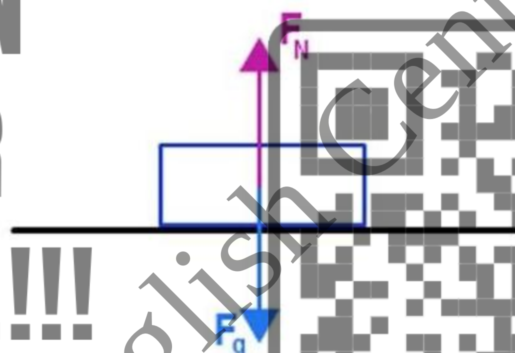
If N is the number of forces acting on a body, the net force formula is given by,

$\mathbf{F_{Net} = F_1 + F_2 + F_3 \ldots. + F_N}$

Where,

$F_1, F_2, F_3 \ldots F_N$ is the force acting on a body.

Net force when a body is at rest:



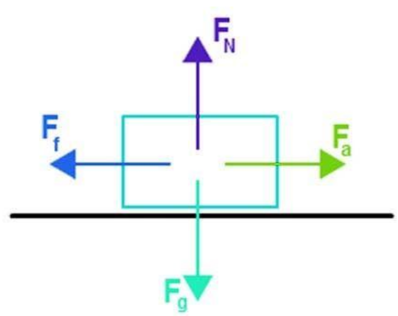
When the body is at rest, the net force formula is given by,

$\mathbf{F_{Net} = F_a + F_g}$.

Where,

- F_a = applied force,
- F_g = gravitational force.

Net force when a body is in motion:



Therefore, the net force formula is given by,

$\mathbf{F_{Net} = F_a + F_g + F_f + F_N}$.

Where,

- F_a is applied force,
- F_g is the gravitational force,
- F_f is the frictional force,
- F_N is a normal force.

5. State the law of conservation of momentum.

Ans) The total linear momentum of an isolated system (in the absence of external force) remains constant.

6. What is the difference between elastic and inelastic collision?

Ans)

Elastic Collision	Inelastic Collision
Total momentum is conserved	Total momentum is conserved
Total kinetic energy is conserved	Total kinetic energy is not conserved
Forces involved are conservative forces	Forces involved are non-conservative forces.
Mechanical energy is not dissipated.	Mechanical energy is dissipated into heat. light, sound etc.

7. How does impulse relate to force and time?

Ans) Impulse is the product of force and time.

Mathematically, impulse is defined as:

$$J = F \cdot \Delta t$$

Where:

- J is the impulse
- F is the force applied to the object
- Δt is the change in time over which the force is applied

8. How does friction influence the motion of an object?

Ans) Friction has a significant impact on the motion of objects, and its effects can be summarized as follows:

1. **Slowing Down Motion:** Friction acts to slow down the motion of objects that are in contact with a surface.
2. **Preventing Motion:** Friction can prevent objects from starting to move in the first place. If an object is at rest on a surface and a force is applied to initiate its motion, the force of static friction comes into play. This force opposes the applied force until a threshold is reached, at which point the object starts moving.
3. **Generating Heat:** Friction between surfaces generates heat energy as a result of the constant interaction and deformation of microscopic irregularities. This heat can sometimes become significant, especially in situations where objects are moving against each other with high force.
4. **Affecting Efficiency:** In many mechanical systems, friction leads to energy loss and reduced efficiency. For example, in machines like engines and vehicles, a portion of the energy is lost as heat due to friction between moving parts, which can lead to decreased overall efficiency.
5. **Influence on Object's Path:** Friction can influence the trajectory of an object in motion. For example, if an object is sliding on a curved surface, friction can cause it to follow a curved path rather than a straight line.

ERQs:

1. Explain Newton's Second Law of Motion and how it relates force, mass, and acceleration. Provide an example to illustrate the concept.

Ans) Acceleration of an object is directly proportional to the applied force

$$a \propto F \longrightarrow (1)$$

Acceleration of an object is inversely proportional to the mass of an object

$$a \propto 1/m \longrightarrow (2)$$

By combining both equations (1) & (2)

$$a \propto \frac{F}{m}$$

$$a = k\frac{F}{m}$$

k = 1

$$a = F/m$$

$$\mathbf{F = ma}$$

Example:

Let's consider pushing a block along a frictionless surface. Suppose you apply a force of 10 Newton’s to a block with a mass of 2 kilograms. According to Newton's Second Law:

$F = m \cdot a$

Given that F = 10N =10N and m=2 kg, we can rearrange the equation to solve for acceleration a:

$a = mF = (2)\ (10) = 5 m/s^2$

So, the block will experience an acceleration of $5 m/s^2$ in the direction of the applied force.

This example illustrates how force, mass, and acceleration are interconnected. If the mass of the block were greater, the acceleration would be smaller for the same applied force. Conversely, if the force were greater, the acceleration would be larger, assuming the mass remains constant.

2. How does the conservation of momentum apply in each type of collision? Give real-life examples of both types of collisions.

Ans) This principle holds true for both elastic and inelastic collisions, but it has different implications for each type of collision.

1. **Example of an Elastic Collision:** Imagine two billiard balls colliding on a pool table. When one ball hits another, they rebound off each other, and the total momentum of the system is conserved. Additionally, if the collision is perfectly elastic, the total kinetic energy of the system remains unchanged, meaning that no energy is lost during the collision.
2. **Example of Inelastic Collision:** Consider a car crashing into a wall and coming to a stop. The collision is perfectly inelastic if the car sticks to the wall after the collision. In this scenario, the two objects (car and wall) move together as a single unit after the collision. The total momentum is conserved because the combined mass of the car and the wall remains the same before and after the collision. However, the total kinetic energy decreases significantly as some of the initial kinetic energy is converted into deformation and heat.

3. Explain how the concept of impulse is related to the change in momentum of an object. Provide an example of an everyday life where impulse plays a significant role.

Ans) Now, let's examine how impulse is related to the change in momentum of an object:

Newton's second law of motion relates force, mass, and acceleration as F=m·a. We can rearrange this equation to express acceleration as a = $\frac{F}{m}$

Acceleration is the rate of change of velocity, and velocity is the rate of change of displacement. In terms of calculus, we can represent acceleration as the derivative of velocity (a = $\frac{dv}{dt}$) and velocity as the derivative of displacement (v = $\frac{dx}{dt}$).

Substituting these relationships, we get:

$$a = \frac{dv}{dt} = \frac{d}{dt} \cdot \frac{dx}{dt} = \frac{d^2x}{dt^2}$$

Now, back to Newton's second law F=m·a. Substituting the expression for acceleration (a = $\frac{d^2x}{dt^2}$) we get:

$$F = m\,\frac{d^2x}{dt^2}$$

$$F \cdot dt = m \cdot d^2x$$

And this is where the concept of impulse comes into play. The left-hand side F·dt is the impulse (J), and the right-hand side $m \cdot d^2x$ is the change in momentum (Δp).

In summary, the relationship between impulse and the change in momentum (Δp) can be expressed as:

$$J = \Delta p$$

One example of an everyday situation where impulse plays a significant role is walking or running.

Let's break down the scenario:

1. **Walking:** When you walk, each step involves a rapid series of impulses. As your foot pushes backward against the ground, the ground's reaction creates an impulse that propels you forward. This process is repeated for each step you take, leading to a continuous change in momentum that enables you to move.
2. **Running or Jumping:** Running and jumping involve more force and a shorter period of time, resulting in larger impulses. This greater impulse leads to a greater change in momentum, allowing you to move faster or jump higher.

Numericals:

1. A car weighing 9800 N is moving with a speed of 40 Km/h. On the Application of breaks it comes to rest after traveling a distance of 50 meters. Calculate the average retarding force?

Data:

W = 9800 N

v = 40 Km/h = $\frac{40 \times 1000}{3600}$ = 11.111 m/s^2

S = 50 m

f = ?

Solution:

$m = \frac{W}{g} = \frac{9800}{9.8} = 1000$ kg

$2aS = v_f^2 - v_i^2$

$2a(50) = 0^2 - 11.111^2$

$a = -1.234\ m/s^2$

The negative sign shows that the velocity of the car is decreasing.

$a = 1.2345\ m/s^2$

f = ma

f = (1000) (1.2345) = 1234.5 N.

2. A helicopter weighs 3920 N. Calculate the force on it if it is ascending up at the rate of 2m/s^2. What will be the force on helicopter if it is moving up with a constant speed of 4 m/s?

Data:

W=3920 N

a = 2 m/sec^2

v = 4 m/sec

F = ?

F' = ?

Solution:

Mass of helicopter = $m = \frac{W}{g}$

$m = \frac{3920}{9.8} = 400$ kg

Since helicopter is ascending up therefore the net force on the helicopter is

$F - W = ma$

$F = ma + W$

$F = 3920 + 400 \times 2$

$F = 3920 + 800$

F= 4720 N

Force on of helicopter when ascending with constant velocity. In this case acceleration of helicopter is zero

$F - W = ma$

$F - W = m(0)$

$F - W = 0$

$F = W$

F= 3920 N

3. A 100 grams bullet s fired from a 10 kg gun with a speed of 100m/s. What is the speed of recoil of the gun?

Data:

$m_1 = 100$ g = $\frac{100}{1000} = 0.1$ kg

$m_2 = 10$ kg

Velocity of bullet before firing = $u_1 = 0$

Velocity of gun before firing = $u_2 = 0$

Velocity of bullet after firing = $v_1 = 1000$ m/s

Velocity of gun after firing = $v_2 = ?$

Solution:

According to the law of conservation of linear momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$(0.1)(0) + (10)(0) = (0.1)(1000) + (10)\,v_2$

$\mathbf{v_2 = -\frac{0.1 \times 1000}{10} = -10\ m/s}$

Negative sign shows that the direction of velocity of recoil of gun is opposite to the direction of velocity of bullet.

4. A machine gun fires 10 bullets per second into a target. Each bullet weighs 20 gram and had a speed of 1500 m/s. Find the necessary force to hold the gun in position?

Data:

Mass of bullet = m = 20g = $\frac{20}{1000}$ = 0.02 kg

Number of bullet = 10

Mass of 10 bullets = M = 10 x 0.02 = 0.2 kg

Time = t = 1s

Initial velocity of bullet = v_i = 0

Final velocity of bullet = v_f = 1500 m/sec

Solution:

Force necessary to hold the gun = change in momentum of bullets per second

$F = \frac{Mv_f - Mv_i}{t}$

$F = \frac{(0.2)(1500) - (0.2)(0)}{1}$

F = 300 N

5. A 50 grams bullet is fired into a 10 kg block that is suspended by a long cord so that it can swing as a pendulum. If the block is displaced so that its center of gravity rises by 10cm, what was the speed of bullet?

Data:

m_1 = 50g

$m_1 = \frac{50}{1000}$ = 0.05Kg

m_2 = 10Kg

h = 10 cm = $\frac{10}{100}$ = 0.1 m

Velocity of block before impact = u_2 = 0

Velocity of bullet after impact = v_1 =V

Velocity of block after impact = V_2 = V

Velocity of bullet before impact =u_1 = ?

Solution:

According to law of conservation of linear momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$(0.05)(u_1) + (10)(0) = 0.05\,v + 10\,v$

$0.05u_1 = 10.05v$

$u_1 = \frac{10.05}{0.05}\,v$

$u_1 = 201\,v \longrightarrow (1)$

According law of conservation of energy

loss of K.E = gain of P.E

$\frac{1}{2}(m_1 + m_2)\,v^2 = (m_1 + m_2)\,gh$

$\frac{1}{2}v^2 = gh$

$v^2 = 2gh$

$v = \sqrt{2gh}$

$v = \sqrt{2(9.8)(0.1)}$

$v = 1.4$ m/s

Putting the value of v is eq (1)

$\mathbf{u_1 = 201 \times 1.4 = 281.4}$ **m/s**

6. A 70-gram ball collides with another ball of mass 140 gram. The initial velocity of the first ball is 9m/s to right while the second ball is at rest. If the collision were perfectly elastic, what would be the velocity of two balls after the collision?

Data:

$m_1 = 70 \text{ gm} = \frac{70}{1000} = 0.07$ kg

$m_2 = 140 \text{ gm} = \frac{140}{1000} = 0.14$ kg

Initial velocity of first ball = $U_1 = 9$ m/s

Initial velocity of second ball = $U_2 = 0$

Final velocity of first ball = V_1 =?

Final velocity of second ball = V_2 =?

Solution:

Final velocity of first ball after collision:

$V_1 = \frac{(m_1 - m_2)U_1}{m_1 + m_2} + \frac{2m_2U_2}{m_1 + m_2}$

$V_1 = \frac{(0.07 - 0.14)(9)}{0.07 + 0.14} + \frac{2(0.14)(0)}{0.07 + 0.14}$

$V_1 = -3$ m/s

Here -ve sign shows that first ball will return back in opposite direction after collision.

Find Velocity of steel ball after collision:

$V_2 = \frac{2m_1U_1}{m_1 + m_2} + \frac{(m_2 - m_1)U_2}{m_1 + m_2}$

$V_2 = \frac{2(0.07)(9)}{0.07 + 0.14} + \frac{(0.14 - 0.07)(0)}{0.07 + 0.14}$

$\mathbf{V_2 = 6}$ **m/s**

7. A truck weighing 2500 kg and moving with a velocity of 21 m/s collides with a stationary car weighing 1000 kg. The truck and car move together after the impact. Calculate their common velocity?

Data:

$m_1 = 2500$ kg

$m_2 = 1000$ kg

$u_1 = 21$ m/s

$u_2 = 0$

$v_1 = v_2 = v = ?$

Solution:

$(2500)(21) + (1000)(0) = (2500)(v) + (1000)v$

$v = 15$ m/s

Unit # 4: Rotational and Circular Motion

Worked Example 4.1

The platter of the hard drive of a computer rotates at 7300 rpm (a) What is the angular velocity of the platter? (b) if the reading head of the drive is located 3.1 cm from the rotation axis , what is the linear speed of the point on the platter just below it? (c) If a single bit requires 0.55 μm of length along the direction of motion, how many bits per second can the writing head write when it is 3.1 cm from the axis?

Data:

rev = f = 7300 rpm

ω = ?

r = 3.1 cm

v = ?

size of bit = 0.55 μm

Size of bits / time = ?

Solution:

Step 1: (a)

$$f = \frac{7300\ rev/min}{60\ s/min} = 121.7\ \frac{rev}{s} = 121.7\ Hz$$

The angular velocity is

$$\omega = 2\pi f = 2\pi \times 121.7 = 764.5\ \frac{rad}{s}$$

Step 2: (b) The linear speed of point 3.1 cm out from the axis is given by

$$v = r\omega$$

$$v = 0.031 \times 764.5 = 23.7\frac{m}{s}$$

Step 3: (c): Each bit requires 0.55×10^{-6} m , so at a speed 23.7 m/s , the number of bits passing the head per second is

$$= \frac{23.7\frac{m}{s}}{0.55 \times 10^{-6}\frac{m}{bit}} = 43 \times 10^{6}\ bits\ per\ second$$

$$or\ 43\ megabits\ /s$$

Worked Example 4.2

A motor cycle wheel turns 3620 times while being ridden for 6.50 minutes. What is the angular speed in rev/min (rpm)?

Data:

t = 6.50 min

no of revolutions = 3620 rev

ω = ?

Solution:

Step 1: formula

$$\omega = \frac{\theta}{t}$$

$$= \frac{3620\, rev}{6.5\, min} = 557\ revolutions\ per\ minutes$$

ω = 557 rpm

Worked Example 4.3

What is the magnitude of force and the centripetal acceleration of a car having mass of 300 kg following a curve of radius 500 m at a speed of 100 km/h? also compare the acceleration with that due to gravity for this fairly gentle curve taken at highway speed.

Data:

m=300kg

r = 500 m

V = 100 km/h = 27.8 m/s

a_c = ?

Solution: Step 1: formula

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{(27.8)^2}{500}$$

a_c = 1.54 m/s^2

Step 2: To calculate the magnitude of force

$$\mathbf{F = ma_c = mv^2/r}$$

$$F = ma_c = (300)(27.8)^2/500$$

$$\mathbf{F = 462N}$$

To compare this centripetal acceleration with acceleration due to gravity by taking ratio, we get

$$\frac{a_c}{g} = \frac{1.54}{9.8}\ g = 0.157\ g$$

0.157 g is noticeable gravity impact especially if you don't wear seat belt.

Worked Example 4.4

Curves on some test tracks and race courses, such as M-1 Islamabad – Lahore Motorway, are very steeply banked. This banking, with the support of tire friction and very stable car configurations, allows the curves to be taken at very high speed. To illustrate, calculate the speed at which a 100 m radius curve banked at 30° should be driven if the road were frictionless.

Approach

We first note that all terms in the expression for the ideal angle of a banked curve except for speed are known; thus, we need only rearrange it so that speed appears on the left-hand side and then substitute known quantities.

Solution:

Step 1:

Data:

r = 100 m

θ = 300

v = ?

Step 2:

$$\tan\theta = \frac{v^2}{rg}$$

$$v = \sqrt{r\,g\,\tan\theta}$$

Step 3:

$$v = \sqrt{100\times 9.8\times \tan 30}$$

v = 23.8 m/s

Worked Example 4.5

The International Space Station orbits at an altitude of 400 km above the surface of the Earth. What is the space station's orbital velocity?

Solution:

Step 1:

The orbital velocity depends on the distance from the center of mass of the Earth to the space station. This distance is the sum of the radius of the Earth and the distance from the space station to the surface:

r = (6.38 x 106 m) + (400 km)

r = 6380000 + 400000 m

r = 6780000 m

Step 2:

The orbital velocity can be found using the formula:

$$V = \sqrt{\frac{G\,M_{central}}{R}}$$

Step 3:

$$V = \sqrt{\frac{6.673\times10^{-11}\times5.98\times10^{24}}{678000}}$$

V=7672 m/s

The orbital velocity of the International Space Station is **7672 m/s.**

Worked Example 4.6

A basketball spinning on the finger of an athlete has angular velocity $\omega = 120.0$ rad/s. The moment of inertia of a sphere that is hollow, where M is the mass and R is the radius. If the basketball has a weight of 0.6000 kg and has a radius of 0.1200 m, what is the angular momentum of this basketball?

Solution:

Data:

$\omega = 120$ rad /s

$m = 0.60$ kg

$r = 0.120$ m

$L = ?$

We can find the angular momentum of the basketball by using the moment of inertia of a sphere that is hollow, and the formula. The angular momentum will be:

$L = I\omega$

$I = 2/3\ MR^2$

$I = 0.66 \times 0.60 \times (0.12)^2$

$I = 5.76 \times 10^{-3}$

$L = 5.76 \times 10^{-3} \times 120$

$L = 0.6912\ kg.m^2/s$

The angular momentum of the basketball that is spinning will be 0.6912 kg. m^2/s.

Worked Example 4.7

The biceps muscle exerts a vertical force on the lower arm, bent as shown in figure. For each case, calculate the torque about the axis of rotation through the elbow joint, assuming the muscle is attached 5 cm from the elbow as shown.

Data:

$F = 700$ N

$r\perp = 5$ cm $= 0.05$ m

Solution: Step 1:

(a) $\tau = r\perp F = 700 \times 0.05 = 35$ N.m

Step 2:

(b) $\tau = r\perp F = r \sin\theta F$

$\tau = (0.05) \sin 60^\circ \times 700$

$\tau = 30$ N.m

The arm exerts less torque at this angle than when it is at 90°. Weight machines at gyms are designed on these parameters.

Section (A): Multiple Choice Questions (MCQs)

1. One radian is about

a) 25^{o}

b) 37^{o}

c) 45^{o}

d) 57^{o}

2. Wheel turns with constant angular speed then:

a) each point on its rim moves with constant velocity

b) each point on its rim moves with constant acceleration

c) the wheel turns through equal angles in equal times

d) the angle through which the wheel turns in each second increases as time goes on

e) the angle through which the wheel turns in each second decreases as time goes on

3. The rotational inertia of a wheel about its axle does not depend upon its

a) diameter

b) mass

c) distribution of mass

d) speed of rotation

4. A force with at given magnitude is to be applied to a wheel. The torque can be maximized by

a) applying the force near the axle, radially outward from the axle

b) applying the force near the rim, radially outward.

c) applying the force near the axle, parallel to a tangent to the wheel.

d) applying the force at the rim, tangent to the rim

5. An object rotating about a fixed axis, I is its rotational inertia and α is its angular acceleration. Its

a) is the definition of torque

b) is the definition of rotational inertia

c) is definition of angular acceleration

d) follows directly from Newton's second law

6. The angular momentum vector of Earth about its rotation axis, due to its daily rotation is, directed

a) tangent to the equator towards east

b) tangent to the equator towards the west

c) north

d) towards the Sun

7. A stone of 2 kg is tied to a 0.50 m long string and swung around a circle at angular velocity of 12 rad/s. The net torque on the stone about the center of the circle is

a) 0 Nm

b) 6Nm

c) 12 Nm

d) 72 N.m

8. A man, with his arms at his sides, is spinning on a light frictionless turntable. When he extends his arms

a) his angular velocity increases

b) his angular velocity remains same

c) his rotational inertia decreases

d) his angular momentum remains the same

9. A space station revolves around the earth as a satellite, 100 km above the Earth's surface. What is the net force on an astronaut at rest inside the space station?

a) equal to her weight on earth

b) a little less than her weight on earth

c) less than half her weight on earth

d) zero (she is weightless)

10. If the external torque acting on a body is zero, then its

a) angular momentum is zero

b) angular momentum is conserved

c) angular acceleration is maximum

d) rotational motion is maximum

KEY:

1. d	2. c	3. d	4. d	5. d
6. c	7. a	8. d	9. d	10. b

Section (B): Structured Questions

CRQs:

1. For an isolated rotating body, what is the relation between angular velocity and radius?

Ans) For an isolated rotating body, the relationship between angular velocity (ω) and radius (r) is given by the formula for tangential velocity (v):

$$v = \omega \cdot r$$

This formula essentially states that the tangential velocity of a point on a rotating body is directly proportional to the angular velocity of the body and the distance of that point from the axis of rotation. In other words, the farther a point is from the axis of rotation, the faster it moves tangentially due to the same angular velocity.

2. When the moment of inertia of a rotating body is halved, then what will be the effect on angular velocity?

Ans) As we know that

$$L = I \cdot \omega$$

$$\omega = \frac{L}{I}$$

When the moment of inertia of a rotating body is halved

$$\omega' = L \div \frac{I}{2}$$

$$\omega' = L \times \frac{2}{I}$$

$$\omega' = 2 \times \frac{L}{I}$$

$$\boldsymbol{\omega' = 2\,\omega}$$

Angular velocity will become twice.

3. Compare kinematics equation of linear motion and circular motion.

Ans)

Linear Motion	Rotational Motion	
$S = V t$	$\theta = \frac{S}{r}$ or $\theta = \omega t$	
$S = V_i t + \frac{1}{2} a t^2$	$\theta = \omega_i t + \frac{1}{2} \alpha t^2$	a constant α constant
$V_{avg} = \frac{V_f + V_i}{2}$	$\omega_{avg} = \frac{\omega_f + \omega_i}{2}$	
$V_f = V_i + at$	$\omega_f = \omega_i + \alpha t$	a constant α constant
$2 a S = V_f^2 - V_i^2$	$2 \alpha \theta = \omega_f^2 - \omega_i^2$	a constant α constant

4. Can a small force ever exert a greater torque than a larger force? Give reason.

Ans) Yes, a small force can exert a greater torque than a larger force, depending on the distance from the axis of rotation at which the force is applied. Torque (τ) is the rotational equivalent of force and is given by the formula:

$\tau = r F \sin(\theta)$

If the force is applied at a larger distance from the axis of rotation (larger r), even if the force itself is smaller, the lever arm effect can result in a greater torque.

5. Give two real world applications of angular momentum.

Ans) Angular momentum, a property of rotating objects, has numerous real-world applications in various fields. Here are two examples:

1. **Gyroscopes in Navigation and Stabilization:** Gyroscopes are devices that utilize angular momentum to maintain stability and provide orientation reference. They have critical applications in aviation, aerospace, navigation, and even everyday technology like smartphones and drones.
2. **Angular Momentum in Sports:** Angular momentum plays a significant role in various sports, particularly those involving rotating motions. Examples include figure skating, gymnastics, diving, and acrobatics. Athletes utilize their angular momentum to perform spins, flips, and twists. By changing their body positions during flight, they can alter their angular momentum and achieve complex maneuvers. Divers, for instance, use angular momentum to execute somersaults and twists in mid-air to achieve graceful and dynamic performances.

6. Derive relationship between torque and angular acceleration.

Ans) Consider a particle of mass m rotating in a circle of radius r at the end of a string whose mass is negligible as compared to mass of string. Assume that a single force F acts on the mass as shown in figure below.

The torque gives rise to the angular acceleration is $\tau = r F$.

Newton's second law of motion is $F = ma$,

Tangential linear acceleration $a = r\,\alpha$

Equation... can be rewritten as

$$\mathbf{F} = m\,\mathbf{r}\,a$$

Multiplying both sides with $\mathbf{r}$

$$\mathbf{r} \times \mathbf{F} = m\,\mathbf{r}^2\,a$$

$$\boldsymbol{\tau} = \mathbf{I}\,\alpha$$

$$\boldsymbol{\tau} \propto \alpha$$

We have a direct relation between the angular acceleration and the applied torque whereas mr^2 is representing rotational inertia which is called moment of inertia.

Now let us consider a rotating rigid object, such as wheel rotating about an axis through its center, which could be an axle. We can think of the wheel as containing of many particles located at various distances from the axis of rotation. We can apply eq.... to each particle of the object, and then sum over all the particles. The sum of the various torques is just the total torque, we get

$$\sum \boldsymbol{\tau} = \left(\sum m\mathbf{r}^2\right) a$$

If each particle is assigned a number (1,2,3, 4....), then

$$\mathbf{I} = \left(\sum m\mathbf{r}^2\right) = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots\ldots\ldots\ldots$$

By combining equations we get

$$\sum \boldsymbol{\tau} = \mathbf{I}\,a$$

This is rotational equivalent of Newton's Second law. It is valid for the rotation of a rigid bodies about a fixed axis.

7. List the moment of inertia dependent factors.

Ans) Here are the moment of inertia-dependent factors listed:

1. **Mass Distribution:** How mass is distributed with respect to the axis of rotation.
2. **Mass:** The total amount of mass the object possesses.
3. **Shape:** The geometric shape and dimensions of the object.
4. **Axis of Rotation:** The specific axis around which the object is rotating.
5. **Distance from the Axis:** The distance between the axis of rotation and the mass elements.
6. **Rotational Symmetry:** The degree of symmetry in the object's shape.
7. **Position of Mass Elements:** The arrangement and position of individual mass elements.
8. **Orientation:** The orientation of the object relative to the axis of rotation.

ERQs:

1. State and explain the law of conservation of angular momentum. Give two examples to illustrate it.

Ans) Statement: The law of conservation of angular momentum states that when no resultant external torque acts on a body, its angular momentum remains constant.

Explanation: In figure 1, consider a particle in uniform circular motion due to a force acting on the particle that is centripetal force required to deflect the particle to keep it in circular path.

The centripetal force always points towards the center of the circle so it produces no torque about an origin at the center of the circle.

The law of conservation of angular momentum then ensures that the angular momentum of the particle is constant. The angular momentum is constant in magnitude ($mr\omega^2$ remains fixed) and constant in direction (the motion is confined to the plane of rotation).

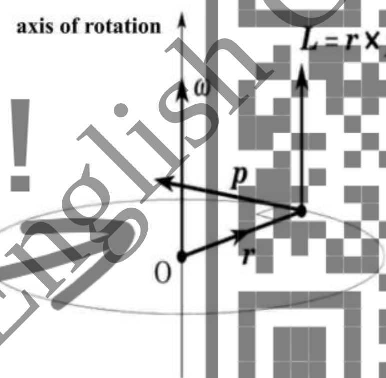


Figure 1

It is worth emphasizing that the above discussion relies on the origin being at the center of the circle. In Figure 2 a different origin O has been chosen, on the axis of rotation but out of the plane of rotation. In this case angular momentum is

$$\vec{L} = r \times p$$
$$\vec{L} = (r_{\parallel} + r_{\perp}) \times p$$
$$\vec{L} = (r_{\parallel} \times p) + (r_{\perp} \times p)$$

that the angular momentum has a component $L_{\perp} = r_{\parallel} \times p$ which is perpendicular to the axis of rotation and is not conserved. This is not a problem because, relative to the origin of Figure 2, the particle experiences a torque $\tau_{\perp} = r_{\parallel} \times F$ where F is the centripetal force acting along $-r_{\perp}$

Moreover, because $r_{\parallel}$ is constant we have

$$\frac{dL_{\perp}}{dt} = r_{\parallel} \times \frac{dp}{dt} = r_{\parallel} \times F = \tau_{\perp}$$

so the rate of change of $L_{\perp}$ is supported by the existence of $\tau_{\perp}$. At the same time, the component of angular momentum parallel to the axis of rotation remains constant because there is no torque in that direction.

$$\frac{dL}{dt} = \tau$$

$$\frac{dL}{dt} = 0 \qquad \text{where } \tau = 0 \text{ (integrating both sides)}$$

$$L = constant$$

Hence, the angular momentum of the particle is conserved if the net torque acting on it is zero.

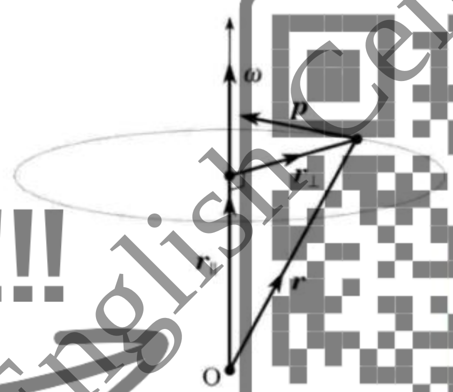


Figure 2

2. Discuss forces action on banked curve and derive relation between curve angle and velocity?

Ans) Forces acting on Banked Curve: On a curve that is not banked, a car traveling along that curve will experience a force of static friction that will point towards the center of the circular pathway restricted by the moving car. This frictional force will be responsible for creating centripetal acceleration, which in turn will allow the car to move along the curve. On a banked curve however, the normal force acting on the object such as a car, will act at an angle with the horizontal, and that will create a component normal force that acts along the x axis. This component normal force will now be responsible for creating the centripetal acceleration required to move the car along the curve. Therefore, for every single angle, there exists a velocity for which no friction is required at all to move the object along the curve. This means that the car will be able to turn even under the most slippery conditions (ice or water).

A free -body diagram is shown in figure below for a car on a frictionless banked curve. If the angle is ideal for the speed and radius r, then the net external force equals the necessary centripetal force. The only two external forces acting on the car are its weight and the normal force of the road N. (A frictionless surface can only exert a force perpendicular to the surface-that is, a normal force.). These two forces must add to give a net external force that is horizontal toward the center of curvature and has magnitude mv^2/r. As it is a crucial force and is horizontal, we use a coordinate system with vertical and horizontal axes. Only the normal force has a horizontal component, so this must equal the centripetal force, that is

$$N \sin\theta = \frac{mv^2}{r}$$

Because the car does not leave the surface of the road, the net vertical force must be zero, meaning that the vertical components of the two external forces must be equal in magnitude and opposite in direction. From figure shown, we note that the vertical component of the normal force is N cosθ, and the only other vertical force is the car's weight. These must be equal in magnitude, thus,

$$N \cos\theta = mg$$

Now we can combine these two equations to eliminate N and get an expression for θ, as desired. Solving the second equation for N = mg/(cosθ) and substituting this into the first yields

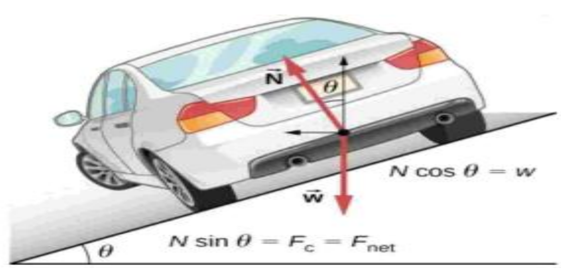
$$mg\frac{\sin\theta}{\cos\theta} = \frac{mv^2}{r}$$

$$mg \tan\theta = \frac{mv^2}{r}$$

$$\tan\theta = \frac{v^2}{r\,g}$$

$$\theta = \tan^{-1}\frac{v^2}{r\,g}$$

This expression can be understood by considering θ how depends on v and r. A large θ is obtained for a large v and a small r. That is, roads must be steeply banked for high speeds and sharp curves. Friction helps, because it allows you to take the curve at greater or lower speed than if the curve were frictionless. Note that θ does not depend on the mass of the vehicle.



3. Derive the formula for centripetal acceleration using fundamental principles and equations, illustrating each step of the derivation.

Ans) Derivation for centripetal acceleration: Following the figure shown, observe that the triangle formed by the velocity vectors and the one formed by the radii r and ΔS are similar. Both the triangles ABC and PQR are isosceles triangle i.e. triangles having two sides identical. The two equal sides of the velocity vector triangles are the speeds $v_1 = v_2 = v$. Using the properties of two identical triangles we get

$$\frac{\Delta v}{v} = \frac{\Delta S}{r}$$

rearranging for Δv, we get

$$\Delta v = \frac{v}{r}\Delta S$$

Dividing both sides Δt

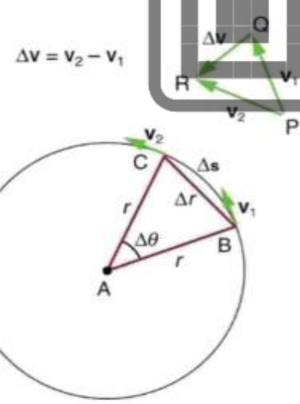
$$\frac{\Delta v}{\Delta t} = \frac{v}{r}\frac{\Delta S}{\Delta t}$$

$\frac{\Delta v}{\Delta t} = a_c$ and $\frac{\Delta S}{\Delta t} = v$, the magnitude of centripetal acceleration is

$$a_c = \frac{v^2}{r}$$

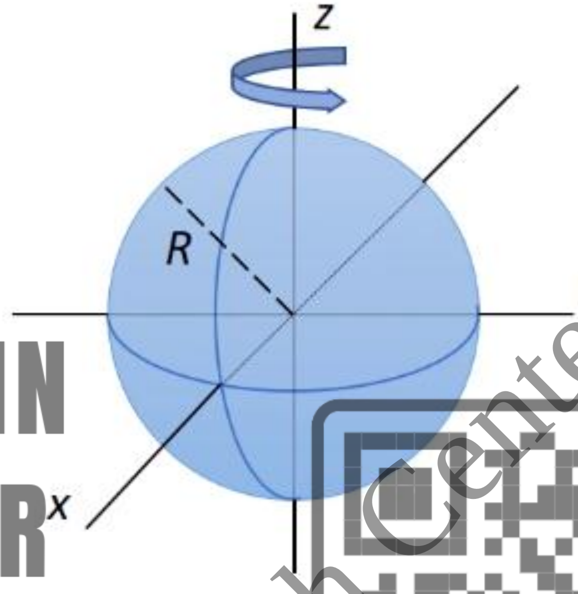
It is useful to show centripetal acceleration in terms of angular velocity. Substituting v = r ω into centripetal acceleration formula, we get

$$a_c = \frac{(r\omega)^2}{r} = r\,\omega^2$$

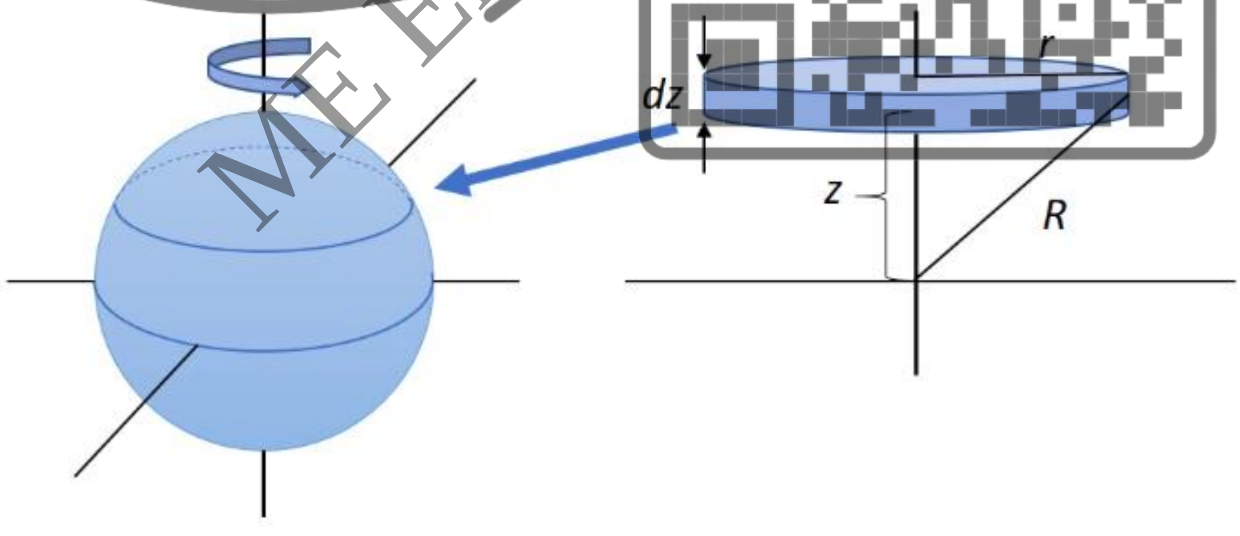


4. Derive the formula for the moment of inertia of a uniform rod or a solid sphere. Clearly illustrate each step of the derivation.

Ans)



$I = \int r^2\, dM = \int dI \longrightarrow (1)$



$dI = \frac{1}{2} r^2\, dM$

$\rho = \frac{M}{V}$ OR $M = \rho V$

$dI = \frac{1}{2} r^2 \rho \, dV$

$dI = \frac{1}{2} r^2 \rho \, dV$

For disk: $dV = \pi r^2 dz$

$dI = \frac{1}{2} r^2 \rho (\pi r^2 dz)$

$dI = \frac{1}{2} \rho \pi r^4 dz$

From second figure

$z^2 + r^2 = R^2$

$r^2 = R^2 - z^2$

$r^4 = (R^2 - z^2)^2$

$dI = \frac{1}{2} \rho \pi (R^2 - z^2)^2 dz$

Put in equation (1)

$I = \int \frac{1}{2} \rho \pi (R^2 - z^2)^2 dz$

$$= \frac{1}{2}\rho\pi \int_{-R}^{R} (R^4 - 2R^2z^2 + z^4) dz$$

$$= \frac{1}{2}\rho\pi \, 2 \int_{0}^{R} (R^4 - 2R^2z^2 + z^4) dz = \rho\pi \left[R^4 z - \frac{2}{3}R^2 z^3 + \frac{1}{5} z^5\right]_0^R$$

$$I = \rho\pi\left[R^5 - \frac{2}{3}R^5 + \frac{1}{5}R^5\right] = \rho\pi\left[\frac{15}{15}R^5 - \frac{10}{15}R^5 + \frac{3}{15}R^5\right] = \rho\pi\frac{8}{15}R^5$$

$$= \rho\pi\frac{4}{3}R^3 \; \frac{8}{15} R^2 \frac{3}{4} = \left(\rho\frac{4}{3}\pi R^3\right)\frac{24}{60}R^2 = (\rho V)\frac{2}{5}R^2 = M\frac{2}{5}R^2$$

$$I = \frac{2}{5} M R^2$$

5. Define torque and explain how does it differ from force in linear motion?

Ans) Torque: Torque is the measure of the force that can cause an object to rotate about an axis.

The key differences between torque and force in linear motion are:

Direction of Effect:

- Torque causes objects to rotate around an axis.
- Force in linear motion causes objects to accelerate along a straight line.

Axis of Rotation vs. Line of Motion:

- Torque involves a specific axis of rotation, around which an object rotates.
- Force in linear motion involves a straight line along which an object moves.

Measurement Units:

- Torque is measured in units such as Newton-meters (Nm) or foot-pounds (ft-lb).
- Force in linear motion is measured in units of Newtons (N) or pounds (lb).

Numericals:

1. A car mechanic applies a force of 800 N to a wrench for the purpose of loosening a bolt. He applies the force which is perpendicular to the arm of the wrench. The distance from the bolt to the mechanic's hand is 0.40 m. Find out the magnitude of the torque applied?

Data:

$F = 800\ N$

$\theta = 90^{o}$ (applies the force which is perpendicular to the arm of the wrench)

$r = 0.40\ m$

$\tau = ?$

Solution:

$\boldsymbol{\tau} = \mathbf{F} \times \mathbf{r}$

$\tau = F\ r \sin(\theta)$

$\tau = (800)\ (0.40) \sin(90)$

$\boldsymbol{\tau = 320\ N.m}$

2. A car accelerates uniformly from rest and reaches a speed of 22 m/s in 9s. If the diameter of a tire is 58 cm, find

(a) The number of revolutions the tire makes during this motion, assuming no slipping and

(b) The final rotational speed of the tire in revolutions per second.

Data:

$V_i = 0$

$V_f = 22$ m/s

$t = 9$s

$d = 58 \text{ cm} = \frac{58}{100} = 0.58$ m

(a) N =?
(b) $\omega_f = ?$

Solution:

(a)

$V_f = V_i + at$

$22 = 0 + a(9)$

$a = 2.444 \text{ m/s}^2$

$S = (0)(9) + \frac{1}{2}(2.444)(9)^2$

$S = 99$ m

Circumference of tire = π d = (π) (0.58) = 1.822 m

So in one revolution it covers 1.822 m, therefore it will make $\frac{99}{1.822}$ = 54.335 rev **≈ 55 rev**

(b)

$a = r\alpha$

$r = \frac{d}{2} = \frac{0.58}{2} = 0.29$ m

$a = r\alpha$

$2.444 = (0.29)\alpha$

$\alpha = \frac{2.444}{0.29} = 8.43\ rad/s^2$

$\omega_f = \omega_i + \alpha t$

$\omega_f = 0 + (8.43)(9) = 75.87\ rad/s$

$\boldsymbol{\omega_f = \frac{75.87}{2\pi} = 12\ rev/s}$

3. An ordinary workshop grindstone has a radius of 7.5 cm and rotates 6500 rev/min.

(a) Calculate the magnitude of centripetal acceleration at its edge in m/s^2 and convert it into multiples of g.

(b) What is the linear speed of a point on its edge?

Data:

$r = 7.5\ cm = \frac{7.5}{100} = 0.075\ m$

$\omega = 6500\ rev/min = \frac{6500 \times 2\pi}{60} = 680.678\ rad/s$

(a) $a_c = ?$, $a_c' = ?$

(b) $V = ?$

Solution:

(a)

$a_c = r\,\omega^2$

$\boldsymbol{a_c = (0.075)(680.678)^2 = 3.47 \times 10^4\ m/s^2}$

Multiplying and dividing right hand side by g

$a_c = 3.47 \times 10^4 \frac{g}{g}$

$a_c = 3.47 \times 10^4 \frac{g}{9.8}$

$\boldsymbol{a_c = 3.55 \times 10^3\ g}$

(b)

$\boldsymbol{V = r\,\omega = 0.075 \times 680.678 = 51.05\ m/s}$

4. A satellite is orbiting the Earth with an orbital velocity of 3200 m/s. What is the orbital radius?

Data:

$V = 3200$ m/s

$R = ?$

Solution:

$$V = \sqrt{\frac{G\,M_{central}}{R}}$$

$$V^2 = \frac{G\,M_{central}}{R}$$

$$(3200)^2 = \frac{6.673 \times 10^{-11} \times 5.98 \times 10^{24}}{R}$$

$$\mathbf{R = 3.897 \times 10^{7}\ m}$$

5. A satellite wishes to orbit the earth at a height of 100 km (approximately 60 miles) above the surface of the earth. Determine the speed, acceleration and orbital period of the satellite. (Given: $M_{earth} = 5.98 \times 10^{24}$ kg, $R_{earth} = 6.37 \times 10^{6}$ m)

Data:

$h = 100$ km

$M_{earth} = 5.98 \times 10^{24}$ kg

$R_{earth} = 6.37 \times 10^{6}$ m

$V = ?$

$a = ?$

$T = ?$

Solution:

$$R = R_{earth} + h = 6.37 \times 10^{6} + 100000 = 6.47 \times 10^{6}\ m$$

$$V = \sqrt{\frac{G\,M_{central}}{R}}$$

$$V = \sqrt{\frac{6.673 \times 10^{-11} \times 5.98 \times 10^{24}}{6.47 \times 10^{6}}}$$

$$\mathbf{V = 7.85 \times 10^{3}\ m/s}$$

$a = \frac{V^2}{R}$

$a = \frac{(7.85 \times 10^3)^2}{6.47 \times 10^6}$ = 9.524 m/s²

$T^2 = \frac{4\pi^2 R^3}{GM_{central}}$

$T^2 = \frac{4\pi^2 (6.47 \times 10^6)^3}{(6.673 \times 10^{-11})(5.98 \times 10^{24})}$

T = 5177.04 s

T = 1.44 hours

6. A thin disk with a 0.3m diameter and a total moment of inertia of 0.45 kg.m² is rotating about its center of mass. There are three rocks with masses of 0.2kg on the outer part of the disk. Find the total moment of inertia of the system?

Data:

$d_{disk} = 0.3m$

$I_{disk} = 0.45\ kg.m^2$

$m_{rock} = 0.2\ kg$

$I_{total} = ?$

Solution:

$I_{total} = I_{disk} + 3I_{rock}$

$I_{rock} = m_{rock} . r_{disk}^2$

$r_{disk} = \frac{d_{disk}}{2} = \frac{0.3}{2} = 0.15\ m$

$I_{rock} = (0.2)\ (0.15)^2 = 4.5 \times 10^{-3}\ kg.m^2$

$I_{total} = 0.45 + 3(4.5 \times 10^{-3})$

$I_{total} = 0.464\ kg.m^2$

7. What is the ideal banking angle for a gentle turn of 1.20 km radius on a highway with a 105 km/h speed limit (about 65 mi/h), assuming everyone travels at the limit?

Data:

$\theta = ?$

$r = 1.20 \text{ km} = 1.20 \times 1000 = 1200 \text{ m}$

$v = 105 \text{ km/h} = \frac{105 \times 1000}{3600} = 29.167 \text{ m/s}$

Solution:

$\tan\theta = \frac{v^2}{rg}$

$\tan\theta = \frac{(29.167)^2}{(1200)(9.8)}$

$\boldsymbol{\theta = 4.14^o}$

8. A 1500-kg car moving on a flat, horizontal road negotiates a curve as shown in Figure 4.21. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.523, find the maximum speed the car can have and still make the turn successfully.

Data:

$m = 1500 \text{ kg}$

$r = 35.0 \text{ m}$

$\mu = 0.523$

$v = ?$

Solution:

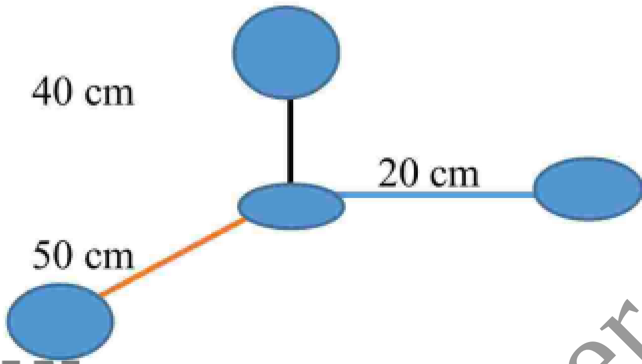
$f = F_c$

$\mu mg = \frac{mv^2}{r}$

$(0.523)(1500)(9.8) = \frac{(1500)\,v^2}{35}$

$\mathbf{v = 13.4 \text{ m/s}}$

9. A system of points shown in figure 4.22. Each particle has same mass of 0.3 kg and they all lie in the same plane. What is the moment of inertia of the system about given axis?



Data:

$m = 0.3$ kg

$I = ?$

Solution:

$I = \Sigma m_i r_i^2 = m \Sigma r_i^2 = 0.3 [0.5^2 + 0.4^2 + 0.2^2]$

$\mathbf{I = 0.135\ kg.m^2}$

10. (a) What is the angular momentum of a 2.9 kg uniform cylindrical grinding wheel of radius 20 cm when rotating 1550 rpm? (b) How much torque is required to stop it in 6s?

Data:

(a)

$m = 2.9$ kg

$r = 20 \text{ cm} = \frac{20}{1000} = 0.2$ m

$\omega = 1550 \text{ rpm} = \frac{1550 \times 2\pi}{60} = 162.32$ rad/s

$L = ?$

(b)

$\tau = ?$

t = 6s

Solution:

(a)

L = I ω

$L = (\frac{1}{2} mr^2)\ \omega$

$L = \frac{1}{2} (2.9)\ (0.2)^2\ (162.32)$

L = 9.415 kg. m^2/s

(b)

$\tau = \frac{L_f - L_i}{t}$

L_i = L = 9.415 kg. m^2/s

$\tau = \frac{(0) - 9.415}{6}$

τ = -1.6 N.m

11. Determine the angular momentum of the Earth (a) about its rotation axis (Assume the Earth as uniform sphere), and (b) in its orbit around the Sun (Take Earth as a particle orbiting the Sun). The Earth has mass 6 x 10^{24} kg and radius 6.4 x 10^{6}m, and is 1.5 x 10^8 km from the Sun.

Data:

Me = 6 x 10^{24} kg

R = 6.4 x 10^6 m

R' = 1.5 x 10^8 km = 1.5 x 10^{11} m

(a)

I = ?

(b)

I' = ?

Solution:

(a)

$I = \frac{2}{5} M_e R^2$

$I = \frac{2}{5} (6 \times 10^{24}) (6.4 \times 10^{6})^2$

$I = 9.8 \times 10^{37}$ kg.m^2

$\omega = \frac{2\pi}{T}$

T = 24 x 60 x 60 = 86400 s

$\omega = \frac{2\pi}{86400}$

$\omega = 7.27 \times 10^{-5}$ rad/s

L = I ω = $(9.8 \times 10^{37})(7.27 \times 10^{-5}) = 7.125 \times 10^{33}$ kg.m^2/s

(b)

$I = M_e R'^2 = (6 \times 10^{24})(1.5 \times 10^{11})^2$

$I = 1.35 \times 10^{47}$ kg.m^2

$\omega = \frac{2\pi}{T}$

T = 365 x 24 x 60 x 60 = 31536000 s

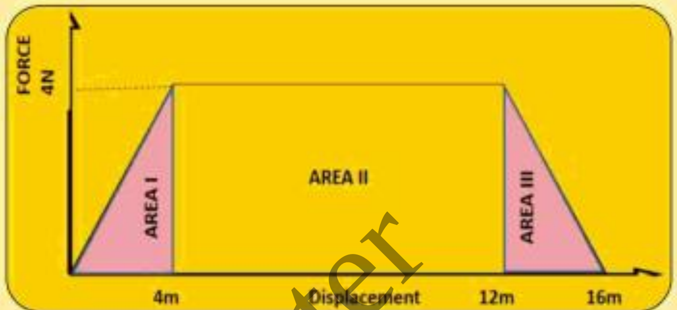
$\omega = \frac{2\pi}{31536000} = 2 \times 10^{-7}$ rad/s

L = I ω = $(1.35 \times 10^{47})(2 \times 10^{-7}) = 2.7 \times 10^{40}$ kg.m^2/s

Unit # 5: Work, Energy and Power

Worked Example 5.1

Calculate the work done from the following force-displacemnt graph.



Solution:

Step 1: Write the known quantities and point out

Force F = 4N

Base of Area I = 4m

Base of Area II = 8m

Base of Area III =4m

Step 2: Write the formula and rearrange if necessary

Area II = Area of rectangle = Force x displacement

Area I & III = Area of triangle = $\frac{1}{2}$ × Force × Displacement

Total workdone = Area I + Area II + Area III

Step 3: Put the value in formula and calculate

Total workdone = W_T = Area I + Area II + Area III

$W_T = \frac{1}{2} \times$ Force × Displacement + Force x displacement + $\frac{1}{2}$ × Force × Displacement

$W_T = \frac{1}{2} \times 4N \times 4m + 4N \times 8m + \frac{1}{2} \times 4N \times 4N$

W_T = 8N.m + 32 N.m + 8 N.m = 48N.m

W_T = 48 J

Worked Example 5.2

A car with a mass of 1,200 kg is traveling at a velocity of 25 m/s. Calculate the kinetic energy of the car.

Solution:

Step 1:

Mass (m) = 1,200 kg

Velocity (v) = 25 m/s

Step 2:

The formula for kinetic energy is given by:

Kinetic Energy $K.E = \frac{1}{2}mv^2$

Step 3:

Substituting the given values into the formula:

$K.E = \frac{1}{2}(1200)(25)^2$

$=$ 375,000 Joules

Therefore, the kinetic energy of the car is 375,000 Joules.

Worked Example 5.3

The mass of the earth is 5.98×10^{24} kg and the mass of the sun is 1.99×10^{30} kg, and the earth is 160 million km away from the sun, calculate the GPE of the earth.

Data:

the mass of the Earth $(m) = 5.98 \times 10^{24}\ kg$ *and mass of the Sun* (M)

$M = 1.99 \times 10^{30}\ kg$

Solution:

Step 1: *The gravitational potential energy is given by:*

$$U = \frac{-GMm}{r}$$

Step 2: $U = \frac{6.673 \times 10^{-11} \times 5.98 \times 10^{24} \times 1.99 \times 10^{30}}{160 \times 10^{9}}$

$$U = 4963 \times 10^{30}\ J$$

Worked Example 5.4

A person riding their bike has a mass of 120 kg; they are riding at 10m/s. suddenly a dog crosses the road and to avoid hitting the dog the bicyclist brakes applying a braking force of 500 N for a distance of 10 meters. What is the final velocity of the bicyclist when they stop braking?

Step 1: *Identify the mass of the object. The mass is 120 kilograms.*

Step 2: *Identify the initial velocity.*

The initial velocity is 10m/s.

Step 3: *Identify or calculate the work done on the object.*

The force on the object is 500 newtons over a distance of 10 meters.

Since the force is a braking force it is resisting the motion of the object making the work done negative.

$W = \vec{F} \times \Delta x = -500 \times 10 = -5{,}000\ Joules$

Step 4: *Identify or calculate the initial energy of the object.*

Using the kinetic energy formula.

$K.E_{initial} = \frac{1}{2} m v_{initial}^2$

$K.E_{initial} = \frac{1}{2}(120)(10)^2 = 6{,}000\ Joules$

Step 5: *Add the result from Step 4 with the result from Step 3.*

$6{,}000\ Joules + (-5{,}000\ Joules) = 1{,}000\ Joules$

Step 6: *Using the result from Step 5 equate it to the equation of kinetic energy and solve for velocity to receive the final velocity of the object.*

$K.E_{final} = 1{,}000\ Joules$

$\frac{1}{2} m v_{final}^2 = 1{,}000$

$\frac{1}{2}(120) v_{final}^2 = 1{,}000$

$v_{final} = \sqrt{16.7} = 4.1 m/s$

The final velocity of the bicyclist when they stop braking is 4.1m/s.

Section (A): Multiple Choice Questions (MCQs)

1. Work done by centripetal force is always:

(a) Maximum (b) Minimum

(c) Zero (d) None of these

2. A body of mass 5 kg is moving with a momentum of 10 kg m/s. A force of 0.2 N acts on it in the direction of motion of the body for 10 seconds. The increase in kinetic energy is:

(a) 2.83 (b) 3.2 J

(c) 3.8 J (d) 4.4 J

3. The kinetic energy of a light and a heavy object is same. Which object has maximum momentum?

(a) Light object (b) Heavy object

(c) Both have same momentum (d) N.O.T

4. Two bodies of mass 1 kg and 2 kg have equal momentum. Then the ratio of their kinetic energies is:

(a) 2:1 (b) 3:1

(c) 1:3 (d) 1:1

5. A body fallen from height h. After it has fallen a height h/2, its will possess:

(a) only potential energy (b) only kinetic energy

(c) half potential half kinetic energy (d) more kinetic and less potential

6. Which of the following quantity can be calculated by multiplying force and velocity?

(a) acceleration (b) power

(c) torque (d) work

7. The minimum velocity given to an object so that it emerges out from the gravitational field of earth is about -----------------:

(a) 11.2 km/s (b) 15.3 km/s

(c) 5 km/s (d) 9.8 km/s

8. When one joule of work is done on a body in one second, power of body is said to be:

(a) One watt (b) 0.5 watt

(c) zero (d) 100 watt

9. The absolute potential energy of an object depends on:

(a) The object's mass and height (b) The object's mass and speed

(c) The object's shape and size (d) The object's color and temperature

10. The escape velocity of a planet depends on which of the following factors?

(a) The mass of the planet only

(b) The radius of the planet only

(c) Both the mass and the radius of the planet

(d) The density of the planet

KEY:

1. c	2. d	3. b	4. a	5. c
6. b	7. a	8. a	9. a	10. c

Section (B): Structured Questions

CRQs:

1. How does work relate to the transfer of energy?

Ans) Work-Energy Theorem: The work-energy theorem states that the net work done on an object is equal to the change in its kinetic energy. In other words, work transfers energy from one form to another. Mathematically, the work-energy theorem can be expressed as:

$$W_{net} = \Delta KE$$

Where:

- W_{net} is the net work done on the object.
- ΔKE is the change in kinetic energy of the object.

2. How is power related to work and time?

Ans) Power, work, and time are related in terms of being inversely and directly proportional.

1. **Directly Proportional:** Power is directly proportional to the amount of work done (or energy transferred). This means that as the amount of work done increases, the power required to do that work also increases, assuming the time taken remains constant. Similarly, if the energy transferred increases, the power involved also increases.

   Mathematically: $P \propto W$

2. **Inversely Proportional:** Power is inversely proportional to the time taken to do the work (or transfer the energy). This means that as the time taken to perform a certain amount of work increases, the power required decreases, assuming the amount of work done remains constant. Conversely, if the time taken decreases, the power involved increases.

   Mathematically: $P \propto \frac{1}{t}$

3. What is the difference between average power and instantaneous power?

Ans)

Aspect	Average Power	Instantaneous Power
Definition	Average rate of energy transfer over a specific time interval.	Power at a particular moment in time.
Formula	$P_{avg} = \frac{W}{t}$	$P_{inst} = F \cdot v$ (linear motion) $P_{inst} = \tau \cdot \omega$ (rotational motion)
Time Frame	Considered over a defined time interval.	Pertains to an exact instant in time.
Usage	Provides an overall view of efficiency or performance over time.	Gives insight into power fluctuations at different moments.
Example	Calculating the power consumption of a device over an hour.	Measuring the power generated by a car engine at a specific RPM.

4. How does gravitational potential energy change with height and mass?

Ans) The change in gravitational potential energy with respect to height and mass can be understood through the formula for gravitational potential energy:

$PE = mgh$

1. **Change with Height (h):** Gravitational potential energy is directly proportional to the height above the reference point. As an object is lifted to a greater height, its gravitational potential energy increases. The higher the object is lifted, the more work is done against gravity to move it against the force of gravity. This work done gets stored as gravitational potential energy.
   Mathematically: $PE \propto h$
2. **Change with Mass (m):** Gravitational potential energy is directly proportional to the mass of the object. A heavier object possesses more gravitational potential energy at a given height compared to a lighter object. This is because a heavier object requires more work to be lifted to that height.
   Mathematically: $PE \propto m$

5. What is the law of conservation of energy?

Ans) Energy can neither be created nor destroyed. It can only be transformed from one form to another. A loss in one form of energy is accompanied by an equal increase in the other forms of energy. The total energy remains constant.

6. How does energy efficiency play a role in various energy transformations?

Ans) Energy efficiency is a crucial factor in various energy transformations as it determines how effectively energy is converted from one form to another while minimizing energy losses. Here's how energy efficiency plays a role in these transformations:

1. **Power Generation:** In power plants, energy transformations convert primary energy sources (like coal, natural gas, or sunlight) into electricity. Maximizing energy efficiency in these transformations is vital to ensure that a higher percentage of the input energy is converted into usable electrical energy, reducing waste and environmental impact.
2. **Transportation:** Vehicles, whether powered by internal combustion engines or electric motors, undergo energy transformations. Energy efficiency in transportation is critical to optimize fuel or electricity usage, extending travel range and reducing emissions.

7. What is the work-energy theorem and how is it expressed mathematically?

Ans) It states that total work done on the body is equal to the change in kinetic energy (Provided body is confined to move horizontally and no dissipating forces are operating).

Mathematically; $\mathbf{W = K.E_2 - K.E_1}$

8. How is the work done by a variable force calculated?

Ans) In this condition we consider the variable force to be variable for any elementary displacement ds as shown in figure below, and work done in that elementary displacement is evaluated. Total work is obtained by integrating the elementary work from initial to final limits.

$$\Delta W = \vec{F} \cdot \overrightarrow{\Delta ds}$$

$$W = \sum \vec{F} \cdot \overrightarrow{\Delta ds}$$

ERQs:

1. How the work done in gravitational field is independent of path?

OR

4. Show that the work done in gravitational field is independent of path

Ans) To prove the statement that the work done in the gravitational field is independent of path. Lets us take a closed triangular path ABC in gravitational field shown in Fig: for simplicity the base BC is taken perpendicular to the gravitational force "mg" initially the body is at A.

$$W_{A\to B} = \mathbf{F.S_1} = F.S_1 \cos \beta$$

Since, $S_1 \cos \beta = h$ and $F = mg$

$$W_{A\to B} = mgh$$

$$W_{B\to C} = \mathbf{F.S_2} = FS_2 \cos 90^o = 0$$

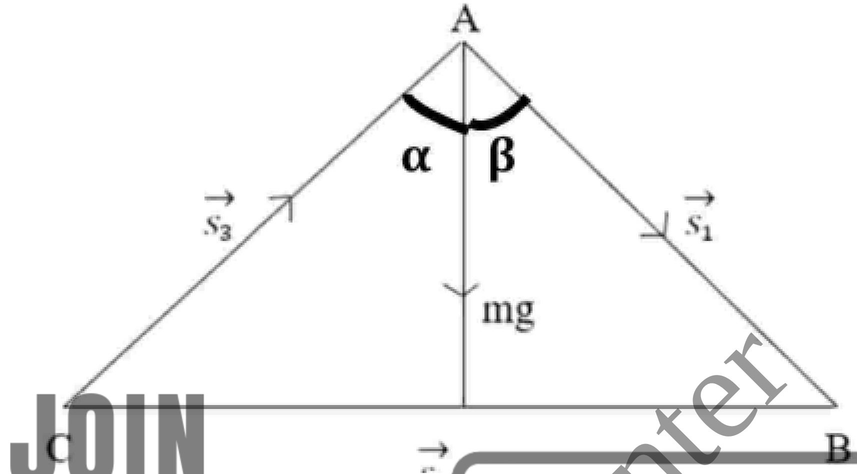
$$W_{C\to A} = \mathbf{F.S_3} = FS_3 \cos (180^o - \beta) = FS_3 (-\cos \beta) = -FS_3 (\cos \beta)$$

Since, $S_3 \cos \beta = h$ and $F = mg$

$$W_{C\to A} = -mgh$$

Thus, total work done along path ABCA

$$= (mgh) + (0) + (mgh) = 0$$



Thus, either the body is moved from A to B and B to C or it is moved directly from A to C, in both the cases the work done is same, such type of field or force in which the work done is independent of path is called conservative field and in conservative field work done in a closed path is always zero.

2. Calculate gravitational potential energy at a certain height due to work against gravity.

Ans) Consider a body of mass 'm' which is taken every slowly to small height 'h' in the gravitational field such as that the acceleration of the body is zero. The work done in moving the body is given by:

$$\text{Work done} = F_{ex}\,h = F_{ex}\,h \cos\theta$$

Where F is the external force applied on the body. Since the external force applied on the body and the displacement are along the same direction, therefore work done by external force “W_{ex}” is given by:

$$W_{ex} = F_{ex}\,h \quad (\text{since } \cos 0^o = 1)$$

As the acceleration of the body is zero therefore magnitude of external force is equal to that of the force of gravity i.e.

$$F_{ex} = mg$$

Therefore $$W_{ex} = mgh \rightarrow (1)$$

Work done "W_g" by the gravitational force "F_g” is given by

$W_g = \mathbf{F_g.h} = F_g h \cos 180^o$ (Since F_g and h in opposite direction then angle between them is 180^o)

Since $F_g = mg$

$$W_g = -mgh$$

OR

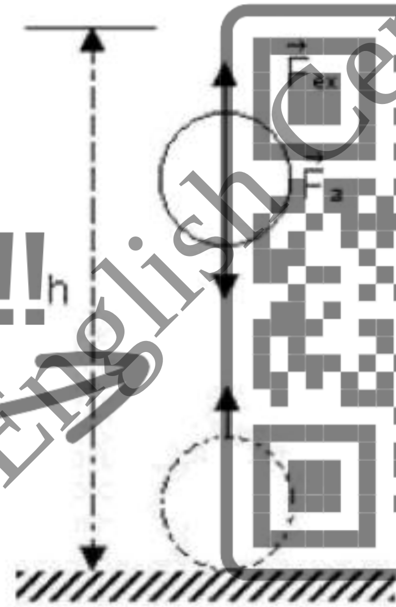
$$-W_g = mgh \longrightarrow (2)$$

Comparing eqn (1) and (2)

$$W_{ex} = -W_g$$

By putting the value of W_g from eq. (2), we get

$$W_{ex} = -W_g = (-mgh) = mgh$$



3. Describe that the gravitational PE is measured from a reference level and can be positive or negative, to denote the orientation from the reference level.

Ans) The measurement of gravitational potential energy involves the use of a reference level, and its sign can be either positive or negative, indicating the orientation of the object relative to the reference level.

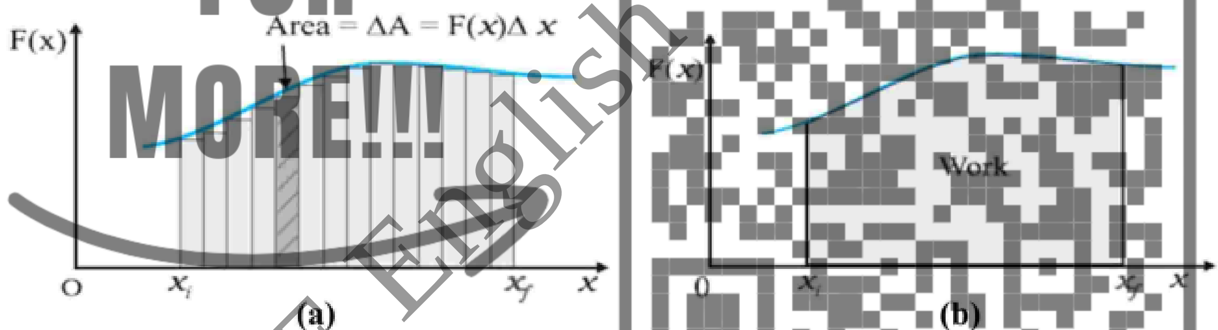
The reference level serves as a baseline point from which the height or distance of the object is measured. Typically, this reference level is chosen to be at the surface of the Earth or another defined point within the gravitational field. When an object is located above the reference level, its gravitational potential energy is positive. This signifies that the object has the potential to do work as it falls toward the reference level under the influence of gravity. The greater the height above the reference level, the higher the potential energy of the object.

Conversely, when an object is positioned below the reference level, its gravitational potential energy becomes negative. This negative sign indicates that the object is already in a lower energy state and would require an input of energy to move it back up to the reference level. Objects closer to or below the reference level have decreasing negative potential energy as they move deeper into the gravitational field.

5. Define work by variable force. Calculate the work done from the force-displacement graph.

Ans) Work done by a variable force refers to the energy transferred or expended when an object is subjected to a force that changes in magnitude and direction as the object undergoes displacement.

The work done from the force-displacement graph: Consider a body covers displacement from x, to xy, when a variable force acts on it. To clarify the situation, we plot the graph between force and displacement covered by the body as shown in figure; 5.6 (a and b).



(a) (b)

the area under the covered line is representing the work done by the body. To calculate the work done by the body we divide the covered displacement into small segments

$\Delta x_1, \Delta x_2, \Delta x_3 \ldots\ldots \Delta x_n$ and the corresponding forces for each segment are $\overrightarrow{F_{x1}}, \overrightarrow{F_{x2}}, \overrightarrow{F_{x3}} \ldots\ldots \overrightarrow{F_{xn}}$ as shown in figure 5.6.

We know the work done for each segment will be $W_1, W_2, W_3, \ldots\ldots W_n$.

The total work done in this case will be $W_T = W_1 + W_2 + W_3 + \cdots W_n = \sum_{i=1}^{n} W_i$

$$W_T = \overrightarrow{F_{1x}}.\overrightarrow{\Delta x_1} + \overrightarrow{F_{2x}}.\overrightarrow{\Delta x_2} + \overrightarrow{F_{x3}}.\overrightarrow{\Delta x_3} + \cdots + \overrightarrow{F_{xn}}.\overrightarrow{\Delta x_n} = \sum_{i=1}^{n} \overrightarrow{F_{xi}}.\overrightarrow{\Delta x_i}$$

$W_T = \sum_{i=1}^{n} \overrightarrow{F_{xi}}.\overrightarrow{\Delta x_i}$

$W_T = \sum_{i=1}^{n} F_{xi}\, \Delta x_i \cos \varphi$

Numericals:

1. A man pulls a trolley through a distance of 10 m by applying a force of 50 N which makes an angle of 60° with the horizontal. Calculate the work done by the man?

Data:

F = 50 N

s = 10 m

$\theta = 60°$

W = ?

Solution:

$W = Fs\cos\theta$

$\mathbf{W = (50)(10)\cos 60 = 250\ J}$

2. A 100 kg man runs up a long flight of stairs in 9.8 second. The vertical height of the stair is 10 m. Calculate its power?

Data:

m = 100 kg

t = 9.8 s

h = 10m

P = ?

Solution:

$P = \frac{W}{t}$

Since, W = mgh

$P = \frac{mgh}{t}$

$P = \frac{(100)(9.8)(10)}{9.8}$

$\mathbf{P = 1000\ W}$

3. When an object is thrown upwards. It rises to a height 'h'. How high is the object in terms of h? when it has lost one third of its original kinetic energy?

Data:

$H = ?$

$K.E_{final} = \frac{2}{3} K.E_{initial}$

Solution:

$\frac{1}{2} mv_f^2 = \frac{2}{3}\frac{1}{2} mv_i^2$

$v_f^2 = \frac{2}{3} v_i^2 \longrightarrow (1)$

Now, if h is the maximum height achieved by the ball when v_f=0 then using the kinematic equation give

$K.E_{final} = K.E_{initial} - P.E_{final}$

$\frac{1}{2} mv_f^2 = \frac{1}{2} mv_i^2 - mgh$

$v_f^2 = v_i^2 - 2gh$

$0 = v_i^2 - 2gh$

$v_i^2 = 2gh$

Now for the height H when $K.E_{final} = \frac{1}{3} K.E_{initial}$,we use the kinematic equation using euation (1)

$\frac{2}{3} v_i^2 = v_i^2 - 2gH$

$2gH = \frac{1}{3} v_i^2$

$2gH = \frac{1}{3} (2gh)$

$\mathbf{H = \frac{h}{3}}$

4. A 70 kg man runs up a hill through a height of 3 m in 2 seconds.

a) How much work does he do against gravitational field?
b) What is the average power output?

Data:

m = 70 kg

h = 3 m

t = 2 s

(a) W = ?
(b) P_{av} = ?

Solution:

(a)

W = mgh = (70) (9.8) (3) = 2058 J

(b)

$P_{av} = \frac{W}{t}$

$\mathbf{P_{av} = \frac{2058}{2} = 1029\ W}$

5. A neutron travels a distance of 12 m in a time interval of 3.6 x 10^{-4} sec. Assuming its speed was constant, Find its kinetic energy? Take the mass of neutron 1.7 x 10^{-27} kg.

Data:

s = 12m

t = 3.6 x 10^{-4} s

m = 1.7 x 10^{-27} kg

K.E = ?

Solution:

$v = \frac{s}{t} = \frac{12}{3.6 \times 10^{-4}}$ = 33333.333 m/s

K.E = $\frac{1}{2}$ m v^2

K.E = $\frac{1}{2}$ (1.7 x 10^{-27}) $(33333.333)^2$

$K.E = 9.444 \times 10^{-19}$ J

$K.E = 9.444 \times 10^{-19} (6.242 \times 10^{18}) = 5.895$ eV

6. A stone is thrown vertically upwards and can reach to a height of 10m, find the speed of stone, when it is just 2m above the ground?

Data:

$h = 10m$

$h' = 2m$

$v' = ?$

Solution:

$2gh = v_f^2 - v_i^2$

$2(-9.8)(10) = 0 - v^2$

$v^2 = 196$

Initial K.E + Initial P.E = Final K.E + Final P.E

$\frac{1}{2}mv^2 + 0 = \frac{1}{2}mv'^2 + mgh'$

$\frac{1}{2}v^2 - gh' = \frac{1}{2}v'^2$

$\frac{1}{2}(196) - (9.8)(2) = \frac{1}{2}v'^2$

$v' = 12.52$ m/s

7. The potential energy of a body at the top of a building is 200 Joules when it is dropped its kinetic energy just before striking the ground is 160 Joules, find the work done against the air resistance?

Data:

P.E = 200 J

K.E = 160 J

W = ?

Solution:

K.E = P.E – W

160 = 200 – W

W = 200 – 160 = 40 J

8. Find the energy equivalent of 1 gram?

Data:

m = 1 g = $\frac{1}{1000}$ = 0.001 kg

E = ?

Solution:

$E = mc^2$

$E = (0.001)\ (3 \times 10^{8})^{2}$

$\mathbf{E = 9 \times 10^{13}\ J}$

8. A 1Kilowatt motor pump, pumps the water from the ground to a height of 10 m. Find, how much litres of water it can pump in one hour?

Data:

$P = 1\ kW = 1000W$

$h = 10m$

$\frac{V}{t} = ?$

Solution:

$P = \frac{W}{t}$

$P = \frac{mgh}{t}$

$\frac{P}{gh} = \frac{m}{t}$

$\frac{m}{t} = \frac{1000}{(9.8)(10)}$

$\frac{m}{t} = 10.2\ kg/s \longrightarrow \quad (1)$

1 kg water = 1 litre water

eqn (1) becomes

$\frac{V}{t} = 10.2$

$\frac{V}{t} = 10.2$ litre/s

$\mathbf{\frac{V}{t} = 10.2 \times 3600 = 3.672 \times 10^4}$ **litre/h**

10. A rocket of mass 2kg is launched in air, when it attains height of 15m the 400 Joules of its chemical fuel burns. Find the speed of rocket at maximum height?

Data:

m = 2 kg

h = 15 m

W = 400 J

V_f = ?

Solution:

Gain in K.E + Gain in P.E = W

$\frac{1}{2}mv_f^2 + mgh = W$

$\frac{1}{2}(2)\,v_f^2 + (2)\,(9.8)\,(15) = 400$

$\mathbf{v_f = 10.3\ m/s}$

11. A motor pumps the water at the rate 500 gram/minute to the height of 120 m. If the motor is 50% efficient then how much input electric power is needed?

Data:

$\frac{m}{t}$ = 500 gram/minute = $\frac{500 \times 60}{1000}$ = 30 kg/s

h = 120m

η = 0.5

P = ?

Solution:

$P = \frac{W}{t}$

$P = \frac{P.E}{t}$

$P = \frac{mgh}{t}$

$P = \frac{m}{t}\,gh$

P = (30) (9.8) (120) = 35280 W

$P' = P \times \eta$

$$P' = (35280)(0.5) = 17640 \text{ W}$$

OR

$$P' = \frac{17640}{746} = 23.646 \text{ hp}$$

Unit #6: Fluid Statics

Worked Example 6.1

A hydraulic system consists of two connected cylinders, Cylinder A and Cylinder B. Cylinder A has a piston with a radius of 5 cm and Cylinder B has a piston with a radius of 10 cm. A force of 200 N is applied to Cylinder A. Calculate the force exerted by Cylinder B.

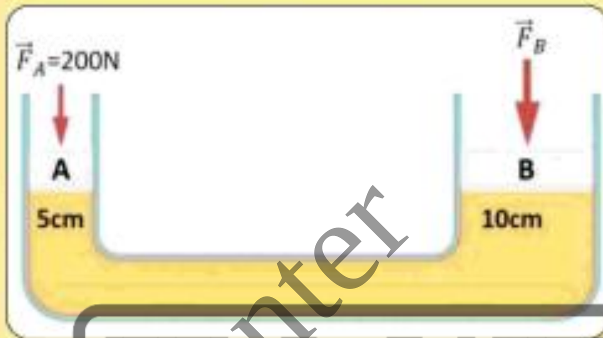
Solution:

Data:

Radius of Cylinder A = 5 cm = 0.05 m

Radius of Cylinder B = 10 cm = 0.1 m

Force applied to Cylinder A $\vec{F}_A$=200N



Step 1: According to Pascal's law, the pressure exerted on the fluid in Cylinder A is transmitted equally to the fluid in Cylinder B. Therefore, the pressure in both cylinders will be the same.

Step 2: The formula for pressure is:

Pressure (P) = Force (F) / Area (A)

The area of a cylinder can be calculated using the formula:

Area (A) = πr^2

Let's calculate the pressure in Cylinder A:

Area of Cylinder A = πA^2

= $(3.14)(0.05)^2$ = 0.00785 m^2

Pressure in Cylinder A $P_A = \vec{F}_A / A$

= 200 N / 0.00785 m²

≈ 25477 Pa

Step 3: Since the pressure is the same in both cylinders, we can calculate the force exerted by Cylinder B using the pressure and the area of Cylinder B:

Area of Cylinder B = πB^2

Area of Cylinder B = $3.14\,(0.1m)^2$

= 0.0314 m²

Step 4: Force exerted by Cylinder B ($\vec{F}_B$) = Pressure (P_A) × Area (B)

= 25477 Pa × 0.0314 m^2

≈ **800 N**

Therefore, the force exerted by Cylinder B is approximately **800 N**, in accordance with Pascal's law.

Worked Example 6.2

When a crown of mass 14.7 kg is submerged in water, an accurate scale reads only 13.4 kg. Is the crown made of gold?

Approach:

If the crown is gold, its density and specific gravity must be very high.

Solution:

Step 1: Formula: $W_a = F_r = W - F_B$

$W - W_a = F_B$

Step 2: Let V be the volume of completely submerged object and ρ_o the object's density (So ρ_o V is its mass) , and let ρ_f is fluid density (water) . Then ρ_f V is the weight of fluid displaced, then

$W = mg = \rho_o Vg$

$W - W_a = F_B = \rho_F V g$

By dividing them

$$\frac{W}{W - W_a} = \frac{\rho_o Vg}{\rho_F Vg} = \frac{\rho_o}{\rho_F}$$

Step 3: Density of fluid i.e. water is 1×10^3 kg/m^3

$$\frac{\rho_o}{\rho_F} = \frac{W}{W - W_a} = \frac{(14.7\ kg)g}{(14.7\ kg - 13.4\ kg)g} = 11.3$$

This shows that crown is not made of gold rather made of lead.

Worked Example 6.3

What volume V of a helium is needed if a balloon is to lift a load of 185 kg?

Step 1: M = 185 kg

The buoyant force on the helium balloon F_b, which is equal to the weight of displaced air, must be at least equal to the weight of the helium plus the weight of balloon and load, the density of helium is 0.179 kg/m^3 density of air is 1.29 kg/m^3.

Step 2: $F_b = (m_{hel} + 185)\ g$

This equation is written in terms density by using Archimedes' principle

$\rho_{air}\ V g = (\rho_{hel}\ V + 185)\ g$

Step 3: Solving for V, we get

$$V = \frac{185}{\rho_{air} - \rho_{hel}}$$

$$V = \frac{185}{1.29 - 0.179}$$

$V = 166.5$ m^3

Worked Example 6.4

A solid, square pine wood raft measures 4.5 m on a side and is 0.35 m thick. (a) Determine whether the raft floats in water and (b) if so how much of the raft is beneath the surface thee distance h?

Step 1:

Side = 4.5 m

Thickness = 0.35 m

Step 2:

To determine whether the raft floats, we will compare the weight of the raft to the maximum possible buoyant force and see whether there could be enough buoyant force to balance the weight. If so, then the value of the distance h can be obtained by utilizing the fact that the floating raft is in equilibrium, with the magnitude of the buoyant force equalizing the raft's weight.

Step 3:

(a)

$W_{raft} = m_{pine}\, g$ where m_{pine} can be calculated by Density × Volume.

$W_{raft} = 550 \times 7.1 \times 9.8$

$W_{raft} = 38269$ N

$F_B = (\rho V g)_{water} = 1000 \times 7.1 \times 9.8 = 69580$ N

Since the maximum possible buoyant force exceeds the 38269 N weight of the raft, the raft will float only partially submerged at a distance h beneath the water.

(b)

The buoyant force balances the raft's weight, so $F_B = 38269$ N. Using the density of water and volume of water 4 m × 4 m × h.

$38269\text{ N} = W_{water} = \rho_{water} \times (4.5\text{ m} \times 4.5\text{ m} \times h) \times 9.8$

Solving for h, we get

$h = 0.19$ m

Worked Example 6.5

The base of an insect's leg is approximately spherical in shape, with a radius of about 2.1×10^{-5} m. The 3.2×10^{-6} kg mass of insect is supported by equally by its six legs. Estimate the angle for an insect on the surface of water. Assume water temperature is 20°C.

Step 1:

$r = 2.1\times10^{-5}$ m

$m = 3.2\times10^{-6}$ kg

$\theta = ?$

Since the insect is in equilibrium, the upward surface tension force is equal to the effective pull of gravity down ward on each leg.

Step 2:

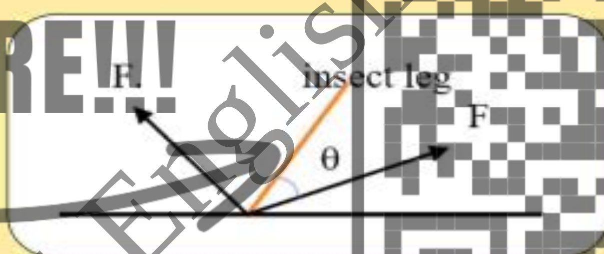
For each leg, we assume the surface tension force acts all around the circle of radius r, at an angle θ. Only the vertical component, $\gamma\ \text{Cos}\ \theta$, acts to balance the weight mg. So, we set the length L equal to circumference of the circle $L = 2\pi r$. Then the net upward force is due to surface tension is $F_y = (\gamma\ \text{Cos}\theta)\ L = (\gamma\ \text{Cos}\theta)\ 2\pi r$. We set this surface tension force γ equal to one-sixth the weight of the insect since it has six legs:

Step 3:

$2\pi\ r\ \gamma\ \text{Cos}\ \theta = 1/6\ mg$

$2\pi\ (2.1\times10^{-5})\ (0.072)\ \text{Cos}\ \theta = 1/6\ (3.2\times10^{-6})\ (9.8)$

$\theta = 56^0$



Worked Example 6.6

Water flows through a fire hose of diameter 6.5 cm at a speed of 5.99 m/s. Find the flow rate of the fire hose in L/min.

Step 1:

$v = 5.99$ m/s

$r = 3.25$ cm $= 0.0325$ m

$A = \pi r^2 = 3.14\ (0.0325)^2$

flow rate=?

Step 2:

Flow Rate = Av

Flow Rate = $(5.99)\times\pi(0.0325)^2$

Flow Rate = $0.01988\ m^3/s \times (1000\ L/m^3)\ (60\ s/1\ min)$

Flow Rate = 1192 L/min

Section (A): Multiple Choice Questions (MCQs)

1. A completely submerged object always displaces its own -----------.

a) weight of fluid
c) density of a fluid
b) volume of a fluid
d) Area of fluid

2. The pressure exerted on the ground by a man is greatest when:

a) he stands with both feet flat on the ground
c) he stands on the toes of one foot
b) he stands flat on one foot
d) he lies down on the ground

3. In a stationary homogeneous liquid

a) pressure is the same at all points

b) pressure depends on the direction

c) pressure is independent of any atmospheric pressure on the upper surface of the liquid

d) pressure is the same at all points at the same level

4. One piston in a hydraulic lift has an area that is twice the area of the other. When the pressure at the smaller piston is increased by Δp the pressure at the larger piston.

a) increases by $2\Delta p$
c) increases by Δp
b) increases by $\Delta p/2$
d) increases by $4\Delta p$

5. In a vacuum, an object has

a) No buoyant force
c) no weight
b) no mass
d) none of these

6. The pressure at the bottom of a pond does NOT depend on

a) Water density
c) the surface area of the pond
b) the depth of the pond
d) none of these

7. A rock suspended by a weighing scale weighs 5N out of water and 2N when submerged in water. What is the buoyant force on the rock?

a) 3N
c) 8N
b) 5N
d) 15 N

8. "An object completely submerged in a fluid displaces its own volume of fluid". This is:

a) Pascal's paradox
c) Pascal's principle
b) Archimedes' principle
d) true, but none of the above

9. Salt water has greater density than freshwater. A boat floats in both freshwater and saltwater. The buoyant force on the boat in salt water is ------------- that in freshwater.

a) equal to
c) larger than
b) smaller than
d) same as

10. You fill a tall glass with ice and then add water to the level of the glass's rim, so some fraction of the ice floats above rim. When the ice melts, what's happens to the water level?

a) the water overflows the rim

b) the water level drops below the rim

c) the water level stays at the rim

d) it depends on the difference in density between water and ice

KEY:

1. b	2. c	3. d	4. c	5. a
6. c	7. a	8. b	9. c	10. c

CRQs:

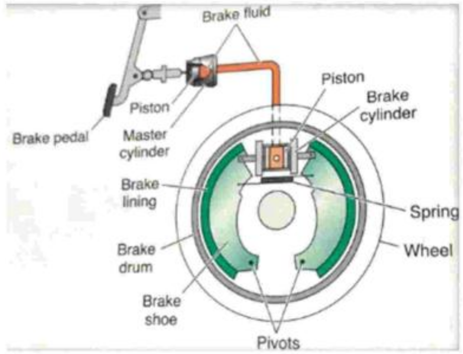
1. State Pascal's principle. Describe its two applications.

Ans) Pascal's principle: "When a change in pressure is applied at any point to any static fluid, it is transmitted perfectly to all portions of the fluid and to the wall of the container."

Applications:

1. **Automobile Hydraulic Brake System:** The rear wheel hydraulic brake system of a front-wheel-drive shown in figure shown is an application of Pascal's principle. When driver pushes the brake pedal,
"**The pressure on the piston in master cylinder is transmitted through the brake fluid to the two pistons in the brake cylinder**"
This transmitted pressure then forces the brake-cylinder pistons to push the brake shoes against the brake drum and stop the automobile. Now by releasing the brake pedal releases the pressure on the pistons in the brake cylinder. The

spring pulls the brake shoes away from the brake drum, which allows the wheel to turn freely again.



2. **Hydraulic Lift or Hydraulic Jack:** Hydraulic lift is able to raise up large weight up to relatively short distance.
The hydraulic lift, or jack is applications of hydraulics being used as a simple machine to multiply force.
It contains an incompressible fluid in a U-shaped tube which is narrower at start and becomes wider area at end which is fixed with a movable piston on each side. If a small force F_1 is applied to the small piston of the hydraulic lift as shown in figure shown, the pressure is transmitted with in all directions. The pressure on the large piston is the same as the pressure on the small piston; **however, the force F; on the large piston is greater because of its large surface area which used to uplift the car.**
From Pascal's principle, the force needed to lift the automobile is less than the weight of the automobile.

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

OR

$$\frac{F_2}{F_1} = \frac{A_2}{A_1}$$

. Mechanical advantage (M.A) = $\frac{F_2}{F_1}$, which is equal to the ratios of areas.

For example, if the area of output piston is 20 times that of the input cylinder, the force multiplied by a factor of 20. Thus, a force of 200 lbs. could lift a 4000 lbs. of weighing automobile.

2. (a) Why must a liquid and not a gas be used as the 'fluid' in a hydraulic machine?

(b) On what other important property of a liquid do hydraulic machines depend?

Ans)

(a) A liquid is preferred over a gas as the fluid in a hydraulic machine because liquids are incompressible, provide efficient energy transmission, are safer and more stable, have quicker response times, and offer consistent properties. Gases are compressible, less stable, and can lead to unpredictable behavior, making them unsuitable for efficient hydraulic operations.

(b) In addition to incompressibility, another important property of liquids that hydraulic machines depend on is the fact that liquids transmit pressure equally in all directions. This is often referred to as Pascal's principle or Pascal's law. According to Pascal's principle, when pressure is applied to a confined liquid, that pressure is transmitted undiminished to all portions of the liquid and to the walls of the container. This uniform transmission of pressure in all directions enables hydraulic systems to achieve mechanical advantage by using a small force to generate a larger force in a controlled manner.

3. Why don't ships made of iron sink?

Ans) Ships made of iron and steel float due to buoyancy, which is determined by the principle that an object will float if it is less dense than the fluid it displaces. The ship's shape and design displace a large volume of water, creating an upward buoyant force that counters the weight of the ship. The overall density of the ship, including its structure and any enclosed air spaces, is carefully managed to ensure that it remains less dense than the water it sits in, allowing it to float.

4. Why do you float higher in salt water than in fresh water?

Ans) Saltwater is denser than freshwater because the dissolved salt particles add to its mass without significantly increasing its volume. As a result, the buoyant force exerted by saltwater is stronger than that exerted by freshwater. This increased buoyant force causes you to float higher in saltwater than in freshwater.

5. What is the difference between being immersed and being submerged in water?

Ans) "Immersed" describes an object partially or fully in contact with a fluid, such as water, without necessarily being completely covered. It suggests a presence within the fluid but not complete submersion. For instance, an ice cube in a glass of water is immersed because part of it is above the water's surface. "Submerged," on the other hand, refers to an object being entirely covered and surrounded by the fluid, typically beneath the fluid's surface. When an object is submerged, it is fully hidden from view within the fluid. A scuba diver underwater is an example of something submerged, as the diver's entire body is beneath the water's surface."

6. Define surface tension and give its any two applications.

Ans) The force per unit length acting on either side of the imaginary line drawn on the liquid surface at rest. The direction of force is tangential to the surface and perpendicular to the line.

Applications:

1. When a pin is placed gently over the surface of water, it floats as shown in figure 6.9(a). There is a depression of the surface which behaves like a stretched membrane. As needle is pressed with a finger, the layer of surface tension breaks and the needle sink in the water.

2. Some insects (with long legs) can walk over the water surface. Surface tension of water supports the weight of insect. Actually insect sinks slightly into the fluid so its weight becomes effective weight of that insect, its true weight is less than the buoyant force.

7. A swimmer dives off a raft in a pool. Does the raft rise or sink in the water? What happens to the water level in the pool? Give reasons for your answer.

Ans) When a swimmer dives off a raft in a pool, the raft will temporarily sink slightly into the water, and the water level in the pool will also temporarily rise. Here's why:

1. **Raft Sinking:** As the swimmer dives off the raft, their weight is removed from the raft, causing a decrease in the downward force applied to it. According to Newton's third law of motion, every action has an equal and opposite reaction. The raft, no longer experiencing the downward force from the swimmer's weight, will experience a slight upward force from the water known as buoyancy. This buoyant force will cause the raft to rise slightly in the water, indicating that it has become less submerged.
2. **Water Level Rise:** When the swimmer dives into the water, they displace a certain volume of water equal to their body's volume. The water level in the pool will temporarily rise due to the volume of water displaced by the swimmer. As the swimmer resurfaces and moves through the water, the displaced water will settle back down, and the water level will return to its original position.

8. Distinguish between flotation and upthrust.

Ans) Flotation is when an object remains on the surface of a fluid without sinking, achieved through a balance between the object's weight and the buoyant force exerted by the fluid. Upthrust, also known as buoyant force, is the upward force exerted by a fluid on an immersed object. This force results from pressure differences and makes objects feel lighter in a fluid compared to air. Upthrust enables flotation by counteracting the object's weight, allowing it to stay afloat. In summary, flotation describes an object's state on the fluid's surface, while upthrust is the force responsible for this state by providing an upward balancing force.

ERQs:

1. Discuss buoyancy in liquids and gases.

Ans) In liquids:

- If the weight of the submerged object is greater than buoyant force, the object sinks.
- If the weight of object is equal to the buoyant force acting upwards on the submerged object, it remains at any level in fluid, like a fish.
- If the buoyant force is greater than the weight of object which is completely submerged, it rises to the surface and floats.

For example, a ship that is launched into sea, sinks into the ocean until the weight of the water it displaces is just equal to its own weight. As the ship is loaded, it sinks deeper, displacing more water, and so the magnitude of the buoyant force continuously matches the weight of the ship and its cargo.

In gases:

We live at the bottom of ocean of air and look upward at balloons and other lighter than air objects are drifting above us. Air pressure acting upward against an object immersed in air is greater than the pressure above pushing down. The buoyancy is equal to the weight of fluid displaced. So, Archimedes' principle also implies to air which is stated as: “An object surrounded by air is buoyed up by a force equal to the weight of the air displaced.”

A cubic meter of air at normal atmospheric pressure and room temperature has a mass of about 1.2 Kg. so its weight becomes around 12 N.

- If the mass of the 1 m^3 object is greater than 1.2 Kg, it falls to the ground, when released.
- If an object of this size has mass of less than 1.2 Kg, buoyant force is greater than weight and it rises in air.

Because gas (Hydrogen or Helium) filled in balloons that rises in air are less dense than air. Air becomes less dense at high altitude; a lesser weight of air is displaced per given volume as the balloon rises. When the weight of displaced air equals the total weight of the balloon, upward motion of balloon ceases. It can be stated as when the buoyant force on the balloon equals its weight, the balloon ceases to rise.

2. Describe law of flotation in liquids and gases.

Ans) The law of flotation, also known as Archimedes' principle, states that any object submerged in a fluid (liquid or gas) experiences an upward buoyant force equal to the weight of the fluid displaced by the object.

1. **Law of Flotation in Liquids:** When an object is placed in a liquid, it displaces an amount of liquid equal to its own volume. The upward buoyant force acting on the object is proportional to the weight of the liquid displaced. If the buoyant force is greater than or equal to the weight of the object, the object will float; if it's less, the object will sink. This is why objects with lower density than the liquid they're placed in float, while those with higher density sink.
2. **Law of Flotation in Gases (Air):** The same principle applies to gases like air, though there are a few differences due to the compressibility of gases compared to liquids. When an object is placed in a gas, it displaces an amount of gas equal to its own volume. However, since gases are compressible, their density can change significantly with pressure and temperature variations. This can influence the behavior of floating or sinking objects in gases.

In both cases, the key concept is that the buoyant force opposes the force of gravity acting on the object. If the buoyant force is greater than or equal to the gravitational force, the object will be positively buoyant and will rise to the surface or remain floating. If the gravitational force is greater, the object will be negatively buoyant and will sink.

3. Discuss surface tension with at least three experiments.

Ans)

Surface tension: The force per unit length acting on either side of the imaginary line drawn on the liquid surface at rest. The direction of force is tangential to the surface and perpendicular to the line.

1. When a pin is placed gently over the surface of water, it floats as shown in figure 6.9(a). There is a depression of the surface which behaves like a stretched membrane. As needle is pressed with a finger, the layer of surface tension breaks and the needle sink in the water.

2. Some insects (with long legs) can walk over the water surface. Surface tension of water supports the weight of insect. Actually insect sinks slightly into the fluid so its weight becomes effective weight of that insect, its true weight is less than the buoyant force.

3. A drop of Olive oil is dropped through a pipette, gently inside a mixture of alcohol and water, which forms a perfectly spherical shape. The density of the mixture is same as of olive oil. Due to surface tension, the tendency of a liquid surface is to keep its area minimum. For a given volume, the surface area of a sphere is minimum. Due to this reason, the raindrop is in spherical shape.

4. Explain Archimedes principle and find gold purity by using density.

Ans) Archimedes' principle: It states that when an object is immersed into a liquid, it experiences upward thrust which is equal to the weight of the liquid displaced by that object.

The magnitude of the upward force depends upon:

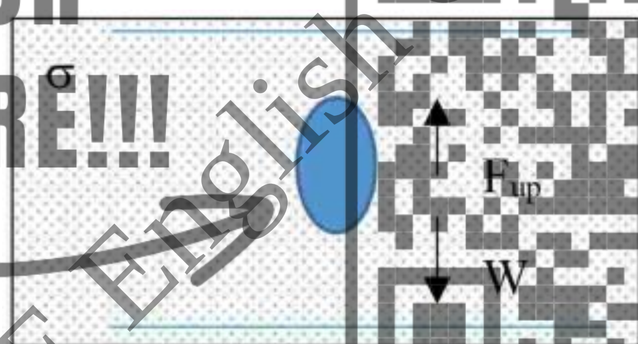
- Volume of the body- more fluid that is displaced, the greater the upthrust.
- Density of fluid- the greater the density, greater is upthrust.

It should be clear from the above that a floating body will displace its own weight of fluid such that there is no vertical resultant force on the body.

Therefore, if a sphere of material density ρ with radius r is fully immersed into a liquid of density σ; shown in figure below, the apparent weight of sphere is given by

Apparent Weight = Actual Weight- upthrust

The fluid may either be a liquid or air which have low density (about 1 Kg/m³), the upthrust in air is usually small, but sufficient to support a helium filled balloon uplift.



Numericals:

1. In a hydraulic press a force of 20 N is applied to a piston of area 0.20 m². The area of the other piston is 2.0 m². What is (a) the pressure transmitted through the fluid, (b) the force on the piston?

Data:

$F_1 = 20$ N

$A_1 = 0.20$ m²

$A_2 = 2.0$ m²

(a) P = ?
(b) F_2 = ?

Solution:

(a)

$P = \frac{F_1}{A_1} = \frac{20}{0.2}$

P = 100 Pa

(b)

$\frac{F_1}{A_1} = \frac{F_2}{A_2}$

$\frac{F_2}{F_1} = \frac{A_2}{A_1}$

$\frac{F_2}{20} = \frac{2}{0.2}$

F_2 = 200 N

2. The pressure in a water pipe in the ground floor of a building is 4x 10^5 Pa but three floors up it is only 2x 10^5 Pa. What is the height between the ground floor and the third floor? The water in the pipe may be assumed to be stationary; density of water =1x10^3 Kg/m^3.

Data:

$P_1 = 4\times 10^5$ Pa

$P_2 = 2\times 10^5$ Pa

h = ?

$\rho = 1\times10^3$ Kg/m^3

g = 10 m/s^2

Solution:

$\Delta P = \rho g h$

$4\times 10^5 - 2\times 10^5 = (1\times10^3)(10)(h)$

h = 20 m

3. The small piston of hydraulic press has an area of 10.0 cm² If the applied force is 50.0 N. what must the area of the large piston to exert a pressing force of 4800N?

Data:

$A_1 = 10.0$ cm²

$F_1 = 50.0$ N

$F_2 = 4800$N

A_2 = ?

Solution:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\frac{F_2}{F_1} = \frac{A_2}{A_1}$$

$$\frac{4800}{50} = \frac{A_2}{10}$$

$A_2 = 960\ cm^2$

4. Mechanical advantage of a hydraulic jack is 420. Find the weight of the heaviest automobile that can be lifted by an applied force of 55 N.

Data:

M.A = 420

F_{in} = 55 N

F_{out} = ?

Solution:

M.A = $\frac{F_{out}}{F_{in}}$

420 = $\frac{F_{out}}{55}$

F_{out} = 23100 N

5. A flat-bottom river barge is 30 ft wide, 85 ft long and 15 ft deep. (a) how many m^3 of water will displace while the top stays 1 m above the water? (b) What load in tons will the barge contain under these conditions if the empty barge weighs 160 tons in dry dock?

Data:

w = 30 ft = $\frac{30}{3.28}$ = 9.15 m

l = 85 ft = $\frac{85}{3.28}$ = 26 m

h = 15 ft = $\frac{15}{3.28}$ = 4.6 m

h' = 4.6 – 1 = 3.6m

m_{empty} = 160 tons = 160 x 1000 = 160000 kg

(a) V (in m^3) = ?
(b) Load the barge can carry = ?

Solution:

(a)

V = w x l x h' = (9.15) (26) (3.6) = 856.44 m^3

(b)

Buoyant force = V x ρ x g

Buoyant force = (856.44) (1000)(9.8) = 8.4×10^6 N

Weight the barge can carry = Buoyant force - Weight of empty barge = 8.4×10^6 - 160000 x 9.8 =6.832×10^6 N

Load the barge can carry = 6.832×10^6 / 9.8 = 6.971×10^5 kg = 6.971×10^5 /1000

Load the barge can carry = 697.1 tons

6. A canal lock gate is 20m wide and 10m deep. Calculate the thrust acting on it assuming that the water in the canal is in level with the top of the gate. Density of water is 1000 kg/m^3.

Data:

w = 20m

h = 10m

ρ = 1000 kg/m^3

F_{thrust} = ?

Solution:

F_{thrust} = P x A

F_{thrust} = ½ ρ g h x A

F_{thrust} = ½ (1000) (9.8) (10) (10 x 20)

$\mathbf{F_{thrust} = 9.8 \times 10^6}$ **N**

7. A tank 4 m long, 3 m wide and 2 m deep is filled to the brim with paraffin (density 800 kg/m^3). Calculate the pressure on the base? What is the thrust on the base?

Data:

l = 4m

w = 3m

h = 2m

P = ?

F_{thrust} = ?

Solution:

$P = \rho g h$

$P = (800)(10)(2) = 16000$ Pa

$F_{thrust} = P \times A = P \times (l \times w)$

$F_{thrust} = 16000 \times (4 \times 3)$

$F_{thrust} = 192000$ N

8. A rectangular boat is 4.0 m wide, 8.0 m long, and 3.0 m deep. (a) How much water will it displace if the top stays 1.0 m above the water? (b) What load will the boat contain under these conditions if the empty boat weighs 8.60×10^4 N in dry dock?

(Weight density of water 9800 N/m^3)

Data:

w = 4 m

l = 8m

h = 3m

h' = 3 – 1 = 2m

Weight of empty boat = 8.60×10^4 N

(a) V = ?
(b) Weight the boat can carry = ?

Solution:

(a)

$V = l \times w \times h' = 8 \times 4 \times 2 = 64$ m^3

(b)

Buoyant force = $V \times \rho \times g$

Buoyant force = $(64)(1000)(9.8) = 6.272 \times 10^5$ N

Weight the boat can carry = Buoyant force - Weight of empty boat = $6.272 \times 10^5 - 8.60 \times 10^4 = 5.412 \times 10^5$ N

9. A hot air balloon has a volume of 2200 m^3. The density of air at temperature of 20 °C is 1.205 kg/m^3. The density of the hot air inside the balloon at a temperature of 100 °C is 0.946 kg/m^3. How much weight can the hot air can lift?

Data:

$V = 2200\ m^3$

$\rho\_cold = 1.205\ kg/m^3$

$\rho\_hot = 0.946\ kg/m^3$

$W = ?$

Solution:

$W = (V \times \rho\_cold - V \times \rho\_hot) \times g$

$W = (2200 \times 1.205 - 2200 \times 0.946) \times 9.8$

$W = 5584.04\ N$

10. A spherical balloon has a radius of 7.15 m and is filled with helium. How large a cargo can it lift, assuming that the skin and structure of the balloon have a mass of 930 kg? Neglect the buoyant force on the cargo volume itself.

Data:

$r = 7.15\ m$

$\rho_{He} = 0.179\ kg/m^3$

$\rho_{air} = 1.29\ kg/m^3$

$m_{ballon} = 930\ kg$

$m_{cargo} = ?$

$W_{cargo} = ?$

Solution:

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (7.15)^3 = 1531.111\ m^3$

$F = \rho_{air} V g = (1.29)(1531.111)(9.8) = 1.94 \times 10^4\ N = 1.98 \times 10^4\ N$

$m_{He} = \rho_{He} V = (0.179)(1531.111) = 274.1\ kg$

$F = m_{cargo}\, g + m_{balloon}\, g + m_{He}\, g$

$1.94 \times 10^4 = m_{cargo} (9.8) + (930) (9.8) + (274.1) (9.8)$

$\mathbf{m_{cargo} = 775.5\ kg}$

$\mathbf{W_{cargo} = m_{cargo}\, g = 775.5 \times 9.8 = 7.6 \times 10^3}$

Unit #7: Fluid Dynamics

Worked Example 7.1

Calculate the terminal velocity of a raindrop of radius 0.2 cm. (Density of water 1000 kg/m^3 and that of air 1 kg /m^3).

Solution:

Step 1:

r = 0.2 cm

V_t = ?

Step 2:

$$v_t = \frac{2gr^2}{9\eta}(\rho - \sigma)$$

Step 3:

$$v_t = \frac{2\times9.8\times(0.2^{-3})^2\times999}{9\times10^{-3}}$$

V_t = 8.7 m/s

Worked Example 7.2

The volume rate of an air conditioning system to be 3.84×10^{-3} m^3/s. The air is sent through an insulated, round conduit with a diameter of 18 cm. This calculation assumed laminar flow. (a) Was this a good assumption? (b) At what velocity would the flow become turbulent?

Solution:

Step 1:

Volume rate = 3.84×10^{-3} m^3/s.

η of air = 0.0181 m Pa.s

ρ of air = 1.23 kg/m^3

diameter = 18 cm

Velocity =?

Step 2:

Volume rate = Av

$3.84\times10^{-3} = \pi\,(0.09)\,v$

v = 0.15 m/s

Step 3:

$$R_e = \frac{2\rho v r}{\eta}$$

$$R_e = \frac{2\times(1.23)\times0.15\times0.09}{0.0181\times10^{-3}}$$

R_e = 1835

Since the Reynolds number is 1835 < 2000, the flow is laminar and not turbulent. The assumption that the flow was laminar is valid.

Step 4:

To find the maximum speed of the air to keep the flow laminar, consider the Reynolds number

$$R_e = \frac{2\rho\ \ r}{\eta} \leq 2000$$

$$v = \frac{2000(0.0181\times10^{-3})}{2(1.23)(0.09)}$$

$$v = 0.16\ m/s$$

Significance:

When transferring a fluid from one point to another, it desirable to limit turbulence. Turbulence results in wasted energy, as some of the energy intended to move the fluid is dissipated when eddies are formed. In this case, the air conditioning system will become less efficient once the velocity exceeds 0.16 m/s, since this is the point at which turbulence will begin to occur.

Worked Example 7.4

Water leaves the jet of a horizontal hose at 10 m/s, if the velocity of water within the hose is 0.4 m/s, calculate the pressure P within the hose. (Density of water 1000 kg/m^3 and atmospheric pressure is 100000 Pa).

Solution:

Step 1:

$V_1 = 10$ m/s

$V_2 = 0.4$ m/s

P = ?

Density of water = 1000 kg/m^3

Atmospheric Pressure = 100000 Pa

Step 2:

Here $h_1 = h_2$, so Bernoulli's equation becomes

$P_1 + ½ \rho V_1^2 = P_2 + ½ \rho V_2^2$

Step 3:

$100000 + ½ \times 1000 \times 100 = P_2 + ½ \times 1000 \times 0.16$

$P_2 = 1.5 \times 10^5$ Pa

Section (A): Multiple Choice Questions (MCQs)

1. For an incompressible fluid, the flow rate is

a) equal for all surfaces

b) constant throughout the pipe

c) greater for the larger parts of the pipe

d) none of the above

2. Bernoulli's principle states that for horizontal flow of a fluid through a tube, the sum of the pressure and energy of motion per unit volume is

a) increasing with time c) constant

b) decreasing with time d) varying with time

3. Which of the following is associated with the law of conservation of energy in fluids?

a) Archimedes' principle c) Pascal's principle

b) Bernoulli's principle d) equation of continuity

4. As the speed of a moving fluid increases, the pressure in the fluid

a) increases

b) remains constant

c) decreases

d) may increase or decrease, depending on the viscosity

5. If the cross-sectional area of a pipe decreases, what happens to the fluid velocity?

a) Increases

b) Decreases

c) Remains the same

d) Depends on the fluid density

6. A sky diver falls through the air at terminal velocity. The force of air resistance on him is

a) half his weight

b) equal to his weight

c) twice his weight

d) Cannot be determined from the information given.

7. Wind speeding up as it blows over the top of a hill

a) Increases atmospheric pressure there

b) decreases atmospheric pressure there

c) doesn't affect atmospheric pressure there

d) equal's atmospheric pressure.

8. A fluid is undergoing "incompressible" flow. This means that:

a) the pressure at a given point cannot change with time

b) the velocity at a given point cannot change with time

c) the velocity must be the same everywhere

d) the pressure must be the same everywhere

e) the density cannot change with time or location

9. A fluid is undergoing steady flow. Therefore:

a) the velocity of any given molecule of fluid does not change.

b) the pressure does not vary from point to point

c) the velocity at any given point does not vary with time

d) the density does not vary from point to point

10. The equation of continuity for fluid flow can be derived from the conservation of

a) energy c) volume

b) mass d) pressure

KEY:

1. b	2. c	3. b	4. c	5. a
6. b	7. b	8. e	9. c	10. b

CRQs:

1. What is difference between streamline and turbulent flow?

Ans)

Aspect	Streamline Flow	Turbulent Flow
Flow Patterns	Smooth and ordered	Chaotic and irregular
Velocity Profiles	Consistent	Fluctuating
Mixing	Minimal	Significant
Energy Dissipation	Low	High
Friction	Low	High

2. Would a drinking straw work in space where there is no gravity? Explain.

Ans) In a vacuum environment like space where there is very little or no gravitational force, the behavior of fluids is quite different compared to what we experience on Earth. The operation of a drinking straw, which relies on the principle of atmospheric pressure and gravity, would be altered in such a space environment.

On Earth: When you use a drinking straw on Earth, you create a partial vacuum inside the straw by sucking on it. This reduces the air pressure inside the straw. Atmospheric pressure on the outside is then greater than the pressure inside the straw, causing the liquid to be pushed up the straw and into your mouth. Gravity also plays a role by helping the liquid flow downward.

In Vacuum: In vacuum, there is no significant gravitational force to cause the liquid to naturally flow downward. As a result, the liquid wouldn't be pulled up the straw just by creating a vacuum as you do on Earth. Without the influence of gravity, the liquid wouldn't flow upwards in the same way it does on Earth, making the traditional use of a drinking straw ineffective.

3. Why do airplanes take off into wind?

Ans) Airplanes take off into the wind to optimize takeoff performance and safety. This practice increases airspeed over the wings, enabling the generation of more lift. It results in shorter takeoff distances, crucial in airports with limited runways. Taking off into a headwind also enhances control during takeoff and allows for a steeper climb after becoming airborne. Additionally, it improves the efficiency of the takeoff and reduces the risk in case of an aborted takeoff. Overall, this approach maximizes the aircraft's performance and safety during this critical phase of flight.

4. Describe terminal velocity in liquids.

Ans) Terminal velocity in liquids is the constant speed that an object, such as a solid particle or a droplet, reaches when it falls or rises through the liquid under the influence of gravity. Terminal velocity occurs when the force of gravity pulling the object downward is balanced by the resistance or drag force exerted by the surrounding liquid.

Key points about terminal velocity in liquids:

1. **Balanced Forces:** When an object initially enters a liquid and starts moving, it accelerates due to gravity. However, as it accelerates, the drag force acting on the object also increases. Eventually, a point is reached where the two forces, gravity and drag, become equal and opposite. At this point, the object no longer accelerates and settles into a constant velocity called terminal velocity.
2. **Dependent on Object and Liquid Properties:** Terminal velocity depends on various factors, including the size, shape, and mass of the object, as well as the properties of the liquid (density and viscosity). Larger and denser objects typically have higher terminal velocities in a given liquid.
3. **Drag Force:** The drag force acting on the object in a liquid is determined by the object's speed (faster objects experience greater drag), its shape (streamlined shapes reduce drag), and the viscosity of the liquid (more viscous liquids produce greater drag).
4. **Use in Science and Engineering:** Understanding terminal velocity is crucial in fields such as fluid dynamics, sedimentation, and particle separation. It is often used to determine the settling rate of particles in liquids, which has applications in water treatment, sedimentology, and the study of fluid behavior.
5. **No Acceleration:** At terminal velocity, the object is no longer accelerating; it moves at a constant speed. If the object's properties or the liquid conditions change, its terminal velocity can change accordingly.

5. Discuss the significance of Reynolds number.

Ans) The Reynolds number (Re) is a dimensionless parameter in fluid dynamics that plays a critical role in understanding and predicting the behavior of fluid flow. It is named after the British scientist Osborne Reynolds, who pioneered the study of fluid dynamics in the late 19th century. The significance of the Reynolds number lies in its ability to provide insights into the type of flow regime a fluid will exhibit and to predict the behavior of flow in different situations. Here's why the Reynolds number is significant:

1. **Flow Regime Identification:** The Reynolds number helps classify fluid flow into different regimes:

- **Laminar Flow (Low Re):** At low Reynolds numbers, fluid flows smoothly in well-defined layers or streamlines with minimal turbulence. Laminar flow is characterized by orderly, predictable motion.
- **Turbulent Flow (High Re):** At high Reynolds numbers, fluid flow becomes chaotic and unpredictable, with vortices, eddies, and mixing. Turbulent flow is characterized by disorderly motion and increased energy dissipation.
- **Transitional Flow (Intermediate Re):** In between laminar and turbulent flow, there exists a transition zone where flow behavior can switch between laminar and turbulent depending on factors like disturbances and the specific geometry of the flow.

2. **Flow Predictions:** The Reynolds number is used to predict how fluids will behave in different situations. For example:

- It helps determine when a flow will transition from laminar to turbulent, which is crucial for designing pipelines, determining heat transfer in pipes, and predicting drag forces on objects moving through fluids.
- It aids in assessing the pressure drop and frictional losses in pipelines and channels.
- It guides the design of efficient aircraft wings, propellers, and turbines by indicating whether the flow over these surfaces will be laminar or turbulent.

6. State Bernoulli's principle.

Ans) It states that "the velocity of a fluid is high, the pressure is low, and the velocity is low, the pressure is high."

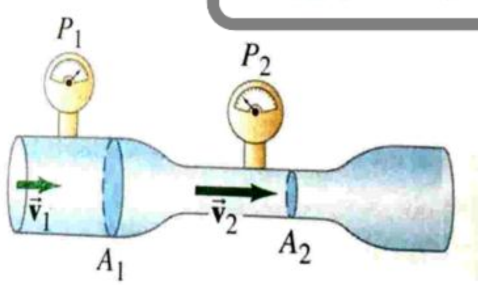
7. Give two applications of Bernoulli's principle.

Ans) **Applications:**

1. **Filter Pump:** In a filter pump, Bernoulli's principle is applied to increase the pressure of the fluid passing through the pump. By reducing the cross- sectional area of the pump's outlet, the fluid's velocity increases according to the principle, leading to a decrease in pressure (as kinetic energy increases). This pressure drop helps draw fluid into the pump and through the filter medium, facilitating the filtration process.

Filter pump

2. **Venturi Meter:** A Venturi meter is a device used to measure the flow rate of a fluid in a pipe. It consists of a gradually narrowing tube (Venturi tube) inserted in the pipe. As the fluid flows through the narrowing section, its velocity increases according to Bernoulli's principle, and the pressure decreases. By measuring the pressure difference between the narrowest section and the wider parts of the pipe, the flow rate can be determined.



Venturi meter

8. "Fluid flow is turbulent rather than laminar", support this statement.

Ans) The statement "Fluid flow is turbulent rather than laminar" can be supported by several observations and considerations:

1. **Reynolds Number:** One of the primary factors determining the flow regime of a fluid is the Reynolds number (Re). When the Reynolds number is high, typically above a critical value (around 2,000 for flow in a pipe), fluid flow tends to be turbulent. In many real-world scenarios, such as fast-moving water in rivers or airflow around aircraft, the Reynolds number is sufficiently high to induce turbulent flow.
2. **Natural Turbulence:** Turbulence is a common and natural occurrence in many fluid systems. For example: In the Earth's atmosphere, turbulence is prevalent and is a significant factor in aviation, causing turbulence during flights.
3. **Practical Applications:** Many engineering systems and industrial processes operate in turbulent flow regimes due to their higher efficiency and mixing capabilities. Examples include:
   - Industrial mixing processes, where turbulent flow enhances the mixing of chemicals and ingredients.
   - Jet engines and gas turbines, where combustion and heat transfer are more efficient in turbulent flows.
   - Water treatment plants and sewage systems, where turbulence aids in the mixing and distribution of chemicals and pollutants.
4. **Laboratory Experiments:** In laboratory settings, researchers often induce turbulence deliberately to study its effects. This is achieved by increasing flow velocities or introducing obstacles into the fluid flow, both of which disrupt laminar flow and promote turbulence.
5. **Common Experience:** In everyday life, we encounter turbulent flow in scenarios like the splashing of water in a fast-moving stream, the swirls and eddies in a cup of stirred coffee, or the gusty winds during a storm. These experiences illustrate the prevalence of turbulent flow in our environment.

9. Discuss importance of Stokes law.

Ans)

1. **Terminal Velocity:** By setting the net force (the drag force minus the gravitational force) equal to zero, you can determine the terminal velocity of a sphere falling or rising through a fluid.
2. **Size of Sphere:** If you know the terminal velocity and the other parameters, you can rearrange Stokes's law to solve for the radius of the sphere.
3. **Density of Sphere or Liquid:** If you have information about the other parameters (such as the terminal velocity and the size of the sphere), Stokes's law can be rearranged to solve for the density of either the sphere or the liquid, depending on which property is unknown.

4. **Viscosity of the Fluid:** Stokes's law can also be used to determine the viscosity (η) of the fluid if you have information about the size and density of the sphere, the terminal velocity, and the other constants. Rearranging the formula can provide an estimate of the fluid's viscosity.

10. Justify spin of ball in Bernoulli's principle.

Ans) The spin of a ball in flight, often seen in sports like baseball and soccer, is justified by the Magnus effect. This effect arises due to the interaction between the spinning ball and the fluid it moves through, adhering to Bernoulli's principle. When a ball spins, it creates a pressure difference on its sides because of the relative airflow caused by rotation. This pressure difference generates a sideways force that makes the ball's path curve. The amount of curve depends on factors like the spin rate, velocity, and fluid properties.

ERQ's:

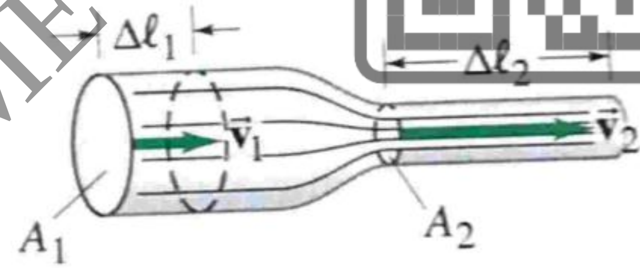
1. Derive equation of continuity. Also show its physical significance.

Ans)

Equation of Continuity: Suppose a steady laminar flow of a fluid through an enclosed tube or pipe as shown in figure the speed of the fluid varies with the diameter of the tube variation. The mass flow rate which follows the law conservation of mass is defined as the mass Δm of fluid that passes at given point per unit time Δt:

$$\text{mass flow rate} = \frac{\Delta m}{\Delta t}$$

In figure shown below,



The volume of a fluid passing through area A_1 in a time Δt is $A_1\Delta l_1$, where Δl_1 is the distance the fluid moves in time Δt. The velocity of fluid passing through A_1 is $V_1 = \Delta l_1/\Delta t$. Then the mass flow rate:

$$\frac{\Delta m_1}{\Delta t} = \frac{\rho_1 \Delta V_1}{\Delta t} = \frac{\rho_1 A_1 \Delta l_1}{\Delta t} = \rho_1 A_1 v_1$$

Where $\Delta V_1 = A_1 \Delta l_1$ is the volume of mass Δm. Similarly, through A_2, the flow rate is $p_2A_2v_2$. Since no fluid flows in or out the sides of the tube, the flow rates through A_1 and A2 must be equal.

$$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t}$$

and

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

This is called the equation of continuity.

If the fluid is incompressible which is an excellent approximation for liquids under most circumstances, then $\rho_1 = \rho_2$, the equation of continuity is

$$A_1 v_1 = A_2 v_2$$

Physical significance: The equation of continuity has important physical significance in fluid dynamics:

1. **Conservation of Mass:** It ensures that mass is conserved within a fluid flow. In other words, the total mass of fluid entering a certain section of a pipe must equal the total mass leaving that section.
2. **Velocity and Area Relationship:** It shows that when the cross-sectional area of a pipe decreases (A2 < A1), the fluid velocity must increase (v2 > v1) to maintain the same mass flow rate. Conversely, when the area increases, the velocity decreases. This relationship is crucial for understanding how fluids behave in pipes, nozzles, and other flow geometries.
3. **Streamline Consistency:** The equation of continuity applies along any streamline within a flow. It ensures that the fluid moves smoothly without any abrupt changes in mass flow rate, which is essential for understanding and predicting fluid behavior in various applications, including engineering and environmental sciences.

2. Derive Bernoulli's equation.

Ans) Bernoulli's equation can be derived from the first principle using the law of conservation of energy. According to energy conservation principles, energy can neither be created nor destroyed. Therefore, during streamline flow, the total mechanical energy remains constant. A few assumptions need to be made before deriving the equation

Assumptions:

- The flow must be steady and streamline.
- The fluid is incompressible the density should remain constant at all points during the flow.
- There are no viscous forces in the fluid, and friction is negligible.

Consider a pipe whose diameter and elevation change as a fluid passes through it. Consider a small mass of the fluid with density ρ that flows from point 1 to 2 as shown in figure below. The work done by a force F on the fluid to displace it by an infinitesimal distance Δx is given by,

$$W = F\Delta x$$

Therefore, at points 1 and 2, the work done are,

$$\Delta W_1 = F_1 \Delta x_1$$
$$\Delta W_2 = F_2 \Delta x_2$$

Total work done when the fluid moves 1 to 2 is,

$$\Delta W = \Delta W_1 - \Delta W_2$$
$$\text{or, } \Delta W = F_1 \Delta x_1 - F_2 \Delta x_2$$

Force is the product of pressure and area

(F = pA), and the volume is the product of length and cross- sectional area

(V = Ax). Therefore,

$$\Delta W = p_1 A_1 \Delta x_1 - p_2 A_2 \Delta x_2 = p_1 \Delta V - p_2 \Delta V = (p_1 - p_2)\Delta V$$

Now, when the fluid moves from point 1 to 2, there is a change of kinetic energy.

$$\Delta K.E = \frac{1}{2} m_2 v_2^2 - \frac{1}{2} m_1 v_1^2 = \frac{1}{2}\rho \Delta V v_2^2 - \frac{1}{2}\rho \Delta V v_1^2 = \frac{1}{2}\rho \Delta V (v_2^2 - v_1^2)$$

Similarly, the change in potential energy is given by,

$$\Delta U = mgh_2 - mgh_1 = \rho \Delta V g(h_2 - h_1)$$

The work done in moving the fluid is the sum of the change in kinetic and potential energies.

$$\Delta W = \Delta K.E + \Delta U$$
$$\text{or, } (p_1 - p_2)\Delta V = \frac{1}{2}\rho \Delta V (v_2^2 - v_1^2) + \rho \Delta V g(h_2 - h_1)$$
$$\text{or, } p_1 - p_2 = \frac{1}{2}\rho (v_2^2 - v_1^2) + \rho g(h_2 - h_1)$$
$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$
$$\text{or, } p + \frac{1}{2}\rho V^2 + \rho g h = \text{constant}$$

This is Bernoulli's equation.

3. Discuss viscous force in fluids.

Ans) Here are some key points regarding viscous force:

1. **Origin of Viscous Force:** Viscous force is primarily the result of interactions between adjacent layers of fluid molecules or particles. As one layer of fluid moves relative to another, these interactions create a resistance force that opposes the motion.
2. **Dependence on Velocity Gradient:** Viscous force is directly proportional to the velocity gradient (du/dy), which represents the rate of change of velocity with respect to distance perpendicular to the direction of flow. The greater the velocity gradient, the higher the viscous force.
3. **Types of Viscous Forces:**
    - **Steady-State Viscous Force:** This occurs when an object moves at a constant velocity through a fluid. In this case, the viscous force opposes the motion and is equal in magnitude but opposite in direction to the applied force.
    - **Transient Viscous Force:** This occurs when an object starts or stops moving through a fluid. Initially, the viscous force is higher, and it gradually decreases as the object reaches a steady-state velocity.
4. **Importance in Fluid Mechanics:** Viscous forces are crucial in fluid mechanics and engineering because they influence the behavior of fluids in various situations. For example:
    - In aerodynamics, viscous drag affects the performance and efficiency of aircraft.
    - In fluid flow through pipes, viscous forces impact the pressure drop and energy losses.
    - In the motion of vehicles, such as cars and ships, understanding and minimizing viscous drag is essential for fuel efficiency.
5. **Reynolds Number (Re):** The Reynolds number is a dimensionless parameter that characterizes the relative importance of viscous forces compared to inertial forces in a fluid flow. It helps determine whether a flow is laminar (dominated by viscous forces) or turbulent (dominated by inertial forces).

4. Define fluid dynamics and explain its significance in the study of fluids. How does it differ from fluid statics?

Ans) **Fluid dynamics:** It is the branch of applied science that is concerned with the movement of liquids and gases.

Significance in the study of fluids:

1. **Description of Fluid Behavior:**
   - **Flow Patterns:** Fluid dynamics explores the different flow patterns that fluids can exhibit, including laminar flow (smooth and orderly), turbulent flow (chaotic and irregular), and transitional flow (a combination of laminar and turbulent).
   - **Boundary Layers:** It examines the formation and properties of boundary layers, which are thin regions near solid surfaces where the fluid's velocity and properties change significantly.
   - **Fluid Properties:** It considers the effects of fluid properties such as viscosity, density, and compressibility on fluid behavior.
2. **Mathematical Modeling:** Fluid dynamics relies on mathematical equations, including the Navier-Stokes equations, Euler's equations, and Bernoulli's principle, to describe and predict fluid behavior. These equations are fundamental tools in the field and are used to model and simulate fluid flows in various scenarios.

Difference between fluid dynamics and fluid statics: Fluid dynamics and fluid statics are two branches of fluid mechanics that focus on different aspects of fluid behavior:

- Fluid dynamics deals with fluids in motion, considering variables like velocity, acceleration, and turbulence. It applies to situations involving fluid flow, such as aerodynamics and hydrodynamics.
- Fluid statics, in contrast, deals with fluids at rest or in equilibrium, focusing on parameters like pressure, density, and buoyancy. It is applied to scenarios like submerged objects and pressure calculations in sealed systems.

5. Discuss the concept of Reynolds number and its significance in fluid dynamics. Explain how Reynolds number relates to the transition between laminar and turbulent flow.

Ans) **Reynolds number:** The Reynolds number is the ratio of inertial forces to viscous forces within a fluid that is subjected to relative internal movement due to different fluid velocities

Mathematically;

$$Re = (\rho * V * L) / \mu$$

Where:

- **Re** is the Reynolds number.
- **ρ** is the density of the fluid.
- **V** is the characteristic velocity of the fluid (usually the velocity of the flow).
- **L** is the characteristic length (typically a linear dimension of the flow channel, such as the diameter of a pipe).
- **μ** is the dynamic viscosity of the fluid.

Significance in fluid dynamics: The Reynolds number is significant in fluid dynamics for several reasons

1. **Flow Regime Prediction:** It predicts the flow regime of a fluid, whether it is laminar, transitional, or turbulent.
2. **Design and Optimization:** Engineers use the Reynolds number to design and optimize fluid systems and components. For example, it helps determine the appropriate size and shape of pipes, channels, and ducts to ensure efficient and cost-effective fluid transport with minimal energy loss.
3. **Flow Stability:** The Reynolds number provides insights into the stability of fluid flows. Understanding the flow regime is crucial for preventing flow separation, pressure drop, and excessive turbulence, which can impact the performance of various systems.
4. **Heat Transfer:** In heat transfer applications, the Reynolds number plays a role in determining whether heat transfer occurs primarily through conduction (in laminar flow) or convection (in turbulent flow), affecting the efficiency of heat exchangers and cooling systems.
5. **Fluid Behavior Analysis:** Researchers and scientists use the Reynolds number to study and model fluid behavior in various applications, including aerodynamics, hydrodynamics, and environmental sciences.
6. **Drag and Lift Prediction:** In aerodynamics, the Reynolds number helps predict the behavior of objects moving through a fluid. It is crucial for understanding drag forces on vehicles and lift forces on aircraft.

Transition between laminar and turbulent flow: The Reynolds number (Re) is a dimensionless parameter that determines whether fluid flow will be laminar, transitional, or turbulent. At low Reynolds numbers (Re < 2,000), viscous forces dominate, leading to smooth, predictable laminar flow. In the transitional range (2,000 < Re < 4,000), flow changes from laminar to turbulent. High Reynolds numbers (Re > 4,000) signify dominant inertial forces, resulting in chaotic, turbulent flow.

Numericals:

1. Two spherical raindrops of equal size are falling through air at a velocity of 0.08 m/s. If the drops join together forming a large spherical drop, what will be the new terminal velocity?

Data:

Velocity of small drops = 0.08 m/s

Velocity of big drop = ?

Solution:

Terminal velocity is directly proportional to the square of the radius of the spherical body.

Since mass is conserved and density remains same, upon joining, the volumes will add up.

Thus, volume of bigger drop = 2 times the volume of each small drop. Since volume is directly proportional to cube of radius, this implies that the radius of the big drop is $\sqrt[3]{2}$ times the radius of each small drop.

Therefore, terminal velocity of the big drop is $\sqrt[3]{2}$ square = $2^{\frac{2}{3}}$ times the terminal velocity of each small drop **= 0.08 x $2^{\frac{2}{3}}$ = 0.13 m/s**

2. Calculate the viscous drag on a drop of oil of 0.1 mm radius falling through air at this terminal velocity. (Viscosity of air = 1.8x10^{-5} Pa.s; density of oil = 850 kg/m³).

Data:

F = ?

r = 0.1 mm = $\frac{0.1}{1000}$ = 0.0001 m

η = 1.8x10^{-5} Pa.s

ρ = 850 kg/m³

Solution:

$v=2r^2 \times (\rho-\rho_o) \times g/9\eta$

$v=2(0.0001)^2 \times (850-0) \times 9.8/9(1.8\times10^{-5})$

v = 1.03 m/s

$F = 6\pi\eta r v$

$F = 6\pi(1.8\times10^{-5})(0.0001)(1.03)$

$\mathbf{F = 3.495 \times 10^{-8}\ N}$

3. What area must a heating duct have if air moving 3.0 m/s along it can replenish the air every 15 minutes in a room of volume 300m^3. Assume air density remains constant.

Data:

$A = ?$

$\Delta v = 3.0\ m/s$

$\Delta t = 15\ mins = 15 \times 60 = 900\ s$

$V = 300 m^3$

Solution:

$\frac{\Delta V}{\Delta t} = A\,v$

$\frac{300}{900} = A\,(3)$

$\mathbf{A = 0.111\ m^2}$

4. Water circulates throughout a house in a hot-water heating system. If the water is pumped at a speed of 0.50 m/s through a 4.0 cm diameter pipe in the basement under a pressure of 3.0 atm, what will be the flow speed and pressure in a 2.6 cm diameter pipe on the second floor 5.0 m above? Assume the pipes do not divide into branches.

Data:

$v_1 = 0.50\ m/s$

$d_1 = 4.0\ cm = \frac{4}{100} = 0.04\ m$

$p_1 = 3.0\ atm = 3 \times 10^5\ Pa$

$v_2 = ?$

$p_2 = ?$

$d_2 = 2.6\ cm = \frac{2.6}{100} = 0.026\ m$

$h_2 = 5\ m$

Solution:

$$A_1 = \frac{\pi}{4} \times d_1^2 = \frac{\pi}{4} \times (0.04)^2 = 1.26 \times 10^{-3}\ m^2$$

$$A_2 = \frac{\pi}{4} \times d_2^2 = \frac{\pi}{4} \times (0.026)^2 = 5.3 \times 10^{-4}\ m^2$$

Since flow rate is same

$$A_1v_1 = A_2v_2$$

$$(1.26 \times 10^{-3})\ (0.5) = (5.3 \times 10^{-4})\ v_2$$

$$\mathbf{v_2 = 1.2\ m/s}$$

$$p_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

$$3 \times 10^5 + \frac{1}{2}(1000)(0.50)^2 + (1000)(9.8)(0) = p_2 + \frac{1}{2}(1000)(1.2)^2 + (1000)(9.8)(5)$$

$$\mathbf{p_2 = 250405\ Pa = \frac{254380}{100000} = 2.5\ atm}$$

5. What is the volume rate of flow of water from a 1.85 cm diameter faucet if the pressure head is 12 m?

Data:

$$d = 1.85\ cm = \frac{1.85}{100} = 0.0185\ m$$

$$h = 12m$$

$$\frac{\Delta V}{\Delta t} = ?$$

Solution:

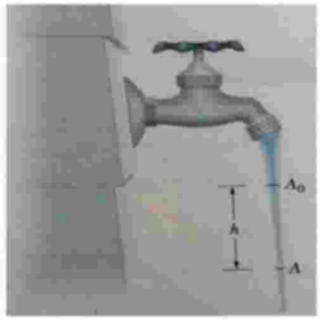
$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 0.0185^2 = 2.7 \times 10^{-4}\ m^2$$

$$v = \sqrt{2gh} = \sqrt{2(9.8)(12)} = 15.34\ m/s$$

$$\frac{\Delta V}{\Delta t} = A\ v$$

$$\mathbf{\frac{\Delta V}{\Delta t} = (2.7 \times 10^{-4})\ (15.34) = 4.142 \times 10^{-3}\ m^3/s}$$

6. The stream of water emerging from a faucet 'neck down' as it falls. The cross-sectional area is 1.2 cm² and 0.35 cm². The two levels are separated by a vertical distance of 45 mm as shown in figure. At what rate does water flow from the tap?



Data:

$A_o = 1.2\ cm^2 = \frac{1.2}{10000} = 0.00012\ m^2$

$A = 0.35\ cm^2 = 0.000035\ m^2$

$h = 45\ mm = \frac{45}{1000} = 0.045\ m$

$\frac{\Delta V}{\Delta t} = ?$

Solution:

At both levels pressure is atmospheric pressure

$p_1 = p_2 = p_{atm}$

$p_1 + ½\ \rho\ v_1^2 + \rho g h_1 = p_2 + ½\ \rho\ v_2^2 + \rho g h_2$

$p_{atm} + ½\ \rho\ v^2 + \rho g(0) = p_{atm} + ½\ \rho\ v_o^2 + \rho g h$

Since flow rate is same

$A_o v_o = Av$

$\frac{A_o v_o}{A} = v$

$½\ \rho\ (\frac{A_o v_o}{A})^2 = ½\ \rho\ v_o^2 + \rho g h$

$½\ (1000)\ (\frac{(0.00012) v_o}{0.000035})^2 = ½\ (1000)\ v_o^2 + (1000)\ (9.8)\ (0.045)$

$5877.551\ v_o^2 = 500\ v_o^2 + 441$

$v_o = 0.29\ m/s$

$\frac{\Delta V}{\Delta t} = A_o v_o$

$\frac{\Delta V}{\Delta t}$ **= (0.00012) (0.29) = 3.48 x 10^{-5} m^3/s = 3.48 x 10^{-5} x 1×10^6 = 34.8 cm^3/s**

7. Water leaves the jet of a horizontal hose at 10 m/s. If the velocity of water within the hose is 0.40 m/s, calculate the pressure within the hose. Density of water is 1000 kg/m³ and atmospheric pressure is 100000 Pa.

Data:

v_2 = 10 m/s

v_1 = 0.40 m/s

p_1 = ?

p_2 = 100000 Pa

ρ = 1000 kg/m^3

Solution:

Assuming both points are at the same level

$h_1 = h_2 = 0$

$p_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$

$p_1 + \frac{1}{2} (1000) (0.4)^2 + \rho g(0) = (100000) + \frac{1}{2} (1000) (10)^2 + \rho g(0)$

$\mathbf{p_1 = 1.5 \times 10^5}$ **Pa**

8. What is the maximum weight of an aircraft with a wing area of 50 m² flying horizontally, is the velocity of the air over the upper surface of the wing is 150 m/s and that the lower surface 140 m/s? Density of air is 1.29 kg/m³.

Data:

A = 50 m^2

v_1 = 140 m/s

v_2 = 150 m/s

ρ = 1.29 kg/m^3

W = ?

Solution:

There is no too much difference between the points therefore we are going to neglect heights

$h_1 = h_2 = 0$

$p_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$

$p_1 + \frac{1}{2} \rho v_1^2 + \rho g(0) = p_2 + \frac{1}{2} \rho v_2^2 + \rho g(0)$

$p_1 + \frac{1}{2} \rho v_1^2 = p_2 + \frac{1}{2} \rho v_2^2$

$p_1 - p_2 = \Delta p = \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2$

$\Delta p = \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2$

$\Delta p = \frac{1}{2} (1.29) (150)^2 - \frac{1}{2} (1.29) (140)^2 = 1870.5 \text{ Pa}$

$F = W = \Delta p A = (1870.5)(50) = 9.35 \times 10^4 \text{ N}$

9. A liquid flows through a pipe with a diameter of 0.50 m at a speed of 4.20 m/s. What is the rate of flow in L/min?

Data:

$d = 0.50 \text{ m}$

$v = 4.20 \text{ m/s}$

$\frac{\Delta V}{\Delta t} = ?$ (in L/min)

Solution:

$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (0.5)^2 = 0.196 \text{ m}^2$

$\frac{\Delta V}{\Delta t} = Av = (0.196)(4.20) = 0.8232 \text{ m}^3/\text{s}$

$\frac{\Delta V}{\Delta t} = 0.8232 \times 1000 \times 60 = 49392 \text{ L/min}$

10. Calculate the average speed of blood flow in the major arteries of the body, which have a total cross-sectional area of about 2.1 cm^2. Use the data of example. (Take $\frac{\Delta V}{\Delta t}$ = 28.56 cm^3/s)

Data:

$A = 2.1\ cm^2$

$\frac{\Delta V}{\Delta t} = 28.56\ cm^3/s$

$v = ?$

Solution:

$\frac{\Delta V}{\Delta t} = Av$

$28.56 = (2.1)\ v$

$v = 13.6\ cm/s$

Unit #8: Electric Fields

Worked Example 8.1

The electron and proton of a hydrogen atom (**figure A**) are separated by approximately $5.3 \times 10^{-11}\ m$. Find the magnitudes of (a) the electric force and (b) the gravitational force between the two particles. (c) What is your conclusion about these forces.

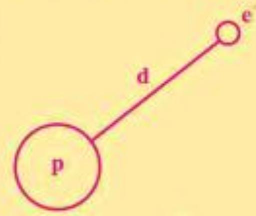


Figure A

Solution:

Step:1 Write down the known quantities and quantities to be found.

$r = 5.3 \times 10^{-11}\ m.$

(a) Charge on electron $= q_e = -1.602 \times 10^{-19} C,$

Charge on proton $= q_p = 1.602 \times 10^{-19} C,$

$k = 8.988 \times 10^9\ N\ m^2/C^2$

Electrostatic force $= F_e = ?$

(b) Mass of electron $= m_e = 9.109 \times 10^{-31}\ kg$

Mass of proton $= m_p = 1.672 \times 10^{-27}\ kg.$

where $G = 6.67 \times 10^{-11}\ N\ m^2/kg^2,$

Gravitational force $= F_g = ?$

Step:2 Write down the formula and rearrange if necessary.

(a) For electrostatic force $F_e = k\frac{q_e q_p}{r^2}$

(b) For gravitational force $F_g = G\frac{m_e m_p}{r^2}$

Step:3 Put the values in formula and calculate.

(a) $F_e = 8.988 \times 10^9\ N\ m^2/C^2 \times \frac{-1.602\times 10^{-19} C \times 1.602\times 10^{-19} C}{(5.3 \times 10^{-11}\ m)^2}$

$$F_e = -8.2 \times 10^{-8} N$$

where negative sign indicates the force is attractive. The magnitude of this electrostatic force is $8.2 \times 10^{-8} N$.

(b) $F_g = 6.67 \times 10^{-11}\ N\ m^2/kg^2\ \frac{9.109\times 10^{-31}\ kg \times 1.672\times 10^{-27}\ kg}{(5.3 \times 10^{-11}\ m)^2}$

$$F_g = 3.6 \times 10^{-47} N$$

Hence, the gravitational force of attraction between the particles is $3.6 \times 10^{-47} N$

(c) Conclusion:

The gravitational force between electron and proton is negligible as compared to the electric force, which implies that electric force is strong force.

Worked Example 8.2

Two positive charge of equal magnitude are placed of in vacuum at distance of 50cm and repel each other with the electric force of 0.1N. **(a)** Find the value of the charge. **(b)** Calculate the force between these two charges if they are places in an insulating liquid whose permittivity is five times that of a vacuum.

Solution:

Step:1 Write down the known quantities and quantities to be found.

$r = 0.5m$

$F = 0.1N$

$\varepsilon = 5\varepsilon_0$

(a) $q_1 = q_2 = q = ?$

(b) $F_{liquid} = ?$

Step:2 Write down the formula and rearrange if necessary.

(a) $F = k\frac{q_1 q_2}{r^2} \Rightarrow q^2 = \frac{F \times r^2}{k}$

(b) $F_{liquid} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} = \frac{1}{5 \times 4\pi\varepsilon_0}\frac{q^2}{r^2}$

$F_{liquid} = \frac{1}{5}F$

Step:3 Put the values in formula and calculate.

(a) $q^2 = \frac{0.1 \times (0.5)^2}{8.988 \times 10^9}$

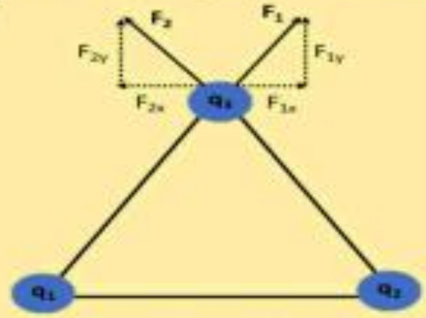
$q = 1.7 \times 10^{-7}C = 0.17\mu C$

(b) $F_{liquid} = \frac{0.1N}{5} = 0.02N$

Hence, the magnitude of electric force for an insulating liquid is $0.02N$.

Worked Example 8.3

Three charges $q_1 = +2\,\mu C$, $q_2 = +3\,\mu C$, and $q_3 = +4\,\mu C$ are placed in air at the vertices of an equilateral triangle of sides $10\ cm$ (see figure). Calculate the magnitude of the resultant force acting on the charge q_3?



Solution:

Step:1 Write down the known quantities and quantities to be found.

$q_1 = +2\,\mu C,\ q_2 = +3\,\mu C,\ q_3 = +4\,\mu C,\ r = 0.1\,m$ and $\theta = 60^0$

Magnitude of the resultant force acting on charge $q_3 = F_{resultant} = ?$

Step:2 Write down the formula and rearrange if necessary.

Force between the charge q_1 and q_3 $F_1 = k\frac{q_1 q_3}{r^2}$

Force between the charge q_2 and q_3 $F_2 = k\frac{q_2 q_3}{r^2}$

Now, resolving the forces F_1 and F_2 into their components and the resultant value of x and y-components (see figure 8.6)

$$F_x = F_1\cos\theta - F_2\cos\theta$$
$$F_y = F_1\sin\theta + F_2\sin\theta$$

The magnitude of the resultant force acting:

$$F = \sqrt{F_x^{\,2} + F_y^{\,2}} = \sqrt{F_1^{\,2} + F_2^{\,2} + 2F_1F_2(\sin^2\theta - \cos^2\theta)}$$

Step:3 Put the values in formula and calculate.

To find the force between the charge q_1 and q_3

$$F_1 = 8.988\times 10^9\ N\,m^2/C^2 \times \frac{2\times 10^{-6}C \times 4\times 10^{-6}C}{(0.1\,m)^2}$$

$$F_1 = 7.2\,N$$

Similarly, the force between the charge q_2 and q_3

$$F_2 = 8.988\times 10^9\ N\,m^2/C^2 \times \frac{3\times 10^{-6}C \times 4\times 10^{-6}C}{(0.1\,m)^2}$$

$$F_2 = 10.8\,N$$

The resultant force.

$$F = \sqrt{(7.2)^2 + (10.8)^2 + 2(7.2)(10.8)\{(\sin 60^0)^2 - (\cos 60^0)^2\}}$$
$$F = 15.69\,N$$

Hence, the magnitude of the resultant force acting on the charge q_3 is 15.69N.

Worked Example 8.4

Find the intensity of electric field at a point such that a proton placed at it would experience a force equal to its weight.

Solution:

Step:1 Write down the known quantities and quantities to be found.

Mass of proton $= m_p = 1.672 \times 10^{-27} kg$,

Charge on proton$= q_p = 1.602 \times 10^{-19} C$ and $g = 9.8\ ^{m}/_{s^2}$

Step:2 Write down the formula and rearrange if necessary.

$$\vec{E} = \frac{F}{q_p}$$

as the electric force equal to its weigh, *i.e.* $F = W = mg$,

$$\vec{E} = \frac{mg}{q_p}$$

Step:3 Put the values in formula and calculate.

$$\vec{E} = \frac{(1.672 \times 10^{-27}\ kg) \times 9.8\ ^{m}/_{s^2}}{1.602 \times 10^{-19} C}$$

$$\vec{E} = 10.22 \times 10^{-8}\ N/C$$

Worked Example 8.5

a) Calculate the magnitude of electric field strength, if a test charge $q_0 = +3.5\ \mu C$ experience a force of $70\ mN$ (filed lines of electric filed shown in figure A).

b) if this test charge is replaced by an electron, then calculate force on an electron and state the direction of force.

Figure A

Solution:

Step:1 Write down the known quantities and quantities to be found.

$F = 70\ mN = 70 \times 10^{-3} N$, $q_0 = 3.5\ \mu C = 3.5 \times 10^{-6} C$

Charge on electron$= q_e = 1.602 \times 10^{-19} C$

(a) $E = ?$

(b) F=?

Step:2 Write down the formula and rearrange if necessary.

$E = \frac{F}{q_0}$ and $F = Eq_e$

Step:3 Put the values in formula and calculate.

(a) $E = \frac{70 \times 10^{-3} N}{3.5 \times 10^{-6} C} = 2.0 \times 10^4\ N/C$

(b) $F = 2.0 \times 10^4\ N/C \times 1.602 \times 10^{-19} C$

$F = 3.204 \times 10^{-15} N$

The magnitude of the force experienced by an electron is $3.204 \times 10^{-15} N$.

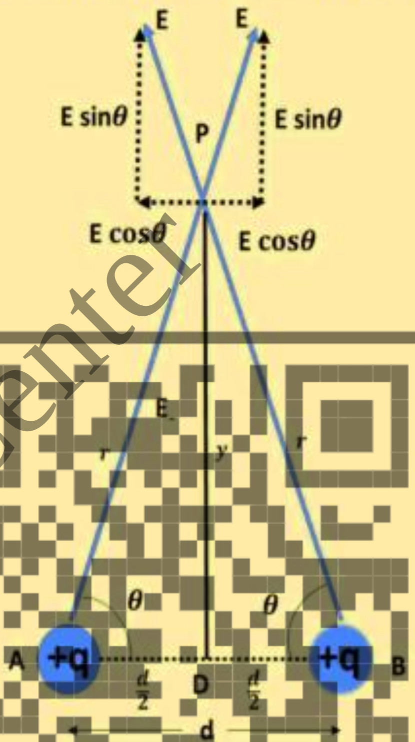
The direction of the force on an electron is directly upwards because field lines is directly downwards and the charge on an electron is negative.

Worked example 8.6

Calculate the electric field intensity at point P of two positive charges of same magnitude which are separated by small distance "d" as given in figure.

Solution:

Resolving the field vectors into the components. From figure, it clear that the horizontal components cancel each other. Therefore, the net electric is due to the vector sum of the vertical components.



$$E_{total} = E\,sin\theta + E sin\theta$$

$$E_{total} = 2E\,sin\theta$$

where $sin\theta = \frac{y}{r}$ and using equation (8.7), we get

$$E = 2 \times \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right)\frac{y}{r}$$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{qy}{r^3}$$

where $r = \sqrt{y^2 + \left({}^{d}/_{2}\right)^2}$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{qy}{\left(y^2 + \left({}^{d}/_{2}\right)^2\right)^{3/2}}$$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{qy}{y^3\left(1 + \left({}^{d}/_{2y}\right)^2\right)^{3/2}}$$

As $d \ll y$, therefore approximate electric field intensity due to two positive charges can be calculated by neglecting the term $\left({}^{d}/_{2y}\right)^2$.

$$E = \frac{1}{2\pi\varepsilon_0}\frac{q}{y^2}$$

Worked Example 8.7

An alpha particle $(+2e)$ in a nuclear accelerator moves from one terminal at the potential of $6.5\ X\ 10^6\ V$ to another terminal at zero potential. What is the corresponding change in the potential energy of the system?

Solution:

Step:1 Write down the known quantities and quantities to be found.

Charge on alpha particle $q = 2e = 2 \times 1.6 \times 10^{-19}\ C$

The electric potential at one terminal $= V_1 = 6.5\ X\ 10^6\ V$

The electric potential at other terminal $= V_2 = 0\ V$

$$\Delta U = ?$$

Step:2 Write down the formula and rearrange if necessary.

From equation (8.11), we have $V_1 = \frac{U_1}{q}$, $V_2 = \frac{U_2}{q}$,

Therefore, the change in potential energy

$$\Delta U = q(V_2 - V_1)$$

Step:3 Put the values in formula and calculate.

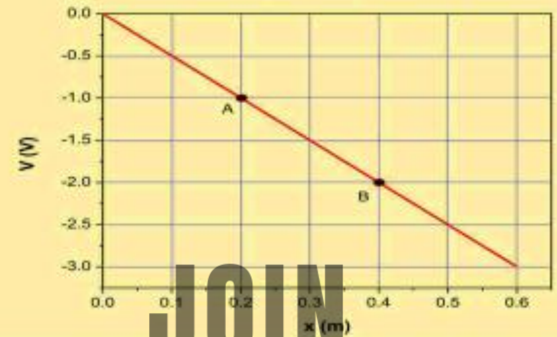
$$\Delta U = 2 \times 1.6 \times 10^{-19}\ C(0 - 6.5 \times 10^6\ V)$$

$$\Delta U = -2.1 \times 10^{-12} J$$

Hence, the potential energy of the system is $2.1 \times 10^{-12} J$.

Worked Example 8.9

An electron is placed in an *xy* plane where the electric potential depends on *x* and *y* as shown in figure. The scale of the vertical axis is set by one square box equal to 500V. Find the electric field (both cases) acting on the electron between two points as shown in the graph?



$$\frac{GmM_e}{r_1 r_2}(r_2 - r_1) = GmM_e\left(\frac{r_2 - r_1}{r_1 r_2}\right)$$

Solution:

Step:1 Write down the known quantities and quantities to be found.

From the graph (a) of the given figure

$$\Delta V = -1000 - (-500) = -500\ V$$
$$\Delta x = 0.4 - 0.2 = 0.2\ m$$

From the graph (b)

$$\Delta V = 750 - 250 = 500\ V$$
$$\Delta x = 0.6 - 0.2 = 0.4\ m$$

Step:2 Write down the formula and rearrange if necessary.

$E_x = -\frac{\Delta V}{\Delta x}$ and $E_y = -\frac{\Delta V}{\Delta y}$

Step:3 Put the values in formula and calculate.

$$E_x = -\left(\frac{-500\ V}{0.2\ m}\right) = 2500\ V/m$$

Similarly, from the graph (b)

$$E_y = -\frac{\Delta V}{\Delta y} = -\left(\frac{500\ V}{0.4\ m}\right) = -1250\ V/m$$

Hence, the value of electric field acting on the electron in both cases are 2500 V/m and -1250 V/m.

Worked Example 8.8

For what value of change in electric potential to remove the six electrons from the carbon atom if the change in potential energy is 120 keV.

Solution:

Step:1 Write down the known quantities and quantities to be found.

$q = 6e = 6 \times 1.6 \times 10^{-19}\ C$

$\Delta U = 120\ keV = 120 \times 1.6 \times 10^{-19}\ kV$

Step:2 Write down the formula and rearrange if necessary.

$$\Delta V = \frac{\Delta U}{q}$$

Step:3 Put the values in formula and calculate.

$$\Delta V = \frac{120 \times 1.6 \times 10^{-19}\ kV}{6 \times 1.6 \times 10^{-19}\ C}$$

$$\Delta V = 20\ kV$$

Hence, the value of change in electric potential is $20\ kV$.

Section (A): Multiple Choice Questions (MCQs)

1. A 2μC point charge is located a distance "d" away from 6μC point charge, what is the ratio of F_{12}/F_{21}?

(a) 1/3 (b) 3

(c) 1 (d) 12

2. The minimum charge on an object cannot be less than:

(a) 1.6×10^{-19} C (b) 3.2×10^{-19} C

(c) 9.1×10^{9} (d) No definite value exist

3. Two charges are placed at a certain distance. If the magnitude of each charge is doubled the force will become

(a) 1/4th of its original value (b) 4 times of its original value

(c) 1/8th of its original value (d) 8 times of its original value

4. Which of the following can be deflected while moving in the electric field?

(a) neutron (b) photon

(c) electron (d) (a) and (b)

5. The flux through a flat surface of area “A” in a uniform electric field "E" is maximum when the surface area is

(a) Parallel to E (b) perpendicular to E

(c) placed 45° to E (d) placed 60° to E

6. The product of charge "q" and small separation "d" between two charges of same magnitude and opposite in nature is known as:

(a) Electric dipole (b) Moment arm

(c) Electric dipole moment (d) Flux of electric field

7. 12 J of work is to be done against an existence electric field to take a charge of 0.01 C from one-point A to another point B. The potential difference between B and A is

(a) 120 V (b) 1200 V

(c) 1.2 V (d) 12 V

8. The force between two charges placed in air is F, if air is replaced by a medium of relative permittivity ε_r then force is reduced to:

(a) F ε_r (b) F/ε_r

(c) ε_r/F (d) ε ε_r

9. The negative gradient of the potential is:

(a) potential energy (b) voltage

(c) electric field intensity (d) electric flux

10. The electric flux through a plane area will be half of its maximum value when area is held at angle of ______ with electric field.

(a) 30° (b) 60°

(c) 45° (d) electric flux

KEY:

1. c	2. a	3. b	4. d	5. a
6. a	7. b	8. b	9. c	10. b

Section (B): Structured Questions

CRQs:

1. Why do most objects tend to contain nearly equal numbers of positive and negative charges?

Ans) Objects tend to contain roughly equal numbers of positive and negative charges due to the conservation of charge and the electrostatic forces between charged particles. This electrical neutrality is a result of protons and electrons in atoms. Objects aim to maintain this balance because it minimizes the creation of strong electric fields and prevents disruptive electrical interference. When objects gain excess positive or negative charges, they can redistribute them through contact or induction until neutrality is restored. This principle of charge conservation ensures that the total charge in a closed system remains constant. Overall, maintaining nearly equal positive and negative charges is essential for stable electrical behavior in our surroundings.

2. When measuring an electric field, could we use a negative rather than a positive test charge?

Ans) When measuring an electric field, it's entirely acceptable to use a negative test charge instead of a positive one. Both positive and negative test charges can be employed to gauge the electric field's characteristics. A positive test charge experiences a force in the direction of the field lines, while a negative test charge experiences a force in the opposite direction when the field originates from a positive source charge. Employing both types of test charges helps confirm the field's strength, direction, and symmetry, particularly when assessing fields generated by stationary point charges.

3. During fair weather, the electric field due to the net charge on Earth points downward. Is Earth charged positively or negatively?

Ans) During fair weather, the electric field near the Earth's surface points downward, indicating that the Earth is negatively charged in this context. This direction of the electric field signifies that a positive test charge placed at the Earth's surface would experience a downward force. The Earth's surface is often considered to be at a potential of zero in electrostatics, and any charge in its vicinity affects the electric field.

4. How the electric flux through a closed surface is independent on the size or shape of the surface enclosed the charge.

Ans) The electric flux through a closed surface, as described by Gauss's Law, is entirely independent of the size or shape of the enclosing surface. It depends solely on the total electric charge enclosed within that surface. The law states that electric flux (Φ_E) equals the enclosed charge (Q_{enc}) divided by the electric constant (ε_o), with no regard for the geometry of the surface. This principle holds true for surfaces of varying shapes or sizes, as long as they are closed. In essence, the law emphasizes that the charge within the closed surface dictates the electric flux, making it a powerful tool in analyzing electric fields in various contexts, especially those with symmetric field distributions.

5. What is electric dipole and electric dipole moment?

Ans) Electric dipole: Electric dipole is a simple system in electromagnetism consisting of two opposite electric charges of equal magnitude, separated by a small distance d.

Electric dipole moment: The electric dipole moment is the product of the magnitude of the charge and the distance between the centres of positive and negative charges.

$$\mathbf{p} = Q \times r$$

Here, Q is charge and r is the distance.

6. A charged particle is seen to be moving in an electric field along a straight line. How it effects the path of motion of the particle?

Ans) When a charged particle moves along a straight line in an electric field, its path is significantly affected by the field's direction and the particle's charge. If the particle is positively charged, it accelerates in the same direction as the electric field, while a negatively charged particle accelerates in the opposite direction. The magnitude of the force and acceleration depends on the charge, electric field strength, and particle mass. A stronger electric field or a higher charge results in greater force and acceleration. In essence, the electric field exerts a force that influences both the direction and speed of the particle's motion along the straight line within the field.

7. An electron has a speed of 10^6 m/s. Find its energy in electron volt.

Data:

$v = 10^6$ m/s

K.E (in eV) = ?

Solution:

$K.E = \frac{1}{2} mv^2$

Since, $m = 9.11 \times 10^{-31}$ kg

$K.E = \frac{1}{2} mv^2$

$K.E = \frac{1}{2} (9.11 \times 10^{-31}) (10^6)^2$

$K.E = 4.555 \times 10^{-19}$ J

$K.E = 4.555 \times 10^{-19} \times 6.242 \times 10^{18}$

K.E = 2.84 eV

ERQs:

1. Define electric charge. Discuss the fundamental properties of electric charge, including the principles of conservation of charge and quantization of charge.

Ans) Electric charge: Electric charge can be defined as a fundamental property of subatomic particles that gives rise to the phenomenon of experiencing force in the presence of electric and magnetic fields.

Fundamental properties of electric charge: There are several fundamental properties of electric charge, including the principles of conservation of charge and quantization of charge:

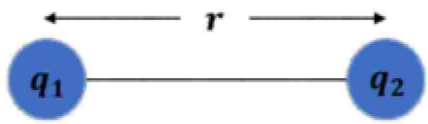
1. **Conservation of Charge**: The principle of conservation of charge states that the total electric charge in a closed system remains constant over time. In other words, electric charge cannot be created or destroyed; it can only be transferred from one object to another.
2. **Quantization of Charge:**
   - Electric charge is quantized, meaning it exists in discrete, indivisible units. The smallest unit of electric charge is the charge of an electron (e) or the charge of a proton, which are equal in magnitude but opposite in sign.
   - The charge of an electron is approximately -1.602×10^{-19} coulombs (C), and the charge of a proton is $+1.602 \times 10^{-19}$ C. These values represent the elementary charges, and all other charges are multiples of this fundamental charge.
   - Mathematically, the charge (Q) of an object can be expressed as $Q = ne$, where n is an integer (positive or negative) representing the number of elementary charges.
3. **Two Types of Charge**: There are two types of electric charge: positive and negative. Like charges (positive-positive or negative-negative) repel each other, while opposite charges (positive-negative) attract each other.
4. **Conservation in Electromagnetic Interactions:** In electromagnetic interactions, such as those in electrical circuits or between charged particles, the total charge before and after the interaction remains the same. This conservation of charge ensures that electrical circuits remain balanced and that charge is neither created nor lost during interactions.
5. Charge Transfer and Redistribution: Objects can acquire electric charge through various processes, including friction, conduction, and induction. During these processes, electrons can be transferred from one object to another, leading to changes in the charge distribution and resulting in electrical phenomena.

2. State and explain Coulombs law. Apply it to calculate the electric field due to an isolated point charge.

Coulombs law:

Statement: The magnitude of the electrostatic force of attraction or repulsion between two point charges is directly proportional to the product of the magnitudes of charges and inversely proportional to the square of the distance between them.

Explanation: The illustration for two charges (q_1,q_2) and separation (r) between the charges is shown in figure below.



The electrostatic force F is directly proportional to the product of magnitudes of the charges q_1 and q_2. and inversely proportional to the square of the distance r between the charges and

$$\vec{F} \propto q_1 q_2$$

$$\vec{F} \propto \frac{1}{r^2}$$

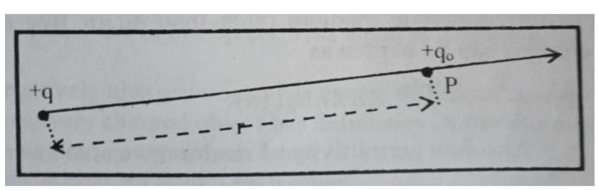
$$\vec{F} = k\frac{q_1 q_2}{r^2}$$

Where k is known as the Coulomb's constant and

$$k = \frac{1}{4\pi\varepsilon_0}.$$

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

Electric field due to an isolated point charge: Consider a test charge q_o is placed at a point of an electric field of source charge "+q" in presence of air of free space between them. Also suppose that the distance between +q and q_o is "r" as shown



According to the Coulomb's law

$$F = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}$$

Where $q_1 = q$ and $q_2 = q_o$

Therefore, $F = \frac{1}{4\pi\varepsilon_o}\frac{q q_o}{r^2}$

Or $\frac{F}{q_o} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$

From definition of intensity

$$E = \frac{F}{q_o}$$

$$\mathbf{E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}}$$

3. Define electric flux and explain its significance in the study of electromagnetism. How does it relate to electric fields and charged particles?

Ans) Electric flux: The total number of electric field lines passing a given area in a unit time is defined as the electric flux.

Significance in the study of electromagnetism: Here's why electric flux is important in electromagnetism

1. **Measuring Electric Field**: Electric flux helps us measure the strength of electric fields passing through a given area or surface. It quantifies how many electric field lines pass through a specific area and is a crucial tool for characterizing electric field distributions.
2. **Gauss's Law**: Electric flux is a key component of Gauss's Law, one of the four fundamental Maxwell's equations that describe electromagnetism. Gauss's Law relates the electric flux through a closed surface to the charge enclosed within that surface. It provides a convenient way to calculate electric fields in symmetric situations, simplifying complex problems.
3. **Charge Distribution Analysis**: Electric flux aids in analyzing charge distributions. By considering the electric flux through surfaces surrounding charged objects or distributions, you can determine the total charge enclosed within those surfaces. This is particularly useful in understanding the behavior of conductors and insulators.
4. **Applications in Engineering**: Electric flux calculations have practical applications in engineering, such as designing electrical circuits, antennas, and other electronic devices. Engineers use electric flux to optimize the performance of these systems by ensuring the proper distribution of electric fields.

5. **Electrostatic Shielding**: Electric flux analysis is instrumental in understanding and designing electrostatic shields, such as Faraday cages.

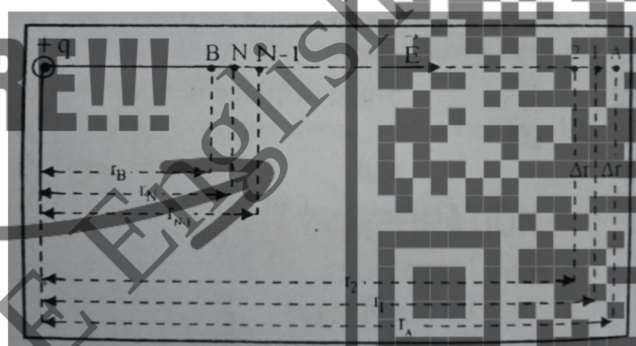
Relates to electric fields and charged particles: Electric flux is a measure of the flow of electric field lines through a specified area or surface. It quantifies the strength and direction of electric fields, with denser field lines indicating a stronger field. It plays a crucial role in relating electric fields and charged particles, especially through Gauss's Law, which links electric flux to the total charge enclosed by a closed surface. Electric flux helps visualize and analyze electric field behavior around charged objects, aiding in the design of electrical systems and understanding the properties of conductors and insulators in electromagnetism.

4. What is electric potential? Drive expression for potential due to an isolated point charge.

Ans) Electric potential: Electric potential is defined as the amount of work needed to move a unit charge from a reference point to a specific point against an electric field.

Derivation of expression for potential due to an isolated point charge:

Consider two points A and B in a straight line at distances r_A and r_B respectively from an isolated point charge +q as shown in the figure.



In order to determine the electric potential at B, a test charge q_o is moved from A to B. For this purpose some work has to be performed on q_o. Since electric intensity does not remain constant through A to B, so we divide the whole distance into a large number of small and equal distances r_1, r_2, ------ r_N, such that electric field which is the geometric mean over any surface assumed to be constant at all surfaces.

First, we determine the work done in moving charge q_o from point A to point 1

$\Delta W = Fd\cos\theta$

Here, $F = q_oE$

$d = \Delta r$

$\theta = 180^o$

$\Delta W_{A \to 1} = q_o E \Delta r \cos 180^o$

$\Delta W_{A \to 1} = -q_o E \Delta r$

$\Delta W_{A \to 1}/q_o = -E \Delta r$

Since, $\Delta W_{A \to 1}/q_o = \Delta V_{A \to 1}$

$\Delta V_{A \to 1} = -E \Delta r$

Electric field intensity due to an isolated point charge is given by

$E = \frac{Kq}{r^2}$

$\Delta V_{A \to 1} = -\, Kq \frac{(r_A - r_1)}{r^2}$

Where 'r' is the geometric of q_o when moved from point A to point 1

$r = \sqrt{r_A r_B}$

$r^2 = r_A r_B$

$\Delta V_{A \to 1} = -\, Kq \frac{(r_A - r_1)}{r_A r_1}$

$\Delta V_{A \to 1} = -\, Kq \left(\frac{r_A}{r_A r_1} - \frac{r_1}{r_A r_1}\right)$

$\Delta V_{A \to 1} = -\, Kq \left(\frac{1}{r_1} - \frac{1}{r_A}\right)$

Similarly $\Delta V_{1 \to 2} = -\, Kq \left(\frac{1}{r_2} - \frac{1}{r_1}\right)$

$\Delta V_{N \to B} = -\, Kq \left(\frac{1}{r_B} - \frac{1}{r_N}\right)$

Total electric potential from A to B is the electric sum of all electric potentials.

$\Delta V_{A \to B} = \Delta V_{A \to B} + \Delta V_{A \to B} + \text{-----------} + \Delta V_{N \to B}$

$\Delta V_{A \to B} = -\, Kq \left(\frac{1}{r_1} - \frac{1}{r_A}\right) - Kq \left(\frac{1}{r_2} - \frac{1}{r_1}\right) - \text{--------------} - Kq \left(\frac{1}{r_B} - \frac{1}{r_N}\right)$

$\Delta V_{A \to B} = -\, Kq \left(\frac{1}{r_1} - \frac{1}{r_A} + \frac{1}{r_2} - \frac{1}{r_1} + \text{--------------} + \frac{1}{r_B} - \frac{1}{r_N}\right)$

$$\Delta V_{A\to B} = -Kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

$$V_B - V_A = -Kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

5. Define an electric dipole. Derive formula for the electric field due to an electric dipole at a point "P" placed on its axial line.

Ans) Electric dipole: Electric dipole is a simple system in electromagnetism consisting of two opposite electric charges of equal magnitude, separated by a small distance d.

Electric field due to an electric dipole at a point on its axial line: AB is an electric dipole of two point charges −q and +q separated by small distance 2d. P is a point along the axial line of the dipole at a distance r from the midpoint O of the electric dipole.

The electric field at the point P due to +q placed at B is,

$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r-d)^2} \quad \text{(along BP)}$$

The electric field at the point P due to −q placed at A is,

$$E_2 = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r+d)^2} \quad \text{(along PA)}$$

Therefore, the magnitude of resultant electric field (E) acts in the direction of the vector with a greater, magnitude. The resultant electric field at P is

$$E = E_1 + (-E_2)$$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r-d)^2} - \frac{1}{4\pi\varepsilon_o}\frac{q}{(r+d)^2} \quad \text{(along BP)}$$

$$E = \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{(r-d)^2} - \frac{1}{(r+d)^2}\right] \quad \text{(along BP)}$$

$$E = \frac{q}{4\pi\varepsilon_o}\left[\frac{4rd}{(r^2-d^2)^2}\right] \quad \text{(along BP)}$$

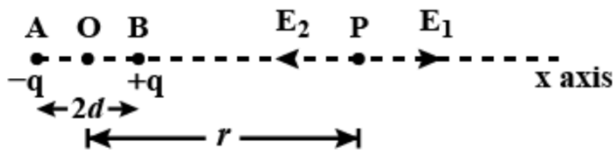
If the point P is far away from the dipole, then $d \ll r$

$$E = \frac{q}{4\pi\varepsilon_o}\frac{4rd}{r^4}$$

$$E = \frac{q}{4\pi\varepsilon_o}\frac{4d}{r^3}$$

Since, Electric dipole moment p = q × 2d

$$E = \frac{1}{4\pi\varepsilon_o}\frac{2p}{r^3} \quad \text{along BP}$$



Electric field at a Point on the axial line

Numericals:

1. (a) Calculate the value of two equal charges if they repel one another with a force of 0.1 N when situated 50 cm apart in a vacuum.

(b) What would be the size of the charges if they were situated in an insulating liquid whose permittivity was ten times that of a vacuum?

Data:

(a) F = 0.1 N, r = 50 cm = 0.5 m, $q_1 = q_2 = q$

(b) q' = ?, $\varepsilon = 10\varepsilon_o$

Solution:

(a)

$$F = k\frac{q_1 q_2}{r^2}$$

$$0.1 = 8.988 \times 10^9 \times \frac{q^2}{0.5^2}$$

q = 1.7 μC

(b)

$$F = k\frac{q_1 q_2}{r^2}$$

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi(10\varepsilon_o)}\frac{q_1 q_2}{r^2}$$

$$F = \frac{1}{10} \times \frac{1}{4\pi\varepsilon_o} \times \frac{q'^2}{r^2}$$

$F = \frac{1}{10} \times k \times \frac{q'^2}{r^2}$

$0.1 = \frac{1}{10} \times 8.988 \times 10^9 \times \frac{q'^2}{(0.5)^2}$

q' = 5.3 μC

2. How far apart must two protons be if the magnitude of the electrostatic force acting on either one due to the other is equal to the magnitude of the gravitational force on a proton at Earth's surface? (mass of proton = 1. 67 x 10^{-27} kg, charge of proton = 1.6 x 10^{-19} C)

Data:

r = ?

$F_g = F_e$

q = 1.6 x 10^{-19} C

m = 1. 67 x 10^{-27} kg

Solution:

$F_g = F_e$

$mg = kq^2/r^2$

$(1.67 \times 10^{-27})(9.8) = 8.988 \times 10^9 (1.6 \times 10^{-19})^2 / r^2$

r = 0.119 m

3. An electron of charge 1.6 x 10^{-19} C is situated in a uniform electric field of intensity 1200-volt cm^{-1}. Find the force on it, its acceleration, and the time it takes to travel 2 cm from rest (electronic mass, m = 9.1 x 10^{-31} kg).

Data:

q = 1.6 x 10^{-19} C

E = 1200-volt cm^{-1} = 120000 V/m

F = ?

a = ?

t = ?

s = 2cm = 0.02 m

$v_i = 0$

$m = 9.1 \times 10^{-31}$ kg

Solution:

$E = \frac{F}{q}$

$120000 = \frac{F}{1.6 \times 10^{-19}}$

$\mathbf{F = 1.92 \times 10^{-14}\ N}$

$F = ma$

$1.92 \times 10^{-14} = (9.1 \times 10^{-31})\, a$

$\mathbf{a = 2.11 \times 10^{16}\ m/s^2}$

$s = v_i t + \frac{1}{2} a t^2$

$0.02 = (0)\, t + \frac{1}{2} (2.11 \times 10^{16})\, t^2$

$\mathbf{t = 1.37 \times 10^{-9}\ s}$

4. An alpha particle (the nucleus of a helium atom) has a mass of 6.64×10^{-27} kg and a charge of 2e. What are the (a) magnitude and (b) direction of the electric field that will balance the gravitational force on the particle?

Data:

$m = 6.64 \times 10^{-27}$ kg

$q = 2e = 2(1.6 \times 10^{-19}) = 3.2 \times 10^{-19}$ C

(a) E = ?

(b) Direction of E = ?

Solution:

(a)

$F_g = mg = (6.64 \times 10^{-27})\,(9.8)$

$F_g = 6.5 \times 10^{-26}$ N

$E = \frac{F_g}{q}$

$E = \frac{6.5 \times 10^{-26}}{3.2 \times 10^{-19}}$

$\mathbf{E = 2.03 \times 10^{-7}\ N/C}$

(b)

Since the force of gravity is downward, then the force due to the electric field must point upward. Since q > 0 in this situation, this implies the electric field must itself point upward.

5. A proton and an electron form two corners of an equilateral triangle of side length 2.0X 10^{-6} m. What is the magnitude of the net electric field these two particles produce at the third corner?

Data:

$r = 2.0 \times 10^{-6}$ m

$E_{net} = ?$

Solution:

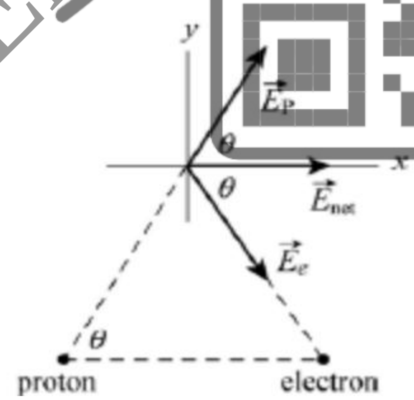
From the figure below we see that,

$$E_e = E_p = \frac{e}{4\pi\varepsilon_o r^2}$$

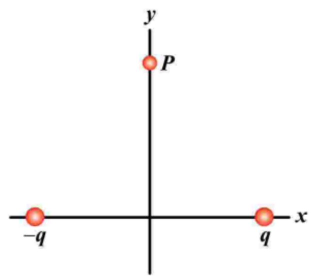
We note that the components along the y axis cancel during the vector summation. With $\theta = 60^o$, the magnitude of the net electric field is obtained as follows:

$$E_{net} = E_x = 2E_e \cos\theta = 2\left(\frac{e}{4\pi\varepsilon_o r^2}\right)\cos\theta$$

$$\textbf{Enet} = 2 \times \frac{1.6\times10^{-19}}{4\pi(8.854\times10^{-12})(2.0\times10^{-6})^2} \times \cos 60 = 3.6 \times 10^2 \text{ N/C}$$



6. Figure shows two charged particles on an x axis: $-q=-3.20\times10^{-19}$ C at x = -3.00 m and $q = 3.2 \times 10^{-19}$ C at x = 3.00 m. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the net electric field produced at point P at y = 4.00 m?



Data:

$-q=-3.20\times10^{-19}$ C

x = -3.00 m

$q = 3.2 \times 10^{-19}$ C

x = 3.00 m

E_{net} = ?

Direction = ?

Solution:

From the figure below we see that,

$E = \frac{q}{4\pi\varepsilon_o r^2}$

$\tan\theta = \frac{4}{3}$

$\theta = 53.13^o$

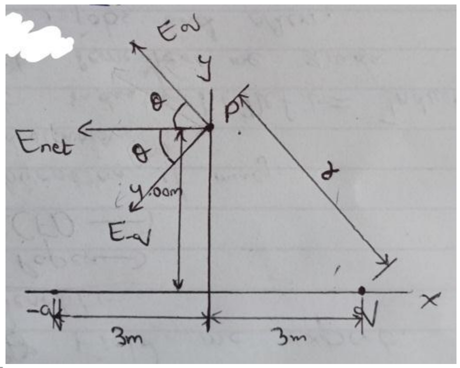
$r = \sqrt{3^2+4^2} = 5m$

We note that the components along the y axis cancel during the vector summation. With $\theta=53.13^o$, the magnitude of the net electric field is obtained as follows:

$E_{net} = E_x = 2E\cos\theta = 2\left(\frac{q}{4\pi\varepsilon_o r^2}\right)\cos\theta$

$$E_{net} = 2 \times \frac{3.2\times10^{-19}}{4\pi(8.854\times10^{-12})(5)^2} \times \cos53.13^o = 1.38 \times 10^{-10}\ N/C$$

The net electric field points in the -x direction



7. A proton and an electron form two corners of an equilateral triangle of side length 5 μm. What is the magnitude of the net electric field these two particles produce at the third corner?

Data:

$r = 5\ \mu m = 5 \times 10^{-6}\ m$

$E_{net} = ?$

Solution:

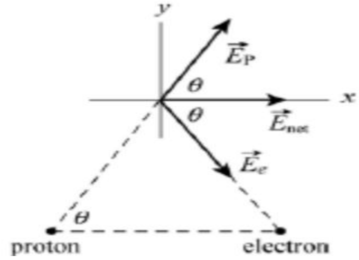
From the figure below we see that,

$$E_e = E_p = \frac{e}{4\pi\varepsilon_o r^2}$$

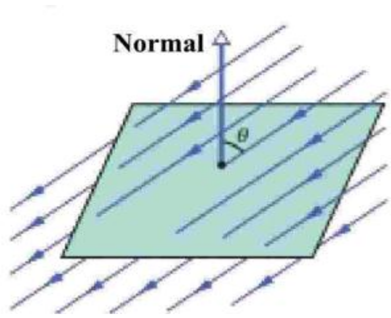
We note that the components along the y axis cancel during the vector summation. With θ=60^o, the magnitude of the net electric field is obtained as follows:

$$E_{net} = E_x = 2E_e \cos\theta = 2\left(\frac{e}{4\pi\varepsilon_o r^2}\right)\cos\theta$$

$$E_{net} = 2 \times \frac{1.6\times10^{-19}}{4\pi(8.854\times10^{-12})(5\times10^{-6})^2} \times \cos60 = 57.52\ N/C$$



8. The square surface shown in figure measures 3.2 mm on each side. It is immersed in a uniform electric field with magnitude E=1800 N/C and with field lines at an angle of e 35° with a normal to the surface, as shown. Take that normal to be directed "outward," as though the surface were one face of a box. Calculate the electric flux through the surface.



Data:

$l = 3.2\text{ mm} = 0.0032\text{ m}$

$E = 1800\text{ N/C}$

$\theta' = 35^o$

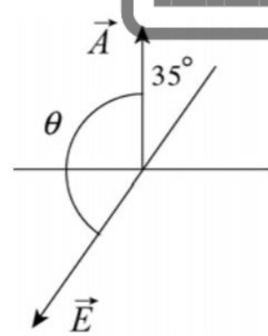
$\theta = 180^o - 35^o = 145^o$

$\varphi_e = ?$

Solution:

$A = l^2 = (0.0032)^2 = 1.024\times10^{-5}\text{ m}^2$

$\varphi_e = EA\cos\theta = (1800)(1.024\times10^{-5})(\cos 145^o) = -1.5\times10^{-2}\text{ N.m}^2\text{/C}$



9. An electron is liberated from the lower of two large parallel metal plates separated by a distance h = 2cm. The upper plate has a potential of 2400 volts relative to the lower. How long does the electron take to reach it?

Data:

h = 2cm = 0.02 m

V = 2400 V

Solution:

$S = v_i t + ½ at^2$

Since, $v_i = 0$

$h = ½ at^2 \longrightarrow (1)$

$a = \frac{F}{m} \longrightarrow (2)$

$F = Eq \longrightarrow (3)$

$E = \frac{V}{h}$

Put the value of E in equation (3)

$F = \frac{Vq}{h}$

Put in equation (2)

$a = \frac{Vq}{hm}$

Put in equation (1)

$h = ½ \frac{Vq}{hm} t^2$

$0.02 = ½ \frac{(2400)(1.6x10^{-19})}{(0.02)(9.11x10^{-31})} t^2$

$t = 1.4 \times 10^{-9}$ s

10. Two large parallel metal plates are 1.5 cm apart and have charges of equal magnitudes but opposite signs on their facing surfaces. Take the potential of the negative plate to be zero. If the potential halfway between the plates is then 5.0 V, what is the electric field in the region between the plates?

Data:

$2d = 1.5$ cm

$d = 0.75$ cm $= 0.0075$ m

$V = 5.0$ V

$E = ?$

Solution:

$E = \frac{V}{d}$

$\mathbf{E = \frac{5}{0.0075} = 6.7\times10^{2}\ V/m}$

Unit #9: Capacitors

Worked Example 9.1

A parallel plate capacitor consists of two plates with an area of 0.01 m^2 each, separated by a distance of 0.002 m. The capacitor is filled with a dielectric material having a relative permittivity (εr) of 4. Calculate the capacitance of this capacitor.

Solution:

Step 1: Formula:

$$C = \epsilon_o \epsilon_r A / d$$

where:

C is the capacitance,

ϵ_o is the vacuum permittivity constant (approximately 8.854×10^{-12} F/m),

ϵ_r is the relative permittivity (given as 4),

A is the area of one plate (0.01 m^2),

d is the separation distance between the plates (0.002 m).

Step 2:

Now, substitute the given values into the formula:

$$C = (8.854 \times 10^{-12} \text{ F/m}) \times 4 \times (0.01 \text{ m}^2) / (0.002 \text{ m})$$

$$C \approx 8.854 \times 10^{-12} \times 4 \times (0.01 / 0.002) \text{ F}$$

$$C \approx 3.5416 \times 10^{-11} \text{ F}$$

So, the capacitance of the parallel plate capacitor is approximately 3.5416×10^{-11} Farads (F).

Section (A): Multiple Choice Questions (MCQs)

1. The capacitance of a capacitor is NOT influenced by

(a) Plate thickness (b) Plate area

(c) Plate separation (d) Nature of the dielectric

2. What is the value of capacitance of a capacitor which has a voltage of 4V and has 16C of charge?

(a) 2F (b) 4F

(c) 6F (d) 8F

3. Capacitors are used in electric power supply system to:

(a) Improve power factor (b) Reduce line current

(c) Provide voltage stability (d) switching

4. In a variable capacitor, capacitance can be varied by:

(a) Turning the rotatable plates in or out (b) Changing the plates

(c) Sliding the rotatable plates (d) Changing the material of plates

5. Energy stored in the capacitor is:

(a) E=1/4 CV (b) $E=1/2\ CV^2$

(c) $E=CV^2$ (d) E=1/2 CV

6. The time constant of a series RC circuit consisting of 100μF capacitor in series with a 1000Ω resistor is.

(a) 0.1 s (b) 0.1 ms

(c) 0.01 s (d) 0.01 ms

7. The charging of a capacitor through a resistance follows

(a) linear law (b) square law

(c) exponential law (d) none of the above

8. When the total charge in a capacitor is doubled, the energy stored

(a) remains the same (b) is doubled

(c) is halved (d) is quadrupled

9. The capacitance C is charged through a resistor R. The time constant of the charging circuit is given by

(a) C/R (b) 1/RC

(c) RC (d) R/C

10. Capacitor blocks:

(a) alternating current (b) direct current

(c) both alternating and direct current (d) neither alternating nor direct current

KEY:

1. a	2. b	3. a	4. a	5. b
6. a	7. c	8. b	9. c	10. d

Section (B): Structured Questions

CRQs:

1. State the factors on which the capacitance of a parallel plate capacitor depends.

Ans) The capacitance of a parallel plate capacitor depends on several factors, including:

1. **Plate Area (A):** The capacitance is directly proportional to the surface area of the plates. A larger plate area results in a higher capacitance because it provides more surface area for the electric field to develop between the plates.
2. **Plate Separation (d):** The capacitance is inversely proportional to the distance between the plates. A smaller distance between the plates (shorter separation) results in a higher capacitance because the electric field lines can interact more closely, increasing the capacitance.
3. **Permittivity of the Dielectric Material (ε_r):** The capacitance is also affected by the dielectric material placed between the plates. The dielectric constant or permittivity (ε) of the material determines how well it can store electric charge and affects the capacitance. A higher permittivity material increases the capacitance.
4. **Permeability of Free Space (μ):** In vacuum or free space, the capacitance depends on the permeability of free space (μ_0), which is a constant in vacuum.
5. **Voltage (V):** The capacitance is directly proportional to the voltage applied across the plates. Increasing the voltage increases the charge stored on the plates and thus increases the capacitance.

2. Explain what is meant by dielectric constant (relative permittivity). State two physical properties desirable in a material to be used as the dielectric in a capacitor.

Ans) Dielectric constant: The dielectric constant, also known as relative permittivity, is a property of a material that measures its ability to store electrical energy in an electric field. It quantifies how much a dielectric material can increase the capacitance of a capacitor when placed between its plates compared to a vacuum or free space.

The dielectric constant is represented by the symbol ε_r (epsilon) and is a dimensionless quantity. It is typically greater than 1, as most dielectric materials have a greater ability to store electric charge than a vacuum. A dielectric constant of 1 indicates that the material has the same electrical properties as a vacuum, meaning it does not affect the capacitance of a capacitor.

Two physical properties of dielectric: When selecting a material to be used as the dielectric in a capacitor, two important physical properties to consider are:

1. **High Dielectric Constant (Relative Permittivity):** A desirable dielectric material should have a high dielectric constant. A high dielectric constant means that the material can significantly increase the capacitance of the capacitor when placed between the plates. This property allows the capacitor to store more electric charge for a given voltage, resulting in a higher energy storage capacity. Materials with high dielectric constants are particularly useful in applications where compactness and efficiency are important, as they allow for smaller and more efficient capacitors.
2. **Low Dielectric Loss:** Another crucial property for a dielectric material is low dielectric loss, often referred to as the dissipation factor (tan δ). Dielectric loss represents the amount of energy that is lost as heat when the capacitor is charged and discharged. A good dielectric material should have minimal dielectric loss to ensure that the stored energy is efficiently retained and released. Low dielectric loss also helps maintain the stability and accuracy of electronic circuits and components, especially in high-frequency applications. Materials with low dielectric loss are preferred to minimize energy wastage and heat generation in the capacitor.

3. Drive expressions for the combined capacitance of two capacitors (a) connected in series, (b) connected in parallel.

Ans) Capacitors in series: Let us suppose capacitors of capacitances C_1, C_2 and C_3 are connected in series between two points 'A' and 'B' as shown in figure. As all capacitors are connected in a single path, therefore when they are charged, each capacitor acquires the same amount of charge irrespective of their capacitances. Also, the potential difference V_{eq} applied across the points A and B is equal to the sum of potential difference across each capacitor

$V_{eq} = V_1 + V_2 + V_3 \rightarrow$ (1)

Since, $V = \frac{Q}{C}$

Therefore: $V_{eq} = \frac{Q}{C_{eq}}$, $V_1 = \frac{Q}{C_1}$, $V_2 = \frac{Q}{C_2}$ and $V_3 = \frac{Q}{C_3}$

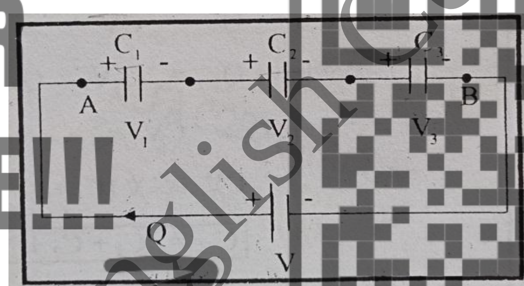
Equation(1) becomes

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

After taking Q common and cancelling, we get

$$\mathbf{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

Where C_{eq} is the net or equivalent capacitance and the reciprocal of which is equal to the sun of reciprocal of all capacitors connected in series. This results implies that in series combination, net capacitance of circuit decreases.



Capacitors in parallel: Let us suppose capacitors of capacitances C_1, C_2 and C_3 are connected in parallel between two points A and B as shown in figure. As each capacitor is connected between the same two points, so when a potential difference V is applied between points A and B, then all capacitors would have the same potential difference across them. On giving charge Q_{eq} to point A, capacitors C_1, C_2 and C_3 will acquire charges Q_1, Q_2 and Q_3 respectively, depending upon their capacitances, hence

$Q_{eq} = Q_1 + Q_2 + Q_3 \rightarrow$ (1)

From Q = CV

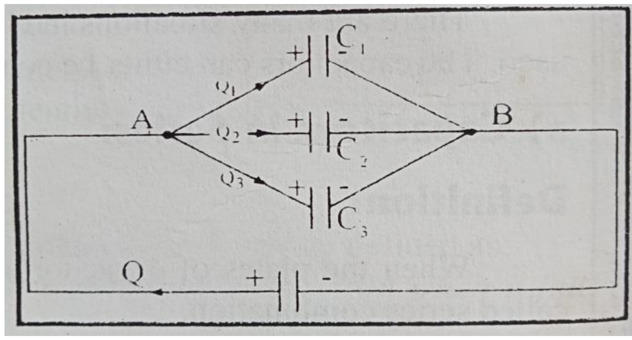
Therefore, $Q_{eq} = CV_{eq}$, $Q_1 = C_1V$, $Q_2 = C_2V$ and $Q_3 = C_3V$

$C_{eq}V = C_1V + C_2V + C_3V$

After taking V common and cancelling, we get

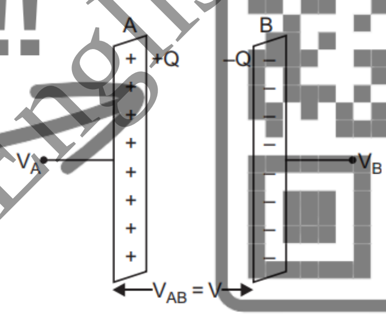
$$\mathbf{C_{eq} = C_1 + C_2 + C_3}$$

Where, C_{eq} is the net or equivalent capacitance which is equal to the sum of capacitances of all capacitors connected in parallel. This results implies that in parallel combination, the net capacitance of circuit increases.



4. Derive an expression for the energy stored in a capacitor C when there is a potential difference V between the plates.

Ans) When a capacitor is charged by a battery, work is done by the charging battery at the expense of its chemical energy. This work is stored in the capacitor in the form of electrostatic potential energy. Consider a capacitor of capacitance C. Initial charge on capacitor is zero. Let a charge Q be given to it in small steps. When charge is given to capacitor, the potential difference between its plates increases. Let at any instant when charge on capacitor be q, the potential difference between its plates V = q/C .



Now work done in giving an additional infinitesimal charge dq to capacitor is

$$dW = V\,dq = \frac{q}{C}\,dq$$

The total work done in giving charge from 0 to Q will be equal to the sum of all such infinitesimal works, which may be obtained by integration. Therefore total work is

$$W = \int_0^Q V\,dq = \int_0^Q \frac{q}{C}\,dq$$

$$= \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{1}{C}\left(\frac{Q^2 - 0}{2}\right) = \frac{Q^2}{2C}$$

If V is the final potential difference between capacitor plates, then Q =CV

$$W = \frac{(CV)^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

This work is stored as electrostatic potential energy of capacitor i.e., Electrostatic potential energy,

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

5. A capacitor gets a charge of 50 C when it is connected to a battery of emf 5 V. Calculate the capacity of the capacitor.

Data:

Q = 50 C

V = 5 V

C = ?

Solution:

$C = \frac{Q}{V}$

$C = \frac{50}{5}$ = 10 F

ERQs:

1. Calculate combined capacitance of capacitors in series and in parallel.

Ans) Capacitors in Series: Suppose you have three capacitors in series with capacitances of 10 µF, 15 µF, and 20 µF.

$$\frac{1}{C_{eq}} = \frac{1}{10} + \frac{1}{15} + \frac{1}{20}$$

$$\frac{1}{C_{eq}} = \frac{13}{60}$$

C_{eq} = 4.62 µF

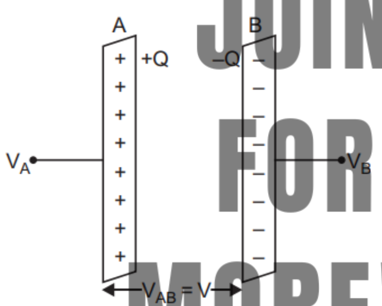
Capacitors in Parallel: Suppose you have three capacitors in parallel with capacitances of 10 µF, 15 µF, and 20 µF.

$C_{eq} = C_1 + C_2 + C_3$

$\mathbf{C_{eq} = 10 + 15 + 20 = 45\ \mu F}$

2. Prove that energy stored in a capacitor is $E = \frac{1}{2} CV^2$.

Ans) When a capacitor is charged by a battery, work is done by the charging battery at the expense of its chemical energy. This work is stored in the capacitor in the form of electrostatic potential energy. Consider a capacitor of capacitance C. Initial charge on capacitor is zero. Let a charge Q be given to it in small steps. When charge is given to capacitor, the potential difference between its plates increases. Let at any instant when charge on capacitor be q, the potential difference between its plates V = q/C .



Now work done in giving an additional infinitesimal charge dq to capacitor is

$$dW = V\,dq = \frac{q}{C}\,dq$$

The total work done in giving charge from 0 to Q will be equal to the sum of all such infinitesimal works, which may be obtained by integration. Therefore total work is

$$W = \int_0^Q V\,dq = \int_0^Q \frac{q}{C}\,dq$$

$$= \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{1}{C}\left(\frac{Q^2 - 0}{2}\right) = \frac{Q^2}{2C}$$

If V is the final potential difference between capacitor plates, then Q =CV

$$W = \frac{(CV)^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

This work is stored as electrostatic potential energy of capacitor i.e., Electrostatic potential energy,

$E = U = \frac{Q^2}{2C} = \frac{1}{2}CV^2$

3. Describe the factors affecting the capacitance of a parallel plate capacitor?

Ans) The capacitance of a parallel plate capacitor depends on several factors, including:

1. **Plate Area (A):** The capacitance is directly proportional to the surface area of the plates. A larger plate area results in a higher capacitance because it provides more surface area for the electric field to develop between the plates.
2. **Plate Separation (d):** The capacitance is inversely proportional to the distance between the plates. A smaller distance between the plates (shorter separation) results in a higher capacitance because the electric field lines can interact more closely, increasing the capacitance.
3. **Permittivity of the Dielectric Material (ε_r):** The capacitance is also affected by the dielectric material placed between the plates. The dielectric constant or permittivity (ε) of the material determines how well it can store electric charge and affects the capacitance. A higher permittivity material increases the capacitance.
4. **Permeability of Free Space (μ):** In vacuum or free space, the capacitance depends on the permeability of free space (μ_0), which is a constant in vacuum.
5. **Voltage (V):** The capacitance is directly proportional to the voltage applied across the plates. Increasing the voltage increases the charge stored on the plates and thus increases the capacitance.

Numericals:

1. The capacitance of air-filled parallel plate capacitor is 1.3 pF. If the separation of the plates is doubled and wax is inserted between them. The new capacitance is 2.6 pF. Find the dielectric constant of the wax.

Data:

C = 1.3 pF

C’ = 2.6 pF

ε_r = ?

Solution:

$C = \frac{\varepsilon_o A}{d}$

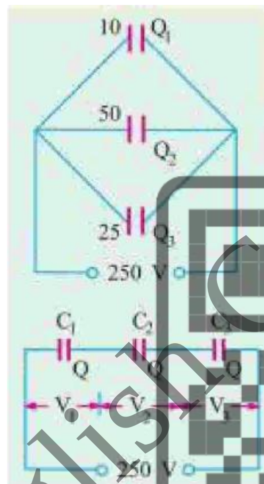
$C' = \frac{\varepsilon_o \varepsilon_r A}{2d}$

$\frac{C'}{C} = \frac{\varepsilon_r}{2}$

$\varepsilon_r = 2 \times \frac{2.6}{1.3}$

$\boldsymbol{\varepsilon_r = 4}$

2. Three capacitors have capacitances 10, 50 and 25 μF respectively as shown in figure. Calculate (i) charge on each when connected in parallel to a 250 V supply (ii) total capacitance and (iii) potential difference across each when connected in series.



Data:

$C_1 = 10\ \mu F$

$C_2 = 50\ \mu F$

$C_3 = 25\ \mu F$

Parallel Connection

(i) $V = 250\ V$
$Q_1 = ?$, $Q_2 = ?$, $Q_3 = ?$

(ii) $C_{eq} = ?$

Series Connection

(iii) $V_{eq} = 250\ V$
$V_1 = ?$, $V_2 = ?$, $V_3 = ?$

Solution:

(i)

$Q_1 = C_1V = (10 \times 10^{-6})(250) = 2.5\times10^{-3}$ C = 2.5 mC

$Q_2 = C_2V = (50 \times 10^{-6})(250) = 12.5\times10^{-3}$ C = 12.5 mC

$Q_3 = C_3V = (25 \times 10^{-6})(250) = 6.25\times10^{-3}$ C = 6.25 mC

(ii)

$C_{eq} = C_1 + C_2 + C_3$

C_{eq} = 10 + 50 +25 = 85 μF

(iii)

$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

$\frac{1}{C_{eq}} = \frac{1}{10} + \frac{1}{50} + \frac{1}{25}$

C_{eq} = 6.25 μF

$Q = C_{eq}V_{eq} = (6.25\times10^{-6})(250) = 1.5625\times10^{-3}$ C = 1.5625 mC

$V_1 = \frac{Q}{C_1}$

$V_1 = \frac{1.5625\times10^{-3}}{10\times10^{-6}}$ = 156.25 V

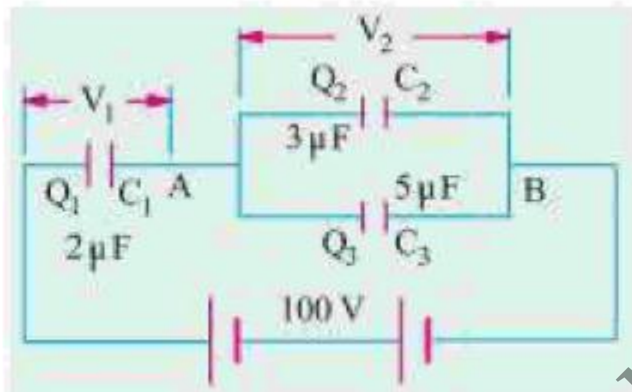
$V_2 = \frac{Q}{C_2}$

$V_2 = \frac{1.5625\times10^{-3}}{50\times10^{-6}}$ = 31.25 V

$V_3 = \frac{Q}{C_3}$

$V_3 = \frac{1.5625\times10^{-3}}{25\times10^{-6}}$ = 62.5 V

3. Three capacitors are connected through a potential difference of 100 volts as shown in figure. Find the charges and potential difference across each capacitor.



Data:

$V = 100\ V$

$Q_1 = ?$, $V_1 = ?$

$Q_2 = ?$, $V_2 = ?$

$Q_3 = ?$, $V_3 = ?$

Solution:

Capacitors 2 and 3 are connected in parallel, while their combination is in series with capacitor 1

$C_A = C_2 + C_3$

$C_A = 3 + 5 = 8\ \mu F$

$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_A}$

$\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{8}$

$C_{eq} = 1.6\ \mu F$

$Q = C_{eq} V$

$Q = (1.6x10-6)\ (100) = 1.6x10^{-4}\ C = 160\ \mu C$

$\mathbf{Q_1 = Q = 160\ \mu C}$

$V_1 = \frac{Q_1}{C_1}$

$\mathbf{V_1 = \frac{160x10^{-6}}{2x10^{-6}} = 80\ V}$

$Q_A = Q = 160\ \mu C$

$V_A = \frac{Q_A}{C_A} = \frac{160x10^{-6}}{8x10^{-6}} = 20\ V$

$\mathbf{V_2 = V_3 = V_A = 20V}$

$Q_2 = C_2 V_2$

$\mathbf{Q_2 = (3x10^{-6})\ (20) = 6x10^{-5}\ C = 60x10^{-6}\ C = 60\ \mu C}$

$Q_3 = C_3 V_3$

$\mathbf{Q_3 = (5x10^{-6})\ (20) = 1x10^{-4}\ C = 100x10^{-6}\ C = 100\ \mu C}$

4. Capacitor is charged through a large non-reactive resistance by a battery of constant voltage V. For this arrangement, if the capacitor has a capacitance of 10 μ F and the resistance is 1 M Ω, calculate the time taken for the capacitor to receive 90% of its final charge.

Data:

C = 10 μ F

R = 1 MΩ

t = ?

Solution:

$\tau = RC = (1x10^{6})\ (10x10^{-6}) = 10$

$Q = Q_o\ (1-e^{-\frac{t}{\tau}})$

$\frac{Q}{Q_o} = 1-e^{-\frac{t}{10}}$

$\frac{0.9\ x\ Q_o}{Q_o} = 1-e^{-\frac{t}{10}}$

$e^{-\frac{t}{10}} = 1 - 0.9 = 0.1$

$-t/10 = \ln(0.1) = -2.3$

$t/10 = 2.3$

t= 23 s

5. What capacitance is required to store energy of 10 kWh at a potential difference of 1000 V?

Data:

$E = 10\ kWh = 10 \times 3.6\times10^6 = 3.6\times10^7\ J$

$V = 1000\ V$

$C = ?$

Solution:

$E = \frac{1}{2} C V^2$

$3.6\times10^7 = \frac{1}{2} C (1000)^2$

$C = 72\ F$

6. A 2.0 μ F capacitor and a 4.0 μ F capacitor are connected in parallel across a 300 V potential difference. Calculate the total energy stored in the capacitors

Data:

$C_1 = 2.0\ \mu F$

$C_2 = 4.0\ \mu F$

$V = 300\ V$

$E = ?$

Solution:

$C_{eq} = C_1 + C_2$

$C_{eq} = 2 + 4 = 6\ \mu F$

$E = \frac{1}{2} C_{eq} V^2$

$E = \frac{1}{2} (6\times10^{-6}) (300)^2 = 0.27\ J$

7. A 12.0 V battery is connected to a capacitor, resulting in 54.0 μC of charge stored on the capacitor. How much energy is stored in the capacitor?

Data:

$V = 12\ V$

$Q = 54.0\ \mu C = 54 \times 10^{-6}\ C$

$E = ?$

Solution:

$E = \frac{1}{2} Q V$

$\mathbf{E = \frac{1}{2} (54 \times 10^{-6}) (12) = 3.24 \times 10^{-4}\ J}$

Unit #10: D.C Circuits

Worked Example 10.1

Calculate the resistance of 100 meter rolls of $2.5mm^2$ copper wire if the resistivity of copper at 20°C is $1.72 \times 10^{-8}\ \Omega$ meter.

Solution:

Step 1: Write the known quantities and point out quantities to be found
Resistivity of copper at 20°C = $\rho = 1.72 \times 10^{-8}\ \Omega$ meter
Coil length = L = 100m
Resistance = R = ?
Cross-sectional area of the conductor = A= $2.5mm^2 = 2.5 \times 10^{-6}\ m^2$.

Step 2: Write the formula and rearrange if necessary

$\rho = R\ A/L$

$R = \rho\ L/A$

Step 3: Put the values in formula and calculate
$R = (1.72 \times 10^{-8}) \times 100/(2.5 \times 10^{-6})$

$R = 0.688\ \Omega$

Result: Resistance will be **688 milli-ohms** or **0.688 Ohms.**

Worked Example 10.2

A 20-meter length of cable has a cross-sectional area of $1mm^2$ and a resistance of 5 ohms. Calculate the conductivity of the cable.

Solution:

Step 1: Write the known quantities and point out quantities to be found
DC resistance = R = 5 ohms
Cable length = L = 20m
Cross-sectional area of the conductor =A =$1mm^2 = 1 \times 10^{-6}\ m^2$.
Conductivity = σ = ?

Step 2: Write the formula and rearrange if necessary

$\sigma = L/RA$

Step 3: Put the values in formula and calculate
$=20/5 \times 1 \times 10^{-6}) = 4 \times 10^{6}$ S/M

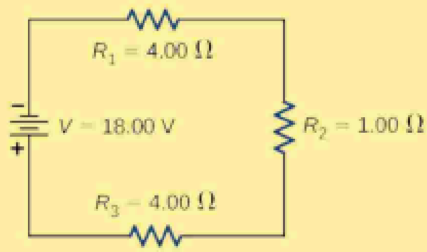
σ =4MS/m

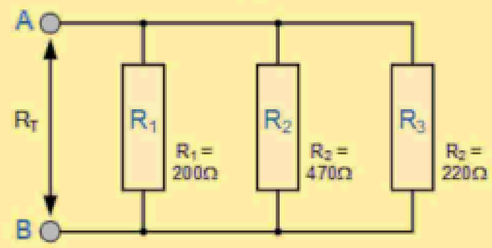
Thus, Conductivity of Cable is 4MS/m

Worked example 3

Determine the equivalent resistance for each of the two circuits shown below:

(a) (b)





Solution:

In Circuit (a) the three resistors R_1, R_2 and R_3 are connected in series.

We know that equivalent resistance of series resistors is given as,

$$R_{eq} = R_1 + R_2 + R_3 + \ldots$$

$$\therefore R_{eq} = 4 + 1 + 4 = 9\ \Omega$$

In Circuit (b) the three resistors R_1, R_2 and R_3 are connected in parallel.

We know that equivalent resistance of parallel resistors is given as,

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{1}{200} + \frac{1}{470} + \frac{1}{220} = 0.0117$$

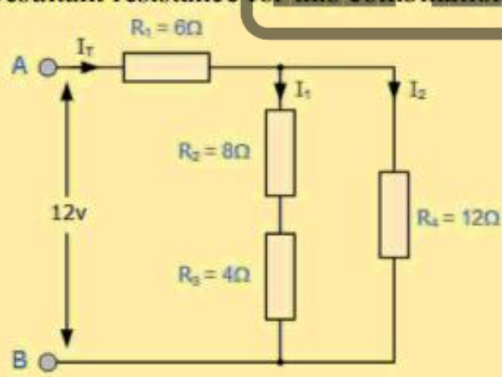
$$\text{therefore: } R_T = \frac{1}{0.0117} = 85.67\Omega$$

Worked example 4:

Find the equivalent resistance of the given circuit

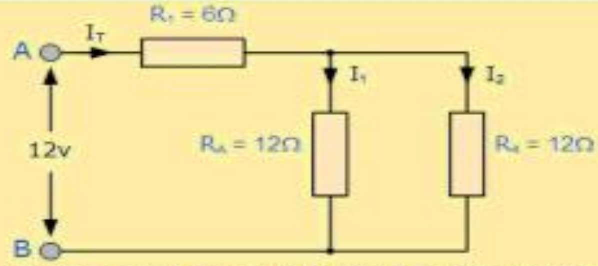
Solution:

We can see that the two resistors, R_2 and R_3 are actually both connected together in a "SERIES" combination. The resultant resistance for this combination would therefore be:



$$R_2 + R_3 = 8\Omega + 4\Omega = 12\Omega$$

So we can replace both resistor R_2 and R_3 above with a single resistor R_A of resistance value 12Ω

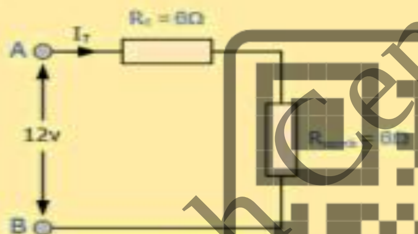


Now our circuit has a single resistor R_A in "PARALLEL" with the resistor R_4. Using the formula for two parallel connected resistors we can find out its equivalent single resistor.

$$R_{(eq)} = \frac{1}{R_A} + \frac{1}{R_4} = \frac{1}{12} + \frac{1}{12} = 0.1667$$

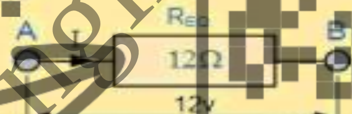
$$R_{(combination)} = \frac{1}{R_{(eq)}} = \frac{1}{0.1667} = 6\Omega$$

The resultant circuit would be:



The two remaining resistances, R_1 and $R_{(comb)}$ are connected together in a "SERIES" combination so resultant will be:

$R_{(ab)} = R_{comb} + R_1 = 6\Omega + 6\Omega = 12\Omega$



Thus equivalent resistance is of 12Ω. It means the original four resistors connected together in the original circuit above can be replaced by a 12Ω resistor.

Section (A): Multiple Choice Questions (MCQs)

1. Kirchhoff's laws are useful in determining:

a) current flowing in a circuit b) emf and voltage drops in a circuit

c) power in a circuit d) both a and b

2. The resistance of a superconductor is:

a) finite b) infinite

c) changes with every conductor d) zero

3. Reciprocal of resistance is called:

a) conductance b) resistivity

c) resonance d) capacitance

4. The graphical representation of Ohms law is:

a) parabola b) hyperbola

c) ellipse d) straight line

5. A potential difference is applied across the ends of a wire. If the potential difference is doubled, then the drift velocity of free electrons will:

a) be quadrupled b) be doubled

c) be halved d) remain unchanged

6. Internal resistance is the resistance offered by:

a) Capacitor b) resistor

c) Conductor d) source of emf

7. Power dissipation in a resistor can be calculated using which formula:

a) $P=V^2/R$ b) $P=I^2xR$

c) $P=VxI$ d) all of these

8. What is a potentiometer primarily used for?

a) Measuring electric current

b) Measuring electric charge.

c) Measuring potential difference (voltage)

d) Measuring electric resistance

9. A heat-sensitive device whose resistivity changes with the change in temperature is called:

a) conductor b) resistor

c) thermistor d) thermometer

10. A wire of uniform area of cross-section A, length L and resistance R is cut into two parts. The resistivity of each part:

a) Becomes zero
b) is halved
c) Is doubled
d) remains same

KEY:

1. d	2. d	3. a	4. d	5. b
6. d	7. d	8. c	9. c	10.d

CRQs:

1. Why is the terminal voltage of a cell less than its emf?

Ans) The terminal voltage of a cell is typically lower than its electromotive force (EMF) due to internal resistance and external factors. Internal resistance (symbolized as "r") inherent in the cell's design causes a voltage drop as current flows through it, reducing the terminal voltage. Ohm's Law, V = IR, explains this relationship. Additionally, when connected to an external circuit with resistance (R), the total resistance in the circuit affects the current flow, leading to further voltage reduction across the load resistance.

2. Why is a potentiometer preferred over a voltmeter for determining the emf of a cell?

Ans) A potentiometer is preferred over a voltmeter for determining the electromotive force (EMF) of a cell due to several advantages. Potentiometers offer high precision, allowing for accurate measurements of small voltage differences without drawing current from the cell. They provide a direct measurement of EMF, eliminating errors related to internal resistance and load effects. Potentiometers have a wide measurement range and minimal impact on the cell's voltage. They can be calibrated for precise results in controlled environments like laboratories. While voltmeters are more practical for quick voltage checks.

3. Nichrome and copper wires of same length and same radius are connected in series. Current I is passed through them. Which wire gets heated up more? Justify your answer.

Ans) In this scenario, the wire that gets heated up more will be the nichrome wire, and this can be justified based on the differences in the electrical resistivity of the two materials. Nichrome has a significantly higher electrical resistivity compared to copper.

When the same current (I) passes through both wires in series, the power dissipated as heat in each wire can be calculated using the formula:

$$P=I^2R$$

Since the length and radius of the nichrome and copper wires are the same, the only significant difference in this context is the electrical resistivity of the materials. Nichrome has a higher resistivity than copper, which means it has a higher resistance (R) for the same length and cross-sectional area. Consequently, when the same current flows through both wires, the power dissipated as heat (P) in the nichrome wire will be greater than that in the copper wire.

4. Explain why the terminal potential of a battery decreases when the current drawn from in increased?

Ans) The terminal potential (voltage) of a battery decreases when the current drawn from it is increased due to its internal resistance. According to Ohm's Law (V = IR), there is a direct relationship between voltage drop (V) and current (I). As the current drawn (I) increases, this voltage drop becomes more significant, resulting in a lower terminal voltage (V_t).

5. What are thermistors? Write their importance.

Ans) Thermistors: Thermistors are temperature-dependent resistors, changing resistance with changes in temperature.

Importance of thermistors: Here are some key points highlighting the importance of thermistors.

1. **Temperature Sensing:** Thermistors are primarily used for temperature sensing and measurement.
2. **Temperature Compensation:** In electronic circuits and devices, thermistors are often used for temperature compensation.
3. **Overheating Protection:** When placed in proximity to heat sources, they can trigger safety mechanisms or shutdown circuits to prevent overheating and potential damage.
4. **Temperature Control:** Thermistors are used in thermostats and temperature control systems to regulate heating or cooling processes.
5. **Medical Applications:** In the medical field, thermistors are used for monitoring body temperature, both externally and internally.
6. **Automotive Industry:** Thermistors play a crucial role in automotive applications, including engine temperature monitoring, climate control, and battery management.
7. **Environmental Monitoring:** Thermistors are used in environmental monitoring equipment to measure temperature variations in natural habitats, climate research, and weather stations.
8. **Industrial Automation:** In industrial processes, thermistors are used to monitor and control temperatures in various machinery and manufacturing processes. They contribute to product quality, efficiency, and safety in industrial settings.

9. **Energy Efficiency:** Thermistors are integral to energy-efficient appliances and devices by enabling temperature-based control, which reduces energy consumption and extends the lifespan of equipment.

6. State Kirchhoff's Laws.

Ans) Kirchhoff's Current Law/ Kirchhoff's First Law: Kirchhoff's current law is based on the conservation of charge because sum of current entering to the junction is equal to sum of current leaving the junction.

Kirchhoff's Voltage Law/ Kirchhoff's second Law: The sum of voltage rises equals the sum of voltage drops around a loop.

7. If Copper and Aluminum wires of the same length have same resistance, which has the larger diameter? And why?

Ans) If copper and aluminum wires have the same resistance and the same length, the relationship between their diameters can be understood by considering their electrical resistivities and the fundamental formula for resistance:

$$R=\rho \cdot L / A$$

Since both wires have the same resistance and the same length, the equation becomes:

$$R_{copper} = R_{aluminum}$$
$$\rho_{copper} \cdot L / A_{copper} = \rho_{aluminium} \cdot L / A_{aluminium}$$

Now, let's consider the difference in electrical resistivities:

- Copper has a lower electrical resistivity (ρcopper) compared to aluminum (ρaluminum).

Since both sides of the equation are equal and the lengths (L) and resistances (R) are the same, we can conclude that for aluminum to match the resistance of copper, it must have a larger cross-sectional area (A) to compensate for its higher resistivity. Consequently, aluminum has larger than copper.

8. What is the difference between potential difference and emf?

Ans)

Aspect	Potential Difference (Voltage)	Electromotive Force (EMF)
Definition	Represents electric potential energy per unit charge between two points.	Represents the total energy supplied per unit charge by an energy source.
Source	Arises within a circuit due to potential gradients caused by components like resistors, capacitors, etc.	Associated with the source of electrical energy (e.g., battery, generator).
Direction of Energy Transfer	Indicates the energy transfer or work done on charges as they move within a circuit. Determines the direction of current flow (from higher to lower potential).	Represents the potential energy supplied by the source and does not dictate the direction of current flow within a circuit.
Role in Circuits	Essential for circuit operation, driving current flow through conductors, and determining the behavior of circuit components.	Provides the initial push or driving force for current in a circuit, ensuring current flow and maintaining a potential difference.

ERQs:

1. State the principle of working of a meter bridge. Explain how it is used to find an unknown resistance.

Ans) Principle of meter bridge: The Wheatstone bridge works on the principle of null deflection. In normal conditions, current flows through the galvanometer and the bridge is said to be in an unbalanced condition. Adjusting the known resistance and variable resistance a condition is achieved when no current flows through the galvanometer ie a balanced condition.
If key K_1 and K_2 are open no current I_g flows through the galvanometer. We can however arrange conditions under which no current flows through the galvanometer even keys K_1 and K_2 are closed in this condition the bridge is said to be balanced.

Let us consider the path of the current through different resistors of bridge.

The current I starting from the positive terminal of the battery reaches the terminal A, where it finds two paths, one through resistor R_1 is I_1 and the one through R_3 is I_2 when the current I_1 reaches the terminal B, it also finds the two paths, the one through the galvanometer which is I_g and the other through the resistor R_2. The current I_2 passing through R_1 when reaches the points D, it also finds two paths, one through galvanometer and the other through R_4 when we adjust the resistance in such a way that.

When the bridge is balanced, then no current flows through the galvanometer (I_g=0). This means that the potential at B and D are same i.e. $V_{BA} = V_{BA}$ hence

$$I_1R_1 = I_2R_3 \longrightarrow (1)$$

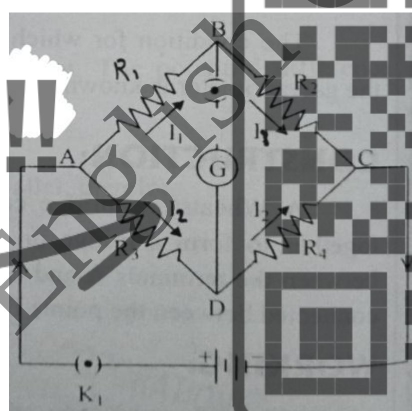
Also the current through R_2 is I_1 and through R_4 is I_2

$$I_1R_2 = I_2R_4 \longrightarrow (2)$$

Dividing (1) by (2), we get

$$\frac{I_1R_1}{I_1R_2} = \frac{I_2R_3}{I_2R_4}$$

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$



Determination Of Unknown Resistance: To determine the unknown resistance X, it is connected in the arm containing resistance R_4. The values of R_1, R_2, and R_3 are so adjusted that the bridge is balanced, hence

$$\frac{R_1}{R_2} = \frac{R_3}{X}$$

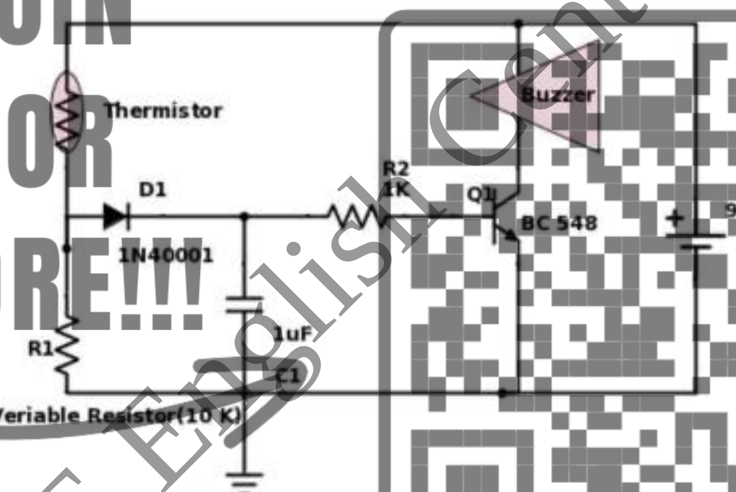
$$X = \frac{R_3}{R_1} \times R_2$$

By knowing the values of R_1, R_2 and R_3 the value of X can be calculated.

2. Define thermoelectricity. Explain the working of a Fire Alarm system using thermistor.

Ans) Thermoelectricity: Thermoelectricity is the electricity generated from heat energy.

Working of a fire alarm system using thermistor: A thermistor is a variable resistor whose resistance changes with temperature. Its temperature detection can be used in fire alarms for the detection of fires. Fire Alarm Circuit is a simple circuit that detects the fire and activates the Siren Sound or Buzzer. In this fire alarm circuit, the resistance of the thermistor is approximately in kilo-ohms at normal temperature. During fire, the resistance reduces to a few ohms as the temperature increases which switches ON the transistor. Once the transistor is turned ON, the current from V_{cc} starts to flow via buzzer which produces a beep sound. For unidirectional conduction a Diode is used and the use of capacitor removes sudden transients from the thermistor.



Fire Alarm

3. Define resistivity. Explain dependence of resistivity on temperature.

Ans) Resistivity: Resistivity is the electrical resistance of a conductor of unit cross-sectional area and unit length.

Resistivity Depends on Temperature: Resistivity does not depend on the size or shape of the material, but it does depend on temperature. Two factors primarily determine the resistivity of a metal.

1. The number of conduction electrons per unit volume and the rate of collisions between an electron and an ion.
2. The sensitive to changes in temperature. At a higher temperature, the internal energy is greater, the ions vibrate with larger amplitudes. As a result, the electrons collide more frequently with the ions.

With less time to accelerate between collisions, they acquire a smaller drift speed, thus, the current is smaller for a given electric field. Therefore, as the temperature of a metal is raised, its resistivity increases. The metal filament in a glowing incandescent light bulb reaches a temperature of about 3000K; its resistance is significantly higher than at room temperature. For many materials, the relation between resistivity and temperature is linear over a fairly wide range of temperatures (about 500°C);

$$\rho_t = \rho_o (1 + \alpha \Delta T)$$

Where, ρ_t = resistivity at temperature T ^oC

ρ_o = resistivity at temperature T_o ^oC

α = linear temperature coefficient of resistivity and has SI units $^oC^{-1}$ or K^{-1}.

Note that for semiconductors, $\alpha < 0$. A negative temperature coefficient means that the resistivity decreases with increasing temperature. It is still true, as for metals that are good conductors that the collision rate increases with temperature. However, in semiconductors the number of carriers (conduction electrons or holes) per unit volume increases with increasing temperature: with more carriers, the resistivity is smaller.

4. What is a rheostat? How can we use a rheostat as a potential divider?

Ans) Rheostat: A rheostat is a variable resistor, used for controlling the flow of electric current by increasing or decreasing the resistance.

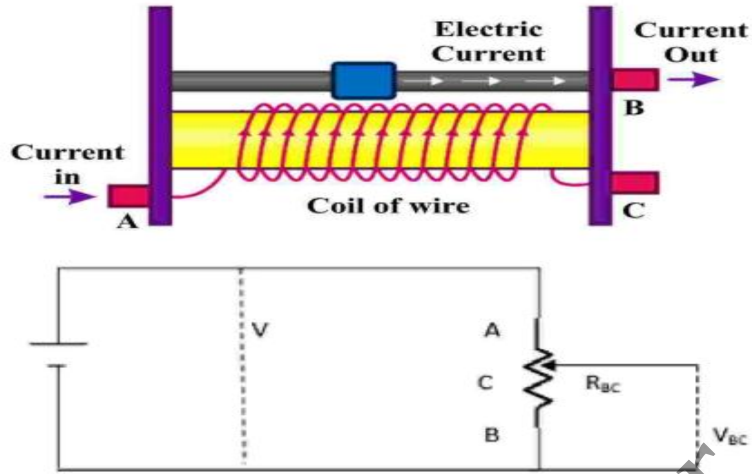
Working of rheostat as a potential divider: A potential divider is used for getting a variable potential from a fixed potential. A potential difference V is provided with the help of a battery across the ends of a variable resistor. If R is the resistance of the wire, the current flowing in the resistor is

$$I = V/R$$

Let R_{BC} be the resistance of the portion BC of the wire. The current passing through this portion is also I. The potential difference between the points B and C is given by

$$V_{BC} = I R_{BC}$$

R_{BC} as we go away from A and decreases as C comes close to A. This changes the ratio RBC to R and hence a variable voltage can be obtained. If V is regarded as input potential difference to the potential divider and V_{BC} as the output potential difference, then VBC can be tapped off and applied to another circuit.



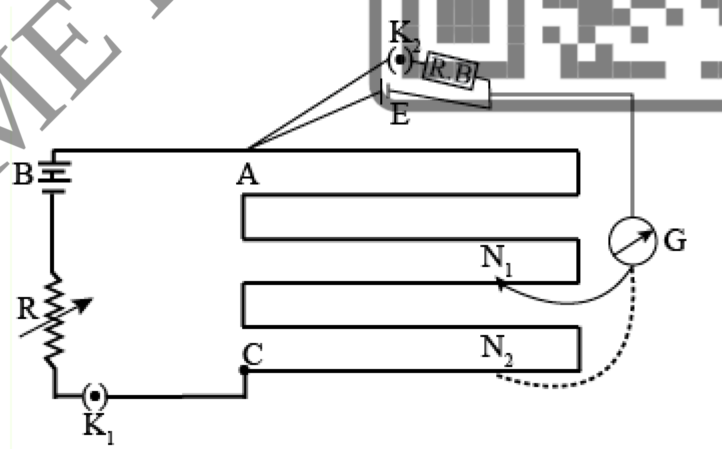
Rheostat as a potential divider

5. State the underlying principle of a potentiometer. Describe briefly, giving the necessary circuit diagram, how a potentiometer is used to measure the internal resistance of a given cell?

Ans) Principle of a potentiometer: The potential difference across any length of a wire of uniform area of cross section is directly proportional to its length when a constant current flows through it.

$V \propto L$ (at constant current)

Measurement of internal resistance of a cell using potentiometer is shown in figure. The cell of emf, E is connected across a resistance box R through key K_2.



When K_2 is open, balance length is obtained at length $AN_1 = l_1$

$E \propto L_1$

$E = \varphi L_1 \longrightarrow (1)$

When K_2 is closed:

Let V be the terminal potential difference of cell and the balance is obtained at $AN_2 = L_2$

$V \propto L_2$

$V = \varphi L_2 \longrightarrow (2)$

From equations (1) and (2), we get

$\frac{E}{V} = \frac{I_1}{I_2} \longrightarrow (3)$

$E = I(r+R)$

$V = IR$

$\frac{E}{V} = \frac{r+R}{R} \longrightarrow (4)$

From equations (3) and (4), we get

$\frac{R+r}{R} = \frac{L_1}{L_2}$

$r = R\left(\frac{L_1}{L_2} - 1\right)$

We know L_1, L_2 and R, so we can calculate r.

Numericals:

1. The storage battery of a car has an emf of 12 V. If the internal resistance of the battery is 0.5Ω, what is the maximum current that can be drawn from the battery?

Data:

$E = 12\ V$

$r = 0.5\Omega$

$I = ?$

Solution:

$E = Ir$

$12 = I\,(0.5)$

$\mathbf{I = 24\ A}$

2. A negligibly small current is passed through a wire of length 12m and uniform cross- section $4.0\times10^{-7}\ m^2$. and its resistance is measured to be 6.0 Ω. What is the resistivity of the material at the temperature of the experiment?

Data:

L = 12m

$A = 4.0\times10^{-7}\ m^2$

R = 6.0 Ω

ρ = ?

Solution:

R = ρL/A

$6 = \rho\ (12)/\ 4.0\times10^{-7}$

$\boldsymbol{\rho = 2\times10^{-7}\ \Omega m}$

3. In a potentiometer arrangement, a cell of emf 1.20 V gives a balance point at 40.0 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 74.0 cm, what is the emf of the second cell?

Data:

$E_1 = 1.2\ V$

$L_1 = 40\ cm = \frac{40}{100} = 0.4\ m$

$L_2 = 74\ cm = \frac{74}{100} = 0.74\ m$

$E_2 = ?$

Solution:

$\frac{E_1}{E_2} = \frac{L_1}{L_2}$

$\frac{1.2}{E_2} = \frac{0.4}{0.74}$

$\mathbf{E_2 = 2.22\ V}$

4. (a) Three resistors 1Ω, 2Ω, and 3Ω are combined in series. What is the total resistance of the combination?

b) If the combination is connected to a battery of emf 24 V and negligible internal resistance, obtain the potential drop across each resistor.

Data:

(a) $R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, series

(b) $V_{eq} = 24$ V (because internal resistance is negligible), $V_1 = ?$, $V_2 = ?$, $V_3 = ?$

Solution:

(a)

$R_{eq} = R_1 + R_2 + R_3$

$\mathbf{R_{eq} = 1 + 2 + 3 = 6\Omega}$

(b)

$V_{eq} = IR_{eq}$

$24 = I(6)$

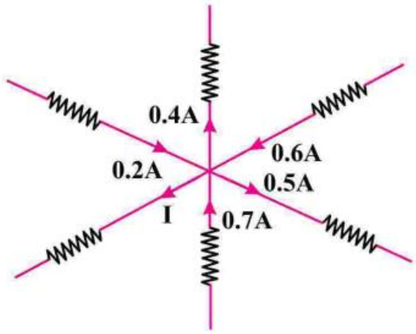
$I = 4$ A

$\mathbf{V_1 = IR_1 = (4)(1) = 4V}$

$\mathbf{V_2 = IR_2 = (4)(2) = 8V}$

$\mathbf{V_1 = IR_1 = (4)(3) = 12V}$

5. From the given circuit find the value of I.



Data:

I = ?

Solution:

$\Sigma I_{in} = \Sigma I_{out}$

$0.2 + 0.6 + 0.7 = 0.4 + 0.5 + I$

$I = 0.6$ A

6. In a meter bridge with a standard resistance of 15Ω in the right gap, the ratio of balancing length is 5:3. Find the value of the other resistance.

Data:

$\frac{L_1}{L_2} = 5:3$

$R = 15\Omega$

$X = ?$

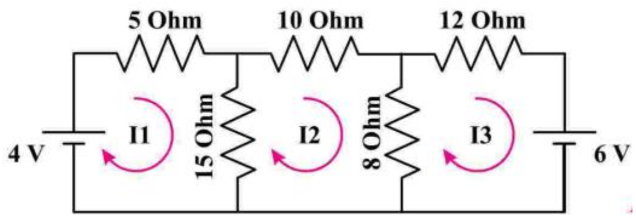
Solution:

$\frac{X}{R} = \frac{L_1}{L_2}$

$\frac{X}{15} = \frac{5}{3}$

$X = 25\Omega$

7. By using KVL find Current flowing through 10Ω resistance.



Data:

$I_2 = ?$

Solution:

For left loop

Voltage rise = Voltage drop

$4 = 5\ I_1 + 15(I_1 - I_2)$

$20\ I_1 - 15\ I_2 = 4 \longrightarrow$ (1)

For middle loop

Voltage rise = Voltage drop

$0 = 15(I_2 - I_1) + 10\ I_2 + 8\ (I_2 - I_3)$

$-15\ I_1 + 33\ I_2 - 8\ I_3 = 0$

$15\ I_1 - 33\ I_2 + 8\ I_3 = 0 \longrightarrow$ (2)

For right loop

$-6 = 8\ (I_3 - I_2) + 12\ I_3$

$-8\ I_2 + 20\ I_3 = -6$

$8\ I_2 - 20\ I_3 = 6 \longrightarrow$ (3)

By using calculator

$I_2 = 0.0323$ A (since sign is positive therefore I_2 in clockwise direction)

Unit #11: Oscillations

Worked Example 11.1

The spring used in one such device shown in Fig.11.12 has a spring constant of 606 N/m, and the mass of the chair is 12.0 kg. The measured oscillation period is 2.41 s. Find the mass of the astronaut.

Step 1: Write the known quantities and point out the quantities to be found.

Spring Constant; $k = 606\ Nm^{-1}$

Mass of chair; $m_{chair} = 12.0$ Kg

Time period; $T = 2.41$ s

Step 2: Write the formula and rearrange if necessary.

Since the astronaut and chair are oscillating in simple harmonic motion, the total mass ($m_{total} = m_{chair} + m_{astronaut}$) of the two is related to the angular frequency ω which is related (Eq.11.20) to period of oscillation as

$$T = 2\pi\sqrt{\frac{m_{total}}{k}}$$

Squaring on both sides and rearranging the above equation.

$$m_{total} = \frac{T^2 k}{4\pi^2}$$

Hence the expression for mass of astronaut;

$$m_{astronaut} = \frac{T^2 k}{4\pi^2} - m_{chair}$$

Step 3: Put the values in the formula and calculate.

$$m_{astronaut} = \frac{2.41^2 \times 606}{4 \times 3.142^2} - 12.0 = \mathbf{77.2\ Kg}$$

Astronaut **Millie Hughes-Fulford**
a body-mass measurement device developed
for determining mass in orbit.

Worked Example 11.2

A block is kept on a horizontal table. The table is undergoing simple harmonic motion of frequency 3 Hz in a horizontal plane. The coefficient of static friction between the block and the table surface is 0.72. Find the maximum amplitude of the table at which the block does not slip on the surface.

Step 1: Write the known quantities and point out the quantities to be found.

Frequency; $f = 3\ Hz$

Coefficient of friction; $\mu = 0.72$

Amplitude; $x_o = ?$

Step 2: Write the formula and rearrange if necessary.

Since $a = \omega^2 x_o$

Maximum force of static friction is given as

$F = \mu mg$

In case that the body does not slip;

$ma = \mu mg$

or $m\,\omega^2 x_o = \mu mg$

and $x_o = \mu g/\omega^2 = \mu g/(2\pi f)^2$

Step 3: Put the values in the formula and calculate.

Amplitude $(x_o) = 0.72 \times 9.8/(2 \times 3.14 \times 3)^2 =$ **0.0198m** or 1.98cm

Section (A): Multiple Choice Questions (MCQs)

1. Two simple pendulums A and B with same lengths, and equal amplitude of vibrations, but the mass of A is twice the mass of B, their period are T_A and T_B and energies are E_A and E_B respectively. Choose the correct statement.

a) $T_A = T_B$ and $E_A > E_B$

b) $T_A < T_B$ and $E_A > E_B$

c) $T_A > T_B$ and $E_A < E_B$

d) $T_A = T_B$ and $E_A < E_B$

2. In order to double the period of a simple pendulum:

a) Its length should be doubled

b) Its length should be quadrupled

c) The mass should be doubled.

d) The mass should be quadrupled

3. A simple harmonic oscillator has amplitude A and time period t. Its maximum speed is:

a) $\frac{4A}{t}$

b) $\frac{2A}{t}$

c) $\frac{4\pi A}{t}$

d) $\frac{2\pi A}{t}$

4. A spring attached by a load of weight W is vibrating with a period T. If the spring is divided in four equal parts and the same load is suspended from one of these parts, the new time period is:

a) $\frac{T}{4}$

b) 2T

c) $\frac{T}{2}$

d) 4T

5. The total energy of a particle executing simple harmonic motion is proportional to:

a) frequency of oscillation

b) maximum velocity of motion

c) amplitude of motion

d) square of amplitude of motion

6. A child swinging on a swing in sitting position, stands up, then the time period of the swing will:

a) Increase

b) decrease

c) remains the same

d) increases if the child is long and decreases if the child is short

7. If a body oscillates at the angular frequency ω_d of the driving force, then the oscillations are called:

a) Forced oscillations

b) Coupled oscillations

c) Free oscillations

d) Maintained oscillations

8. A simple harmonic oscillator with a natural frequency ω_N is force to oscillate with a driving frequency ω_d. The Resonance occurred when:

a) $\omega_N < \omega_d$

b) $\omega_N > \omega_d$

c) $\omega_N = \omega_d$

d) $\omega_N \approx \omega_d$

9. In vehicles, shock absorber reduced the jerks:

a) The shock absorber is the application of damped oscillations.

b) Damping effect is due to the fractional loss of energy

c) Shock absorbers in vehicles reduced jerk

d) All of these

10. A heavily damped system has a fairly flat resonance curve in:

a) An acceleration time graph
b) An amplitude frequency graph
c) Velocity time graph
d) Distance-time graph

KEY:

1. a	2. b	3. d	4. c	5. d
6. b	7. a	8. c	9. d	10. b

CRQs:

1. Explain the concept of periodic motion to oscillatory motion. Discuss the terms period, frequency, and amplitude.

Ans) Periodic motion: Periodic motion is a fundamental concept in physics and refers to a type of motion in which an object repeats its motion over regular intervals of time.

Here is more explanation about it.

1. **Regular Repetition:** In periodic motion, the object's motion repeats itself identically at fixed intervals of time. This means that the object returns to the same position, velocity, and other physical properties after a certain time period.
2. **Equilibrium Position:** Many objects in periodic motion have an equilibrium or rest position, which is a stable point around which the motion occurs. The object moves away from this position and then returns to it during each cycle of motion.
3. **Examples of Periodic Motion:** Many natural and man-made phenomena exhibit periodic motion, including:
    - The swinging of a pendulum back and forth.
    - The rotation of a wheel or a planet around an axis.
    - The vibration of a guitar string producing musical notes.
    - The motion of a child on a swing.
    - The oscillation of a mass attached to a spring (simple harmonic motion).
4. **Applications:** Periodic motion has numerous applications in various fields, including physics, engineering, and technology. It is essential in understanding phenomena such as sound waves, electromagnetic waves, alternating current in electrical circuits, and much more.

Period: The time taken by a particle of medium to complete its one vibration is called the time period of the wave.

Frequency: Frequency, in physics, the number of waves that pass a fixed point in unit time; also, the number of cycles or vibrations undergone during one unit of time by a body in periodic motion.

Amplitude: It is the distance between the resting position and the maximum displacement of the wave.

2. Explain the concept of phase and phase difference in oscillatory motion. Discuss how phase is related to the position and time in an oscillating system.

Ans)

1. **Phase (φ):**
   - Phase is a measure of the position of an oscillating object or wave within its cycle at a specific point in time.
   - It is often represented in radians (or degrees) and is used to describe how much of a full cycle has been completed.
   - A phase of 0 radians (or 0 degrees) corresponds to the starting point of a cycle, while a phase of 2π radians (or 360 degrees) represents one complete cycle.
   - Phase provides information about the current position or state of an oscillatory system.
2. **Phase Difference ($\Delta\varphi$):**
   - Phase difference, denoted as $\Delta\varphi$, quantifies the difference in phase between two oscillating objects or waves.
   - It indicates how much one oscillation is ahead or behind the other in terms of their positions within their respective cycles.
   - Phase difference can be positive, negative, or zero, depending on the relative positions of the oscillations.
   - It is usually measured in radians (or degrees) and can range from $-\pi$ to π radians (or -180 degrees to 180 degrees).

Relation of phase to the position and time: It provides valuable information about where an oscillating object or wave is in its cycle at a given point in time. Let's explore this relationship in more detail:

1. **Phase and Position:** In oscillatory motion, especially in the context of simple harmonic motion (SHM) or waves, an object's position can be represented as a function of time. The phase of the oscillation is used to express this position within the cycle.

   $x(t) = A * \sin(\omega t + \varphi)$

   - $x(t)$ is the position at time t
   - A is the amplitude of the oscillation
   - ω is the angular frequency (related to the oscillation's period), and
   - φ is the phase angle.

2. Phase and Time:

- Phase is intrinsically tied to time in oscillatory systems. It's a measure of how much time has elapsed since the oscillation began or since a specific reference point in the cycle.
- The phase angle (φ) in the equation above represents the phase at a particular moment in time. As time progresses, the phase angle increases linearly with time.
- For example, if $\varphi = 0$ radians (or 0 degrees), it corresponds to the starting point of the cycle ($t = 0$). As time increases, φ also increases, indicating the position of the oscillating object within the cycle at any given time.

3. Explain the concept of damping and its effects on oscillatory motion. Discuss the types of damping, such as over-damping, under-damping, and critical-damping.

Ans) Damping: Damping is a phenomenon in oscillatory motion that involves the gradual reduction of the amplitude (the maximum displacement from equilibrium) of the oscillations over time. It occurs due to the presence of external forces or factors that oppose or dissipate the energy of the oscillating system. Damping has significant effects on oscillatory motion, and these effects depend on the level of damping.

Effects of Damping on Oscillatory Motion:

- Damping reduces the amplitude of oscillations over time, with stronger damping leading to quicker decay.
- The presence of damping affects the period of oscillation. In underdamped systems, the period may be slightly longer compared to the undamped case.
- Damping introduces a phase shift between the oscillatory motion and the driving force or initial conditions, leading to a lag in the response.
- Critical damping ensures the fastest return to equilibrium without overshooting, making it ideal for certain applications.

Types of damping:

1. Overdamping:

- Overdamping occurs when the damping force is relatively strong, causing the oscillations to decay to zero without oscillating back and forth.
- In an overdamped system, the motion returns to the equilibrium position gradually and smoothly, without any overshooting.
- Overdamping is common in scenarios where a quick return to equilibrium without oscillations is desired. Examples include shock absorbers in vehicles or damping systems in engineering applications.
- The time it takes for an overdamped system to return to equilibrium is longer compared to the other damping types.

2. Underdamping:

- Underdamping occurs when the damping force is present but not strong enough to prevent oscillations. As a result, the oscillations persist, but they gradually decrease in amplitude over time.
- In underdamped systems, the motion exhibits oscillatory behavior, with the amplitude decreasing in an exponential manner.
- The time it takes for an underdamped system to return to equilibrium is shorter than for an overdamped system.
- Examples of underdamped systems include simple harmonic oscillators with friction and lightly damped pendulums.

3. Critical Damping:

- Critical damping is a special case where the damping force is adjusted precisely to eliminate oscillations as quickly as possible without overshooting or lingering near the equilibrium position.
- This damping level is often desired in applications where rapid damping without oscillations is essential. Examples include door closers and safety mechanisms in machinery.

4. Discuss the concept of resonance frequency and its relationship to the natural frequency of an oscillating system.

Ans) Resonance frequency: Resonance frequency is a crucial concept in physics and engineering, where a system exhibits maximum oscillation when subjected to an external force matching its natural frequency. This natural frequency depends on the system's physical properties. At resonance, energy transfer is highly efficient, leading to significant oscillation amplification. Examples include musical instruments producing tones, structural elements responding to external forces, and electronic circuits tuned for specific frequencies. Damping can mitigate resonance effects by dissipating energy. Resonance is both a beneficial tool and a potential source of problems, demanding careful consideration and control in various scientific and engineering applications.

Resonance frequency relationship with the natural frequency: The relationship between resonance frequency and the natural frequency is straightforward:

1. **Resonance at Natural Frequency:** When an external force or input is applied to the system at the same frequency as its natural frequency, resonance occurs. The system's response becomes highly amplified, and the amplitude of oscillation can increase significantly. This is because energy is efficiently transferred from the external source to the system.
2. **Resonance Near Natural Frequency:** Resonance can still occur when the applied frequency is very close to, but not exactly at, the natural frequency. In this case, the system's response is still amplified, although not as dramatically as at the exact natural frequency.
3. **Resonance at Harmonic Frequencies:** Resonance can also happen at integer multiples (harmonics) of the natural frequency. For instance, if the applied

frequency is twice the natural frequency, resonance can occur at the second harmonic frequency.

5. Discuss the factors that affect the period of a simple pendulum. Explain how the length of the pendulum, the acceleration due to gravity, mass and the amplitude of oscillation influence the period.

Ans)

1. **Length of the Pendulum (L):** The period of a simple pendulum is directly proportional to the square root of its length. The formula for the period (T) of a simple pendulum is:

   $$T = 2\pi\sqrt{(L/g)}$$

   As you can see, if you increase the length of the pendulum, the period will increase as well, making the pendulum swing more slowly.
2. **Acceleration Due to Gravity (g):** The period of a simple pendulum is inversely proportional to the square root of the acceleration due to gravity. This means that if the value of gravity changes, the period will be affected. For example, if you were on a planet with stronger gravity, the pendulum would have a shorter period, and on a planet with weaker gravity, it would have a longer period.
3. **Mass of the Pendulum Bob (m):** The mass of the bob at the end of the pendulum does not affect the period. This is a fundamental principle of simple pendulum motion, known as the equivalence principle. The period depends only on the length of the pendulum and the acceleration due to gravity, not on the mass of the bob.
4. **Amplitude of Oscillation:** The time period of simple pendulum does not depend on amplitude of oscillations.

ERQs:

1. Define simple harmonic motion (SHM). Discuss the key characteristics of a system undergoing SHM.

Ans) Simple Harmonic Motion: A Simple Harmonic Motion, or SHM, is defined as a motion in which the restoring force is directly proportional to the displacement of the body from its mean position. The direction of this restoring force is always towards the mean position.

1. **Restoring Force**: SHM occurs when a system experiences a restoring force proportional to its displacement from its equilibrium position. This restoring force acts in the opposite direction to the displacement, causing the system to return to its equilibrium position.

2. **Linearity**: The restoring force is directly proportional to the displacement from the equilibrium position. This means that the system follows Hooke's Law, which states that the force is proportional to the displacement: $F = -k\,x$
3. **Periodic Motion**: SHM is a type of periodic motion, which means the system repeats its motion over regular intervals of time. The motion is repetitive and can be described as a sinusoidal function (sine or cosine) of time.
4. **Constant Amplitude**: In SHM, the amplitude remains constant.
5. **Constant Frequency**: The frequency of oscillation is also constant in SHM. It is determined by the characteristics of the system and is independent of the amplitude.
6. **Phase Angle**: The motion of a system undergoing SHM can be described by a sinusoidal function, which includes a phase angle. The phase angle determines the initial conditions of the motion and where the oscillation is in its cycle at a given time.
7. **Energy Conservation**: In the absence of damping forces, the total mechanical energy (the sum of kinetic and potential energy) of a system undergoing SHM remains constant. Energy is exchanged between kinetic and potential energy as the system oscillates, but the total energy remains constant.
8. **Angular and Linear SHM**: SHM can occur in both angular and linear forms. Angular SHM involves the oscillation of an angle (e.g., the motion of a pendulum), while linear SHM involves the oscillation of a linear position (e.g., the motion of a mass on a spring).
9. **Damping and Driving Forces**: Damping forces (e.g., air resistance or friction) and driving forces (e.g., an external force applied periodically) are not considered in SHM.

2. Derive the equation of motion for a mass-spring system in SHM, illustrating each step of the derivation.

Ans) Consider a mass m attached to a spring with a spring constant k. The motion is in one dimension, along the x-axis.

According to Newton's second law

$$F = ma \longrightarrow (1)$$

The restoring force in this case is given by Hooke's Law

$$F = -kx \longrightarrow (2)$$

Comparing equation (1) and (2)

$$ma = -kx$$

$$a = -\frac{k}{m}x$$

Since, $a = \frac{d^2x}{dt^2}$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

This is a standard form of a differential equation for SHM. The solution to this equation is of the form:

$$\mathbf{x(t) = A\cos(\omega t+\varphi)}$$

3. Discuss the concept of energy in SHM. Explain how kinetic energy and potential energy vary throughout the motion of a particle in SHM and how the total mechanical energy is conserved.

Ans) Consider a spring-mass system, when the mass is pulled towards right and released it moves towards the equilibrium position. The speed is greatest as the object passes through the equilibrium position. The object slows down as it reaches to the end points. This phenomenon indicates the inter conversion of kinetic and potential energies of the system at different points but the total mechanical energy of the system at any instant shall remain constant.

E = kinetic energy + Potential energy

Inter Conversion of Kinetic and Potential Energies during SHM:

Kinetic Energy (K) of the Oscillator: Since the kinetic energy at any instant of the system is given by

$$K = \frac{1}{2}mv^2$$

Where, v is the instantaneous velocity of the system.

Using equations,

$$v = \omega x_o\sqrt{1-\frac{x^2}{x_o^2}} \quad \text{and} \quad \omega = \sqrt{\frac{k}{m}}$$

$$K = \frac{1}{2}m\left[x_o\sqrt{\frac{k}{m}}\sqrt{1-\frac{x^2}{x_o^2}}\right]^2$$

$$K = \frac{1}{2}k(x_o^2 - x^2)$$

Since the speed is maximum at equilibrium position i.e. at x= 0. The maximum kinetic energy of the object at equilibrium point is

$$K_{max} = \frac{1}{2}kx_o^2$$

As the object is instantaneously at rest on extreme position, where v = 0 and $x = x_o$. Therefore;

$$K = \frac{1}{2}k(x_o^2 - x_o^2) = 0$$

Potential Energy (U) of the Oscillator:

The net force on the oscillator at equilibrium position is F_o = zero and at extreme point is F_A= kx. The average applied force exerted on the system in displacing it from equilibrium position to extreme position is

$$F_{av} = \frac{F_o + F_A}{2} = \frac{0 + kx}{2} = \frac{1}{2}kx$$

The work done in moving the object from equilibrium position to extreme position, against the elastic restoring force

$$W = F_{av}.x = \frac{1}{2}kx^2$$

This work is stored in the spring-mass system as its elastic potential energy U

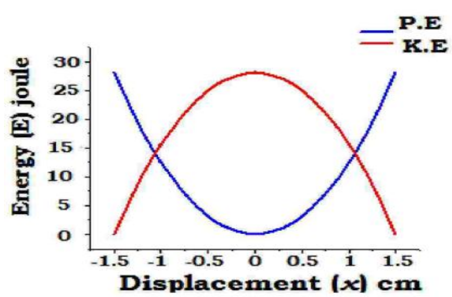
$$U = \frac{1}{2}kx^2$$

This equation expresses the instantaneous elastic potential energy of the object executing simple harmonic motion.

Mathematically, potential energy depends upon x, the instantaneous displacement. Hence the potential energy shall be maximum at the extreme positions.

$$U_{max} = \frac{1}{2}kx_o^2$$

and minimum at equilibrium position where x = 0



The graph shows the elastic potential energy and kinetic energy as a function of position, for a mass oscillating on a spring.

Total Energy (E) of the Oscillator: The expression of total energy of a system executing SHM is

E = kinetic energy + Potential energy

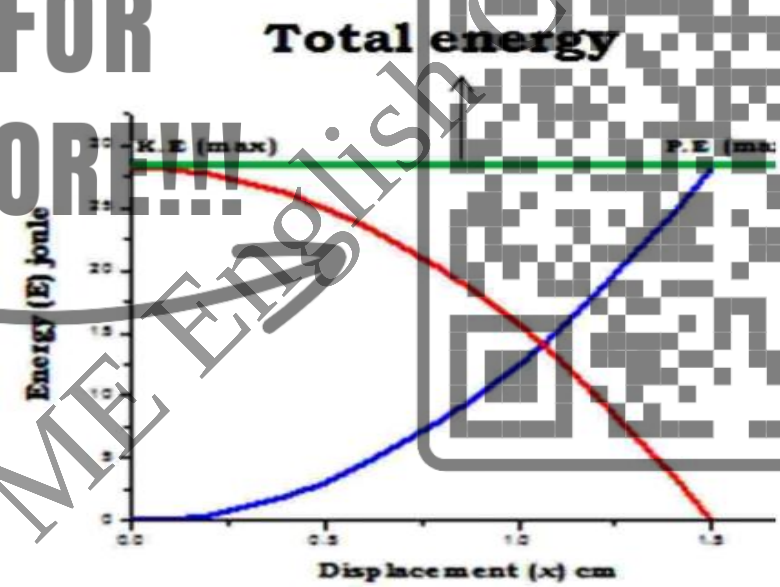
Since, $K = \frac{1}{2}k(x_o^2 - x^2)$ and $U = \frac{1}{2}kx^2$

$$E = \frac{1}{2}k(x_o^2 - x^2) + \frac{1}{2}kx^2$$

$$E = \frac{1}{2}kx_o^2$$

The graphs show that the elastic potential energy is zero where the displacement is zero and maximum at extreme position. Contrary to P.E, kinetic energy is maximum at zero displacement i.e. at equilibrium position and minimum at extreme positions.

The total energy of a body executing SHM at any point is constant.



4. Discuss the concept of resonance in simple harmonic motion. Explain how resonance occurs and its effects on the amplitude and energy transfer in a driven oscillating system.

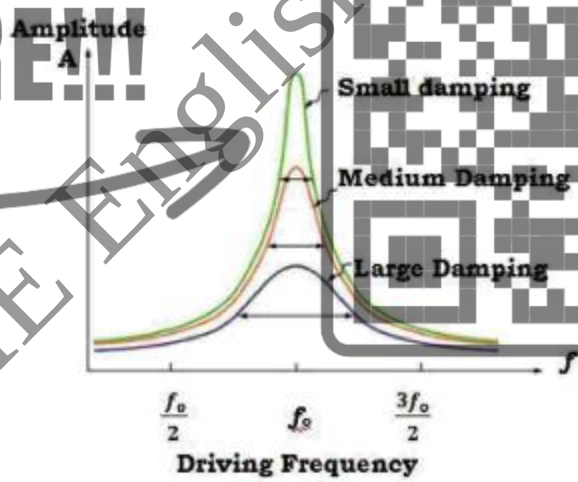
Ans) Resonance:

Definition: If the frequency of external driving force f is continued to increase and if it becomes equal or integral multiple of natural frequency f_o of the system such that

$f_{external} = f_1$ or $= 2f_1$ or $= 3f_1$....... or $= nf_1$...

The amplitude of the motion is maximum, this condition is called resonance.

Effects of resonance on amplitude and energy transfer: At resonance the driving force is always in the same direction as the object's velocity. Since the driving force is always doing positive work, the energy of the oscillator builds up until the dissipation of energy balances the energy added by the driving force. For an oscillator with little damping, the amplitude becomes large (Fig. below) When the driving force is not at resonance, some negative work is stored in the system. Hence the net work done by the driving force decreases as the driving frequency moves away from the resonance. Therefore the oscillator's energy and amplitude is smaller than at resonance.



The curves show a relation between amplitude and driving frequency of a harmonic oscillator. The curves represent the same oscillator with the same natural frequency but with different amounts of damping Resonance occurs when the driving frequency equals the natural frequency

5. Discuss the factors that affect the sharpness of resonance in an oscillatory system. Explain how damping and quality factor influence the width and peak of the resonance curve.

Ans) Factors that affect the sharpness of resonance: The sharpness of resonance depends mainly on two factors: amplitude and damping.

1. **Amplitude (A)**: The amplitude of resonance is directly related to the sharpness of the resonance peak.
   Specifically:
   - **Higher Amplitude**: When the amplitude of resonance is high, it means that the system's response to the driving force is pronounced. This results in a sharper and more prominent resonance peak in the frequency response curve.
   - **Lower Amplitude**: Conversely, when the amplitude of resonance is low, the system's response to the driving force is weaker, leading to a broader resonance curve with less pronounced peak.
2. **Damping (ζ):** The damping ratio (ζ) quantifies the amount of damping in the system. It is typically classified into three regimes:
   - **Underdamped ($\zeta < 1$)**: In this case, damping is relatively low. The resonance peak is sharp, and the system can oscillate for an extended period at its resonant frequency before the amplitude decays.
   - **Critically Damped ($\zeta = 1$)**: This represents a balance between damping and oscillation. The resonance peak is moderately sharp, and the system returns to its equilibrium position relatively quickly.
   - **Overdamped ($\zeta > 1$)**: Here, damping is high, resulting in a broad resonance curve with a shallow peak. The system returns to equilibrium rapidly without significant oscillation.

The influence of damping and quality factor on the width and peak of the resonance curve:

1. **Damping (ζ)**:
   - **Effect on Resonance Width**: Higher damping (ζ) leads to a broader resonance curve. This means that the range of frequencies over which the system responds significantly to an external force is wider.
   - **Effect on Resonance Peak**: Increasing damping reduces the maximum amplitude (peak) of the resonance curve. This is because damping dissipates energy from the system, limiting the maximum amplitude the system can achieve at its resonant frequency. As damping increases, the resonance peak becomes lower, indicating that the system's response to the driving force is weaker.

2. **Quality Factor (Q)**:
   - **Effect on Resonance Width**: A higher quality factor (Q) corresponds to a narrower resonance curve. This means that the system responds significantly to a narrower range of frequencies centered around the resonant frequency. A lower Q value indicates a broader resonance curve, where the system responds over a wider range of frequencies.
   - **Effect on Resonance Peak**: The quality factor (Q) directly affects the height of the resonance peak. A higher Q value results in a taller, more pronounced resonance peak. Conversely, a lower Q value leads to a shorter, less prominent resonance peak.

6. Explain how the period of a mass-spring system can be independent of amplitude, even though the distance travelled during each cycle is proportional to amplitude.

Ans) The independence of the period of a mass-spring system on its amplitude can be understood by following equations.

$T = \frac{1}{f}$ and $T = 2\pi\sqrt{\frac{m}{k}}$

Notice that the amplitude (A) does not appear anywhere in these equation. This mathematical relationship demonstrates explicitly that the period is completely unaffected by changes in the amplitude.

To put it more simply, regardless of whether the amplitude of the oscillation is large or small, the period of the mass-spring system remains the same.

7. A mass hanging vertically from a spring and a simple pendulum both have a period of oscillation of 1s on Earth. The two devices are sent to another planet, where gravitational field is stronger than that of Earth. For each of the two systems, state whether the period is now longer than 1s, shorter than 1s, or equal to 1s. Explain your reasoning.

Ans) On Earth, the period of oscillation for both a mass hanging vertically from a spring and a simple pendulum is 1 second. When these systems are taken to another planet with a stronger gravitational field, we need to consider how the change in gravity affects their periods.

1. **Mass Hanging Vertically from a Spring**: The period will be shorter than 1 second.
   The period of oscillation for a mass hanging from a spring depends on the gravitational field strength (g) and the mass (m) of the object, as given by the formula:

   $T = 2\pi\sqrt{\frac{m}{g}}$

If the gravitational field on the new planet is stronger than that of Earth, 'g' will be larger. Since 'g' is in the denominator of the period formula, an increase in 'g' will result in a shorter period. Therefore, the period will be shorter on the planet with a stronger gravitational field.

2. **Simple Pendulum**: The period will be shorter than 1 second.
   Reasoning: The period of a simple pendulum depends on the gravitational field strength (g) and the length (L) of the pendulum, as given by the formula:

   $$T = 2\pi \sqrt{\frac{L}{g}}$$

   Similar to the mass-spring system, if the gravitational field on the new planet is stronger than that of Earth, 'g' will be larger. Again, since 'g' is in the denominator of the period formula, an increase in 'g' will result in a shorter period. Therefore, the period of the simple pendulum will also be shorter on the planet with a stronger gravitational field.

Numericals:

1. The period of oscillation of an object in an ideal spring and mass system is 0.50s and the amplitude is 5.0 cm. what is the speed at the equilibrium point? and the acceleration at the point of maximum extension of the spring.

Data:

$T = 0.50$ s

$x_{max} = 5.0 \text{ cm} = \frac{5}{100} = 0.05$ m

$v_{max} = ?$

Solution:

$\omega = \frac{2\pi}{T}$

$\omega = \frac{2\pi}{0.5} = 12.566$ rad/s

$v_{max} = \omega\, x_{max}$

$v_{max} = (12.566)(0.05) = 0.628$ m/s

$\mathbf{v_{max} = 0.628 \times 100 = 62.8}$ **cm/s**

$a_{max} = -\omega^2 x_{max}$

$\mathbf{a_{max} = -(12.566)^2 (0.05) = -7.9}$ $\mathbf{m/s^2}$

The negative sign indicates that acceleration is in opposite direction to displacement.

2. A sewing machine needle moves with a rapid vibratory motion, like SHM, as it sews a seam. Suppose the needle moves 8.4 mm from its highest to its lowest position and it makes 24 stitches in 9.0s. What is the maximum needle speed?

Data:

$h = 8.4\text{ mm} = \frac{84}{1000} = 0.0084\text{ m}$

No. of stitches = 24 stitches

$t = 9.0\text{s}$

$v_{max} = ?$

Solution:

$T = \frac{t}{\text{No.of stitches}}$

$T = \frac{9}{24} = 0.375$ stitches/s

$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.375} = 16.755$ rad/s

$x_{max} = 0.0084/2 = 4.2\text{x}10^{-3}$ m

$v_{max} = \omega\ x_{max}$

$v_{max} = (16.755)\ (4.2\text{x}10^{-3}) = 0.07$ m/s

$v_{max} = 0.07 \text{ x } 100 = 7$ cm/s

3. An ideal spring with a spring constant of 15 N/m is suspended vertically. A body of mass 0.60 kg is attached to the unstretched spring and released.

(a) What is the extension of the spring when the speed is a maximum?

(b) What is the maximum speed?

Data:

$k = 15$ N/m

$m = 0.60$ kg

(a) $x = ?$

(b) $v_{max} = ?$

Solution:

(a)

$F = W = k\,x_{max}$

$mg = k\,x_{max}$

$x_{max} = \frac{mg}{k}$

$x_{max} = \frac{(0.6)(9.8)}{15}$

$\mathbf{x_{max} = 0.39\ m}$

(b)

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{15}{0.6}} = 5$ rad/s

$v_{max} = \omega\, x_{max} = (5)(0.39) = 1.95\ m \approx 2\ m$

4. A body is suspended vertically from an ideal spring of spring constant 2.5 N/m. the spring is initially in its relaxed position. The body is then released and oscillates about its equilibrium position. The motion is described by y = (4.0cm) sin[(0.70rad/s) t]. What is the maximum kinetic energy of the body?

Data:

$k = 2.5$ N/m

$y = (4.0cm)\sin[(0.70rad/s)\,t]$

$K_{max} = ?$

Solution:

$y = (4.0cm)\sin[(0.70rad/s)\,t]$

Here $x_o = 4\ cm = \frac{4}{100} = 0.04\ m$

$K_{max} = \frac{1}{2} k x_o^2 = \frac{1}{2}(2.5)(0.04)^2 = 2 \times 10^{-3}\ J = 2\ mJ$

5. The period of oscillation of a simple pendulum does not depend on the mass of the bob. By contrast the period of a mass-spring system does depend on mass. Explain the apparent condition.

Ans) Simple Pendulum: The period of a simple pendulum is determined by the length of the pendulum and the acceleration due to gravity (g). The formula for the period (T) of a simple pendulum is given by:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Where:

- T is the period of oscillation.
- L is the length of the pendulum
- g is the acceleration due to gravity (approximately 9.81 m/s^2 on Earth)

As you can see, the mass of the bob (the object at the end of the pendulum) does not appear in this equation. Therefore, the mass of the bob does not affect the period of the pendulum.

Mass-Spring System: To calculate the period of oscillation (T) for a mass-spring system, you need to consider the mass of the object (m) and the spring constant (k). The formula for the period of a mass-spring system is:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

As you can see, the mass (m) is present in this equation, so the period of oscillation for a mass-spring system depends on the mass of the object attached to the spring.

6. What is the period of a simple pendulum of a 6.0 kg mass oscillating on a 4.0 m long string?

Data:

T = ?

m = 6 kg

L = 4.0 m

Solution:

$T = 2\pi\sqrt{\frac{L}{g}}$

$\mathbf{T = 2\pi\sqrt{\frac{4}{9.8}} = 4.01\ s}$

7. A pendulum of length 75 cm and mass 2.5 kg swings with a mechanical energy of 0.015 J. what is its amplitude?

Data:

$L = 75\ cm = \frac{75}{100} = 0.75\ m$

m = 2.5 kg

E = 0.015 J

Solution:

$\omega = \sqrt{\frac{g}{L}} = \frac{9.8}{0.75} = \sqrt{13.067}$ rad/s

$E = \frac{1}{2} m\omega^2 A^2$

$0.015 = \frac{1}{2} (2.5) (13.067)^2 A^2$

A = 0.03 m = 0.03 x 100 = 3 cm

8. A pendulum of length L_1 has a period of T_1 = 0.950 s. The length of the pendulum is adjusted to a new value L_2 such that T_2 = 1.00 s what is the ratio L_2/L_1.

Data:

T_1 = 0.950 s

T_2 = 1.00 s

L_2/L_1 = ?

Solution:

$$\frac{T_1}{T_2} = 2\pi\sqrt{\frac{L_1}{g}} \div 2\pi\sqrt{\frac{L_2}{g}}$$

$$\frac{T_1}{T_2} = 2\pi\sqrt{\frac{L_1}{g}} \times \frac{1}{2\pi}\sqrt{\frac{g}{L_2}}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{L_1}{L_2}}$$

$$\frac{T_2}{T_1} = \sqrt{\frac{L_2}{L_1}}$$

$$\sqrt{\frac{L_2}{L_1}} = 1.111$$

$$\mathbf{\frac{L_2}{L_1} = 1.234}$$

9. A wire is hanging from the top of a tower such that the top is not visible due to darkness. How do you calculate the height of tower?

Ans) Connect some mass with the end of the wire. Take a stop watch and set the pendulum in vibration. Calculate the time period of the pendulum. You may find the time of, say ten vibrations, and divide it by 10 to find the time period of the pendulum.

Now equation of the simple pendulum is

$$T = 2\pi\sqrt{\frac{L}{g}}$$

π and g are constants. Use the value of T you just calculated in the above equation and find the length of the wire. As the wire hangs from the top of the tower, it will give you the height of the tower.

Note that ℓ gives you the length of the wire and if the wire is above the ground (and principally it should be) add the remaining part to the length of the wire.

10. The amplitude of oscillation of a pendulum decays by a factor of 20.0 in 120 s. By what factor has its energy decayed in that time.

Data:

$t = 120$ s

$A' = \frac{A}{20}$

Solution:

$E = \frac{1}{2} m\omega^2 A^2$

$E' = \frac{1}{2} m\omega^2 A'^2$

$E' = \frac{1}{2} m\omega^2 (\frac{A}{20})^2$

$E' = \frac{1}{400} \frac{1}{2} m\omega^2 A^2$

$\mathbf{E' = \frac{1}{400} E}$

(The energy has decreased by a factor of 400)

Unit #12: Acoustics

Worked Example 12.1

If the velocity of sound in air at 27°C and at a pressure of 76 cm of mercury is 345 ms^{-1}. Find the velocity at 127°C and 75 cm of mercury.

Solution:

There is no effect of change of pressure on the velocity of sound.

Step 1:

Write the known quantities and point out quantities to be found.

$T_1 = 27°C = 27+273 = 300K$

$T_2 = 127°C = 127+273 = 400K$

$v_1 = 345ms^{-1}$

Required: Speed of sound v_2 at 127°C

Step 2:

Write down the formula and rearrange if necessary

Using equation

$$\frac{v_t}{v_o} = \sqrt{\frac{T}{T_o}}$$

Step 3:

Put the values in the formula and calculate.

$$\frac{v_2}{v_1} = \sqrt{\frac{273+127}{273+27}} = \sqrt{\frac{4}{3}}$$

Hence;

$$v_2 = \frac{2}{\sqrt{3}} \times v_1 = \frac{2}{\sqrt{3}} \times 345 = \mathbf{398.4\ ms^{-1}}$$

Worked Example 12.2

In a game of cricket match a spectator is sitting in the stand at a distance of 60.0 m away from the batsman. How long does it take the **sound** of the bat connecting with the ball hit for a six to travel to the spectator's **ears**? if the temperature of air is 27°C.

Solution:

Step 1: Write the known quantities and point out quantities to be found.

$s = 60.0$ m

$T = 27^{\circ}C = 27+273 = 300K$

Required: t,

time in which spectator will hear the sound.

Step 2: Write down the formulae and rearrange if necessary

Using equation $v_t = v_o + 0.61t$ °C

and $s = vt$

Step 3: Put the values in the formula and calculate.

$v_t = 332 + 0.61 \times 27 = 348.47$ ms^{-1}

Using $s = v \times t$ ∴ $t = s/v$

$$t = \frac{60.0}{348.47} = \mathbf{0.172\ s}$$

Worked Example 12.3

A source of sound and listener are moving towards each other with velocities which are 0.5 times and 0.2 times the speed of sound respectively. If the frequency of emitting sound is 2000 Hz, calculate the percentage change in the frequency with respect to the listener.

Solution:

Step 1: Write the known quantities and point out the quantities to be found.

Speed of source; $v_s = 0.5v$

Speed of listener; $v_o = 0.2v$

$f = 400$Hz

Required: $f_o = ?$

Step 2: Write the formula and rearrange if necessary. $f_o = \left(\frac{v + v_o}{v - v_s}\right) f_s$

Step 3: Put the values in the formula and calculate.

$f_o = \left(\frac{v + 0.2v}{v - 0.5v}\right) \times 2000$ taking v common

$f_o = \left(\frac{1.2}{0.5}\right) \times 2000 = \mathbf{4800\ Hz}$

Percentage in Frequency $= \left(\frac{f_o - f_s}{f_s}\right) \times 100\% = \left(\frac{4800-2000}{2000}\right) \times 100\% = \mathbf{140\%}$

Section (A): Multiple Choice Questions (MCQs)

1. The speed v of a wave represented by y= A sin(t - kx) is:

a) k/ω b) ω/k

c) ωk d) 1/ωk

2. Two sound waves are y = A sin(ωt - kx) and y = A cos(ωt - kx). The phase difference between the two waves is:

a) π/2 b) π/4

c) π d) 0

3. If v_a, v_h and v_m are the speeds of sound in air, hydrogen gas, and a metal at the same temperature, then:

a) $v_a > v_h > v_m$ b) $v_m > v_h > v_a$

c) $v_h > v_m > v_a$ d) $v_h > v_a > v_m$

4. The speed of sound in air at STP is 332 m/s. If the air pressure becomes double at the same temperatue, the speed of sound become:

a) 1382 m/s b) 664m/s

c) 332 m/s d) 166 m/s

5. How does the speed of sound v in air depend on the atmospheric pressure P.

a) $v \propto P^o$ b) $v \propto P^{-1}$

c) $v \propto P^2$ d) $v \propto P^1$

6. The speed of sound in a gas is proportional to:

a) Square root of isothermal elasticity b) square root of adiabatic elasticity

c) Isothermal elasticity d) adiabatic elasticity

7. The length of a pipe closed at one end is L. In the standing wave whose frequency is 7 times the fundamental frequency, what is the closest distance between nodes?

a) 1/14 L b) 1/7 L

c) 2/7 L d) 4/7 L

8. A 620 Hz frequency song of a ice cream trolley approaches with speed v to a boy standing at the door of his house is heard with frequency f_1. If the trolley is stopped and the boy approaches to the ice cream trolley with same speed v; the boy now hears the sound with frequency f_2 choose the correct statement:

a) $f_1 = f_2$; both are greater than 620Hz   b) $f_2 > f_1 >$ 620Hz

c) $f_1 = f_2$; both are lesser than 620Hz   d) $f_1 > f_2 >$ 620Hz

9. The speed of sound in a gas in which two waves of wavelength 50 cm and 50.4 cm produce 6 beats per second is:

a) 303 m/s   b) 350 m/s

c) 378 m/s   d) 400 m/s

10. The speed of a wave in a medium is 760 m/s. If 3600 waves are passing through a point in the medium in 2 minutes, then its wave length is:

a) 13.85 m   b) 25.3 m

c) 41.5 m   d) 57.2 m

KEY:

1. b	2. a	3. b	4. c	5. c
6. b	7. c	8. b	9. a	10.b

CRQs:

1. Why sound can not travel through vacuum.

Ans) Sound is a mechanical wave, so to propagate it, some material or medium is required. We know a vacuum is an empty space where no matter particles are present. Sound cannot travel through a vacuum as there are no particles present for vibrations to take place.

2. On what factors speed of sound depends on?

Ans) **Speed of sound depends upon following factors:**

1. **Density of air:**

   From the relation, $v = \sqrt{\frac{\text{Elastic modulus of medium}}{\text{Density of the medium}}}$

   It is clear that speed of sound varies inversely to the square root of density of air.
2. **Moisture:** Moisture is the presence of a liquid, especially water in other media, small amounts of water may be found, for example, in the air (called humidity). The presence of moisture in the air decreases the resultant density of air which

increases the speed of sound in humidity. Hence the speed of sound in damp (wet) air is greater than in dry air.

3. **Pressure:**

$$v = \sqrt{\frac{\gamma RT}{M}}$$

The above equation shows that speed of sound is independent of pressure of gas (air).

4. **Temperature:** Temperature changes do not affect the speed of sound in liquid and solid media quite significantly. But for a gas (air) the rise and fall of temperature at constant pressure significantly increases or decreases the volume of gas, and thus the density of gas is changes inversely which results in the increase of velocity.
5. **Wind:** If v_w is the wind speed then the speed of sound along the direction of wind relative to ground is $(v + v_w)$ and $(v - v_w)$ against the direction of wind.

3. What are the conditions for the interference of waves?

Ans) To observe interference, certain conditions must be met:

1. **Coherence:** The waves must be coherent, which means they have a constant phase relationship. In other words, the crests and troughs of the waves align with each other as they overlap. Coherence is crucial for maintaining a stable interference pattern.
2. **Similar Frequency and Wavelength**: Interfering waves should have similar frequencies and wavelengths. This ensures that the wave patterns overlap consistently. When waves with different frequencies or wavelengths interfere, complex patterns may result, making it more challenging to observe clear interference effects.
3. **Overlapping in Space and Time**: The waves must overlap in both space and time. This means they must intersect at the same location in space simultaneously. If waves arrive at different times or locations, they will not interfere with each other.
4. **Constant Wave Speed**: The waves should travel at a constant speed through the medium. Variations in wave speed can lead to phase differences that may affect interference patterns.
5. **Single Frequency Source (for Two-Source Interference)**: In cases of two-source interference (like the double-slit experiment for light or electron waves), a single-frequency source is required to ensure consistent interference patterns. If the source emits a range of frequencies, the interference pattern may become more complex.

4. Is the energy of a wave is maximum or minimum at nodes?

Ans) In a standing wave, nodes are points with minimal or zero displacement of the medium. Consequently, the energy of the wave is at a minimum at these locations. Nodes represent regions of low energy because the particles in the medium remain at rest or oscillate with minimal amplitude.

5. Why is it possible to understand the words spoken by two people at the same time?

Ans) In the realm of sound, understanding words spoken by two people simultaneously relies on the auditory system's remarkable capabilities. Sound waves from different voices possess unique characteristics, such as varying frequencies and amplitudes, enabling our ears to separate them into distinct streams.

ERQs:

1. Define a sound wave and explain its nature as a longitudinal wave. Discuss the key properties of sound waves, such as frequency, wavelength, amplitude, and speed of propagation.

Ans) Sound wave: Sound is a longitudinal/mechanical wave that is created by a vibrating object, such as a guitar string, the human vocal cords, or the diaphragm of a loudspeaker.

The nature of sound wave as a longitudinal wave: Here's how sound waves exhibit the characteristics of longitudinal waves

1. **Particle Displacement**: In a longitudinal wave, particles of the medium oscillate or vibrate back and forth in the direction of wave propagation. In the case of sound waves, air molecules (or molecules in any other medium through which sound travels) move parallel to the direction of the wave.
2. **Compression and Rarefaction**: Sound waves consist of regions of compression and rarefaction. In the compression phase, the air molecules are pushed closer together, creating a region of high pressure. In the rarefaction phase, the air molecules move farther apart, creating a region of low pressure. These alternating regions of compression and rarefaction propagate through the medium as the sound wave travels.
3. **Propagation of Energy**: Longitudinal waves transfer energy from one point to another by causing particles in the medium to pass energy along. In the case of sound waves, this energy transfer is achieved as the particles of the medium transmit the disturbance (vibrations) to neighboring particles.
4. **Wavefronts**: Longitudinal waves are often represented by wavefronts, which are surfaces connecting points of equal phase (equal displacement). In the case of sound waves, these wavefronts are typically depicted as concentric spheres originating from the sound source (e.g., a vibrating object or a speaker)

5. **Frequency and Pitch**: In longitudinal waves like sound waves, the frequency of the wave corresponds to the pitch of the sound. Higher-frequency sound waves have a higher pitch, while lower-frequency sound waves have a lower pitch.
6. **Amplitude and Loudness**: The amplitude of a longitudinal wave corresponds to the intensity or loudness of the sound. Larger amplitude sound waves carry more energy and are perceived as louder.

2. Discuss the concept of the Doppler Effect in sound waves. Explain how the motion of a source or observer affects the perceived frequency and pitch of a sound wave. Provide examples to illustrate the Doppler Effect in daily life.

Ans) Doppler Effect of Sound: The Doppler effect is a phenomenon in physics that describes the perceived change in frequency of a wave (such as sound or light) when the source of the wave or the observer is in relative motion. In the case of sound waves, the Doppler effect occurs when there is relative motion between the source of the sound and the observer. The key characteristic of the Doppler effect is that the frequency of the sound waves appears higher (or lower) to the observer depending on the direction of motion.

Effects on the perceived frequency and pitch of a sound wave: The motion of either the source of a sound wave or the observer can significantly affect the perceived frequency and pitch of the sound, and this effect is described by the Doppler effect. Let's explore how the motion of the source and the observer affects the perceived frequency and pitch:

1. **Motion of the Source**:
   - **Source Moving Towards the Observer**: If the sound source is moving towards the stationary observer, the perceived frequency ($f_{observed}$) of the sound wave will be higher than the actual emitted frequency (f_{source}). This is because the sound waves are "bunched up" as the source moves closer to the observer, resulting in shorter wavelengths and a higher frequency and thus higher pitch.
   - **Source Moving Away from the Observer**: If the sound source is moving away from the stationary observer, the perceived frequency ($f_{observed}$) of the sound wave will be lower than the actual emitted frequency (f_{source}). This is because the sound waves are "stretched out" as the source moves farther from the observer, resulting in longer wavelengths and a lower frequency and thus lower pitch.
   - **No Relative Motion**: When there is no relative motion between the source and the observer, the perceived frequency ($f_{observed}$) is equal to the emitted frequency (f_{source}). In this case, the observer hears the true pitch of the sound.

2. **Motion of the Observer**:
   - **Observer Moving Towards the Source**: If the observer is moving towards the stationary sound source, the perceived frequency ($f_{observed}$) of the sound wave will be higher than the actual emitted frequency (f_{source}). This is because the observer is encountering sound waves more frequently as they move closer to the source, resulting in a higher perceived frequency and thus higher perceived pitch.
   - **Observer Moving Away from the Source**: If the observer is moving away from the stationary sound source, the perceived frequency ($f_{observed}$) of the sound wave will be lower than the actual emitted frequency (f_{source}). This is because the observer is encountering sound waves less frequently as they move farther from the source, resulting in a lower perceived frequency and thus lower perceived pitch.

Doppler Effect in daily life:

1. The Doppler effect can be observed in everyday situations such as when a vehicle with a siren passes by. As the vehicle approaches, the sound of the siren is heard at a higher pitch (higher frequency) than its actual source frequency. When the vehicle moves away, the sound of the siren is heard at a lower pitch (lower frequency) than its actual source frequency.
2. The Doppler Effect is utilized in satellite communication to maintain accurate transmission of signals between satellites and ground stations. When a satellite is approaching a ground station, the transmitted radio signals experience a Doppler shift, causing the frequency to appear higher. To compensate for this shift and ensure reliable communication, the satellite's receiver adjusts accordingly. In situations where the satellite is stationary or moving away, there is no significant Doppler shift, and the transmitted and received frequencies remain consistent. This application is crucial for stable and continuous satellite communication.

3. What is standing wave? How they are produced? Also elaborate the concept of nodes and antinodes in standing waves. Explain their locations and the relationship between node spacing and wavelength.

Ans) Standing wave: Standing wave, also called stationary wave, combination of two waves moving in opposite directions, each having the same amplitude and frequency.

Production of standing waves: Standing waves are produced when two waves of the same frequency and amplitude travel in opposite directions and interfere with each other within a medium that has fixed boundaries. They are observed in various contexts, such as vibrating strings in musical instruments and sound waves in closed tubes.

Nodes:

- Nodes are points within a standing wave where the displacement (amplitude) of the wave is always zero. In other words, nodes are locations where the particles of the medium do not move from their equilibrium position; they remain stationary.
- Nodes are formed due to destructive interference between the two waves that create the standing wave. When a crest of one wave meets the trough of the other, they cancel each other out, resulting in zero displacement.
- The number and locations of nodes in a standing wave depend on the specific geometry of the medium and the mode of vibration. For example, in a vibrating string fixed at both ends, there will always be nodes at the two endpoints (fixed boundaries), and additional nodes may appear depending on the harmonic (mode) being produced.

Antinodes:

- Antinodes are points within a standing wave where the displacement (amplitude) of the wave is at its maximum. These are regions of the wave where the particles of the medium undergo the most significant displacement from their equilibrium positions.
- Antinodes are formed due to constructive interference between the two waves creating the standing wave. When two crests or two troughs coincide, they reinforce each other, resulting in maximum displacement.
- In a standing wave, antinodes alternate with nodes. The regions between adjacent nodes contain antinodes. The number and positions of antinodes are also determined by the specific characteristics of the medium and the harmonic being produced.

Locations of Nodes and Antinodes:

Nodes: Nodes are points within a standing wave where the displacement (amplitude) of the wave is always zero. These points correspond to positions in the medium where particles do not move from their equilibrium positions; they remain stationary.

Antinodes: Antinodes are points within a standing wave where the displacement (amplitude) of the wave is at its maximum. These regions represent positions in the medium where particles undergo the most significant displacement from their equilibrium positions.

Relationship Between Node Spacing and Wavelength: For a given mode (harmonic) of vibration in a specific medium, the distance between adjacent nodes is equal to half of the wavelength:

$$\text{Distance between nodes} = \frac{\lambda}{2}$$

This relationship holds true for all harmonics. In other words, as you move to higher harmonics (higher modes of vibration), the wavelength decreases, and consequently, the distance between nodes decreases by a factor of two for each successive harmonic.

4. Define the Doppler Effect and Derive the Doppler Effect equation for sound waves in terms of the relative velocity between the source, observer, and the speed of sound.

Ans) Doppler Effect of Sound: The Doppler effect is a phenomenon in physics that describes the perceived change in frequency of a wave (such as sound or light) when the source of the wave or the observer is in relative motion.

Moving Source:

Source is moving towards the observer at rest: If the source is moving at velocity v_s towards a stationary observer on the right, figure (a) shows that the wavelength, the distance between the crests is smaller to the right. Figure (b) shows that at the instant that crest 4 is emitted, crest 3 has travelled outward a distance vT_s from point 3. During the same time interval, the source has advanced a distance v_sT_s. The wavelength as measured by the observer on the right is the distance between crests 4 and 3:

$$\lambda' = vT_s - v_s T_s$$

The frequency at which the crests arrive at the observer is the observed wave frequency f'. The observed time period T' between the arrival of two crests is the time it takes sound to travel a distance $(v-v_s)T_s$.

$$T' = \frac{(v-v_s)\,T_s}{v}$$

The observed frequency is $f' = \frac{1}{T'} = \frac{v}{(v-v_s)} \times \frac{1}{T_s}$

Dividing numerator and denominator by v and substituting $f_s = \frac{1}{T_s}$ yields

$$f' = \left(\frac{v}{v-v_s}\right) \times f_s$$

This shows that the observed frequency is higher than the source frequency, when the source moves in the same direction as the wave towards the observer.

Source is moving away from the observer at rest: If the source is moving at velocity v_s away from the stationary observer on the left, then according to figure (a), the wavelength, the distance between the crests is larger to the left. The wavelength λ' as measured by the observer on the left in fig. (b) is the distance between crests 3 and 4:

$$\lambda' = vT_s + v_s T_s$$

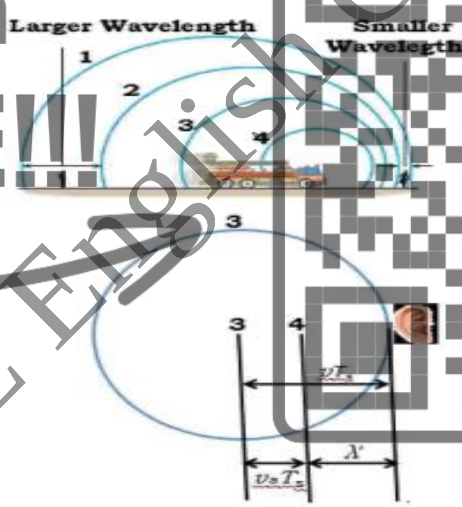
The frequency at which the crests arrive at the observer is the observed wave frequency f'. The observed time period T' between the arrival of two crests is the time it takes sound to travel a distance $(v + v_s)T_s$.

$$T' = \frac{(v+v_s)\,T_s}{v}$$

The observed frequency is $f' = \frac{1}{T'} = \frac{v}{(v+v_s)} \times \frac{1}{T_s}$

Dividing numerator and denominator by v and substituting $f_s = \frac{1}{T_s}$ yields

$$f' = \left(\frac{v}{v+v_s}\right) \times f_s$$



(a) A Fire engine is moving to the right at speed v_s while it blows its siren. The siren emits wave crests at position 1, 2, 3 and 4. Each wave crest moves outward in all directions, from the point at which it was emitted, at speed v. (b) The observed wavelength λ' is the distance between the wave crests.

Moving Observer:

Observer is moving towards the source at rest: A stationary source emits a sound wave at frequency f_s and wavelength $\lambda = v/f_s$, where v is the speed of sound. An observer moving towards the stationary source with speed v_o would observe a shorter time interval between crests. Just as crest 1 reaches the observer, the next crest 2 is at distance λ_s ahead. Crest 2 catches up with the observer with time T' earlier than the time T_s, when the distance vT' the wave crest 2 travels toward the observer is equal to wavelength of sound minus the distance the observer travels towards the wave crest 2 as shown in figure below.

$$vT' = \lambda_s - v_o T'$$
$$\lambda_s = vT' + v_o T'$$
$$\lambda_s = (v + v_o)\, T'$$

Given that f' = 1/T' and $\lambda_s = v/f_s$ replacing T' and λ_s in above equation

$$f' = \left(\frac{v + v_o}{v}\right) f_s$$

An observer is moving towards the stationary source with speed v_o. The observed frequency shall be greater than the source frequency.

Observer is moving away from the source at rest: Now consider an observer moving away from the source at velocity v_o. He observes a longer time interval between crests. Just as crest I reaches the observer, the next crest 2 is a distance λ_s away. Crest 2 catches up with the observer at a time T' later when T' the distance the wave crest travels toward the observer is equal to the distance the observer travels away from the wave crest plus the wavelength (Fig. below)

$$vT' = v_oT' + \lambda_s$$

or

$$(v - v_o)\,T' = \lambda_s$$

$$(v - v_o)\,T' = v/f_s$$

$$f' = \left(\frac{v - v_o}{v}\right)f_s$$

An observer moving away from the source detect a frequency lower than the f_s.

An observer is moving away from the stationary source with speed v_o. The observed frequency shall be lower than the source frequency.

Motion of Both Source and Observer:

Source & listener are moving towards each other: If the source and listener both move towards each other then we combine the two Doppler's shifts. First consider the relative change in frequency with respect to stationary listener as the source is approaching towards the listener. Let f_L be the frequency detected by the listener.

$$f_L = \left(\frac{v}{v - v_s}\right) \times f_s$$

If at some moment the listener starts moving towards the approaching source with velocity v_o then the detected frequency f_o shall be

$$f_o = \left(\frac{v + v_o}{v}\right) f_L$$

Substituting the value of f_L in this equation yields

$$f_o = \left(\frac{v + v_o}{v - v_s}\right) f_s$$

Source & listener are moving away each other: If the source and listener both move away from each other then we combine the two Doppler's shifts. First consider the relative change in frequency with respect to stationary listener as the source is moving away the listener. Let f_L be the frequency detected by the listener.

$$f_L = \left(\frac{v}{v + v_s}\right) \times f_s$$

If at some moment the listener starts moving away from source with velocity v_o then the detected frequency f_o shall be.

$$f_o = \left(\frac{v - v_o}{v}\right) f_L$$

Substituting the value of f_L in this equation yields

$$f_o = \left(\frac{v - v_o}{v + v_s}\right) f_s$$

5. Define standing waves and explain how they are formed.

Ans) **Standing wave:** Standing wave, also called stationary wave, combination of two waves moving in opposite directions, each having the same amplitude and frequency.

Production of standing wave: Here's a step-by-step explanation of how stationary waves are formed:

1. **Two Waves Traveling in Opposite Directions**: The process begins with the generation of two identical waves that travel in opposite directions along the same medium. These waves can be created in various ways, such as by a

source and its reflected wave or by two sources emitting waves that interfere with each other.

2. **Encounter and Overlap**: As the two waves travel through the medium, they encounter each other and overlap. At any given point in the medium, the displacements (amplitudes) of the two waves add together due to the principle of superposition.
3. **Interference**:
   - Interference occurs as the two waves combine. The specific type of interference depends on the relative phases of the waves at each point along the medium.
   - If the crests of one wave coincide with the crests of the other wave (constructive interference), they reinforce each other, resulting in regions of maximum displacement (antinodes).
   - If the crests of one wave coincide with the troughs of the other wave (destructive interference), they cancel each other out, resulting in regions of zero displacement (nodes).
4. **Stable Standing Wave Pattern**: When the two waves interfere in a manner that creates a stable pattern of nodes and antinodes, a stationary wave is formed. These waves appear to "stand still" because the nodes and antinodes do not move; they are stationary in space.
5. **Discuss the concept of harmonics in standing waves. Explain how harmonics are formed and their relationship with the fundamental frequency.**

Ans) Harmonics in standing waves: Harmonics in standing waves refer to the different modes of vibration or patterns of nodes and antinodes that can exist within a single medium. Here's a detailed explanation of the concept of harmonics in standing waves.

Production of harmonics: Harmonics are generated in standing waves when waves of different frequencies, including the fundamental frequency and its overtones, combine through constructive interference. The fundamental frequency is the simplest and lowest-frequency mode, with one antinode and two nodes. Higher harmonics are formed by waves with frequencies that are integer multiples of the fundamental frequency and introduce additional nodes and antinodes.

1. **Fundamental Frequency (First Harmonic)**:
   - The fundamental frequency, often referred to as the first harmonic, is the simplest and lowest-frequency mode of vibration in a standing wave.
   - In the fundamental frequency, there is one antinode (region of maximum displacement) and two nodes (points of zero displacement). For example: In a vibrating string fixed at both ends, the fundamental frequency has one antinode at the center of the string (maximum vibration) and two nodes at the two ends (anchored points).

2. **Higher Harmonics (Overtones)**:
    - Higher harmonics, also known as overtones, represent modes of vibration with frequencies that are integer multiples of the fundamental frequency.
    - Each successive harmonic introduces additional nodes and antinodes compared to the fundamental frequency. The number of nodes and antinodes increases with each higher harmonic.
    - The nth harmonic has n-1 nodes and n antinodes. For example: The second harmonic has two nodes and three antinodes and The third harmonic has three nodes and four antinodes.
3. **Relationship Between Harmonics and Wavelength**: The wavelength (λ) of each harmonic is related to the length of the medium and the harmonic number (n) by the equation:

    $$\lambda n = \frac{2L}{n}$$

    where L is the length of the medium.

    This equation shows that as the harmonic number (n) increases, the wavelength (λn) decreases. Higher harmonics have shorter wavelengths compared to the fundamental frequency.

4. **Harmonics in Music**: In music, harmonics are responsible for the timbre or tone quality of musical instruments. The presence and relative amplitudes of different harmonics in a sound wave give musical instruments their unique sound characteristics.

Relationship of harmonics with the fundamental frequency: Harmonics are frequencies that are integer multiples of the fundamental frequency in a vibrating system or sound wave.

- The frequency of the **first harmonic** (the fundamental frequency) is denoted as f_1.
- The frequency of the **second harmonic** (first overtone) is $2f_1$.
- The frequency of the **third harmonic** (second overtone) is $3f_1$.
- This pattern continues for higher harmonics: the nth harmonic has a frequency of nf_1.

Numericals:

1. The equation of a wave is y(x, t) = 3.5 sin {$\frac{\pi}{3.0}$x -66t} cm where t is in seconds and x and y both are in cm. Find (a) the amplitude and (b) the wavelength of this wave.

Data:

$y(x, t) = 3.5 \sin\{\frac{\pi}{3.0}x - 66t\}$ cm

(a) y_m = ?
(b) λ = ?

Solution:

(a) **y_m = 3.5 cm**

(b) We find λ from

$2\pi/\lambda = \frac{\pi}{3.0}$

λ = 6 cm

2. Why is it that your own voice sounds strange to you when you hear it played back on a tape recorder, but your friends all agree that it is just what your voice sounds like?

Ans) The perception of your own voice sounding strange when played back on a recording device is due to differences in how you hear your voice while speaking compared to how others hear it externally. When you speak, you experience both air conduction and bone conduction, with the latter giving your voice a unique resonance. Listening to a recording only involves air conduction, causing your voice to sound unfamiliar. Your brain has adapted to the way your voice sounds while speaking, and selective attention to content rather than sound contributes to this discrepancy. Friends, who are accustomed to your external voice, perceive the recorded voice as more familiar and natural, as they do not experience this perceptual shift.

3. Why the speed of sound in solids is much faster than the speed of sound in air?

Ans) The speed of sound in solids is significantly faster than in air primarily due to differences in density and stiffness. Solids are denser and have tightly packed particles, allowing sound waves to propagate quickly from one particle to another. Additionally, solids exhibit greater stiffness and stronger intermolecular forces, enabling efficient transmission of sound energy. In contrast, gases have lower density, weaker intermolecular forces, and more widely spaced particles, leading to slower sound wave propagation.

4. An increase in pressure of 100 kPa causes a certain volume of water to decrease by $5\text{x}10^{-3}$ percent of its original volume. Find (a) Bulk modulus of water. (b) What is the speed of sound in water?

Data:

$\Delta P = 100\text{ kPa} = 100000\text{ Pa}$

$\frac{\Delta V}{V} = 5\text{x}10^{-3}\ \% = 5\text{x}10^{-3}/100 = 5\text{x}10^{-5}$

(a) B = ?
(b) v = ?

Solution:

$B = \frac{\Delta P}{\frac{\Delta V}{V}}$

$\mathbf{B = \frac{100000}{5\text{x}10^{-5}} = 2\text{x}10^{9}\ Pa = 2\text{x}10^{9}/1\text{x}10^{6} = 2000\ MPa}$

$\mathbf{v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2\text{x}10^{9}}{1000}} = 1414.213\ m/s}$

5. A uniform string of length 10.0 m and weight 0.25 N is attached to the ceiling. A weight of 1.00 kN hangs from its lower end. The lower end of the string is suddenly displaced horizontally. How long does it take the resulting wave pulse to travel to upper end? (Neglect the weight of string in comparison to hanging mass)

Data:

L = 10.0 m

$W_1 = 0.25$ N

W2 = 1.00 kN = 1000 N

Δt = ?

Solution:

The speed of transverse wave is given by

$v = \sqrt{\frac{FL}{m}}$

In order to evaluate the speed of the wave in the Y-direction, we use the following relation

$v = \frac{\Delta y}{\Delta t}$

$\Delta t = \frac{\Delta y}{v}$

$\Delta t = \frac{\Delta y}{\sqrt{\frac{FL}{m}}}$

$\Delta y = L$

$\Delta t = \frac{L}{\sqrt{\frac{FL}{m}}}$

$\Delta t = \sqrt{\frac{mL}{F}}$

Multiply and divide by g then we get

$\Delta t = \sqrt{\frac{mgL}{Fg}}$

$\Delta t = \sqrt{\frac{W_1 L}{W_2 g}}$

$\mathbf{\Delta t = \sqrt{\frac{(0.25)(10)}{(1000)(9.8)}} = 0.0159\ s \approx 15.9\ ms}$

6. A travelling sine wave is the result of the superposition of two other sine waves with equal amplitudes, wavelengths, and frequencies. The two component waves each have amplitude 5.00 cm. if the resultant wave has amplitude 6.69 cm, what is the phase difference φ between the component waves?

Data:

$y = 6.69$ cm

$\varphi = ?$

Solution:

The given equations for the component waves are:

$y_1 = A \sin(\omega t + kx)$ and $y_2 = A \sin(\omega t + kx - \varphi)$

For resultant

$y = y_1 + y_2 = A \sin(\omega t + kx) + A \sin(\omega t + kx - \varphi)$

Since, $\sin\alpha + \cos\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2}$

$\alpha = \omega t + kx$ and $\beta = \omega t + kx - \varphi$

$y = 2A \sin \frac{(\omega t + kx) + (\omega t + kx - \varphi)}{2} \cos \frac{(\omega t + kx) - (\omega t + kx - \varphi)}{2}$

$y = 2A \sin(\omega t + kx - \frac{\varphi}{2}) \cos\frac{\varphi}{2}$

Since, $A \sin(\omega t + kx - \frac{\varphi}{2}) = 5$

$6.69 = 2(5) \cos\frac{\varphi}{2}$

$\boldsymbol{\varphi = 96^o}$

7. In order to decrease the fundamental frequency of a guitar string by 4.0%, by what percentage should you reduce the tension?

Data:

Decrease in f_1 = 4.0%

Decrease in T = ?

Solution:

$f_1' = f_1 - \frac{4}{100} f_1$

$\frac{f_1'}{f_1} = \frac{24}{25}$

Since, $f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$ and $f_1' = \frac{1}{2L}\sqrt{\frac{T'}{\mu}}$

$\frac{1}{2L}\sqrt{\frac{T'}{\mu}} \div \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{24}{25}$

$\frac{1}{2L}\sqrt{\frac{T'}{\mu}} \text{ x } 2\pi\sqrt{\frac{\mu}{T}} = \frac{24}{25}$

$\sqrt{\frac{T'}{T}} = \frac{24}{25}$

$\frac{T'}{T} = \frac{576}{656}$

$\frac{T'}{T} - 1 = \frac{576}{625} - 1$

$(\frac{T' - T}{T}) \times 100 = 100 \times (\frac{576}{625} - 1)$

$\mathbf{(\frac{T' - T}{T}) \times 100 = -7.8\ \%}$

8. A string 2.0 m long is held fixed at both ends. If a sharp blow is applied to the string at its centre, it takes 0.050s for the pulse to travel to both ends of the string and return to the middle. What is the fundamental frequency of oscillation for this string?

Data:

L = 2.0 m

t = 0.050s

f_1 = ?

Solution:

$v = \frac{L}{t} = \frac{2}{0.05} = 40$ m/s

$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

Since, $v = \sqrt{\frac{T}{\mu}}$

$f_1 = \frac{1}{2L} \times v$

$f_1 = \frac{40}{2 \times 2}$

$f_1 = 10$ Hz

9. A sound source of frequency f_o and an observer are located at a fixed distance apart. Both the source and observer are at rest. However, the propagation medium (through which the sound waves travel at speed) is moving at a uniform velocity in an arbitrary direction. Find the frequency detected by the observer giving physical explanation.

Ans) When the propagation medium is moving at a uniform velocity (v_m), the observed frequency changes due to the Doppler effect.

The Doppler effect equation for sound, when the source is stationary but the medium is moving, is given by:

$$f_{observed} = \frac{f_{source} + v_m}{f_{source} - v_m} \cdot f_{source}$$

Where,

$f_{observed}$ = the frequency observed by the stationary observer.

fsource = the frequency of the source (emitted frequency), which is f_o in this case.

$$f_{observed} = \frac{f_o + v_m}{f_o - v_m} \cdot f_o$$

1. **Medium Moving Toward the Observer** (Positive v_m): If the medium is moving toward the observer, v_m is positive. This means the denominator fo − v_m will be smaller than numerator f_o+v_m, resulting in an observed frequency ($f_{observed}$) that is higher (greater than f_o). This is called a **blue shift**, and the sound will appear to have a higher frequency.
2. **Medium Moving Away from the Observer** (Negative v_m): If the medium is moving away from the observer, v_m is negative. In this case, the denominator $f_o - v_m$ will be larger than numerator f_o+v_m, resulting in an observed frequency ($f_{observed}$) that is lower (less than f_o). This is called a **red shift**, and the sound will appear to have a lower frequency.

So, the frequency detected by the observer ($f_{observed}$) depends on the relative motion of the medium. If the medium is moving toward the observer, $f_{observed}$ will be higher than f_o, and if the medium is moving away from the observer, $f_{observed}$ will be lower than f_o.

10. A train sounds its whistle while passing by a railroad crossing. An observer at the crossing measures a frequency of 219 Hz as the train approaches the crossing and a frequency of 184 Hz as the train leaves. The speed of the sound is 340ms^{-1}. Find the speed of the train and frequency of its whistle.

Data:

When the train approaches the crossing

$f' = 219$ Hz

When the train leaves the crossing

$f' = 184$ Hz

$v = 340ms^{-1}$

$f_s = ?$

$v_s = ?$

Solution:

When the train approaches the crossing

$$f' = \left(\frac{v}{v - v_s}\right) \times f_s$$

$$219 = \left(\frac{340}{340 - v_s}\right) \times f_s \longrightarrow (1)$$

When the train leaves the crossing

$$f' = \left(\frac{v}{v + v_s}\right) \times f_s$$

$$184 = \left(\frac{340}{340 + v_s}\right) \times f_s$$

$$184\left(\frac{340 + v_s}{340}\right) = f_s$$

Put the value of f_s in equation (1)

$$219 = \left(\frac{340}{340 - v_s}\right) \times 184\left(\frac{340 + v_s}{340}\right)$$

$$\frac{219}{184} = \left(\frac{340}{340 - v_s}\right)\left(\frac{340 + v_s}{340}\right)$$

$$\left(\frac{340}{340 - v_s}\right)\left(\frac{340 + v_s}{340}\right) - \frac{219}{184} = 0$$

By using calculator

v_s = 29.5 m/s

Put the value of v_s in equation (1)

$$219 = \left(\frac{340}{340 - 29.5}\right) \times f_s$$

f_s = 200 Hz

Unit #13: Physical Optics

Worked Example 13.1

In a Young's double slit experiment a beam of light consisting of two wavelengths, 6500 A° and 5200 A°, is used to obtain interference fringes on a screen 120 cm away from two slits 2 mm apart. (i) Find the distance of the third bright fringe on the screen from the central maxima for the wavelength 6500 A° (ii) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide.

Solution:

Step 1: Write the known quantities and point out quantities to be found.

Distance of the screen from slits; $L = 120\text{ cm} = 120 \times 10^{-2}\text{ m}$

Slits spacing; $d = 2\text{ mm} = 2 \times 10^{-3}\text{ m}$

Wavelength of light: $\lambda_1 = 6500A° = 6500 \times 10^{-10}\text{ m}$

Wavelength of light; $\lambda_2 = 5200A° = 5200 \times 10^{-10}\text{ m}$

Required:

a) Distance of third bright fringe from central maxima for 6500 A°; $\Delta x = ?$

b) Least distance from the central maximum where the bright fringes due to both the wavelengths coincide; $x = ?$

Step 2: Write down the formula and rearrange if necessary

$$Y_m = \frac{L}{d} m \lambda; \text{ for } m = 3$$

$$Y_3 = \frac{L}{d} 3 \lambda$$

$$\Delta x = Y_3 - Y_0 = \frac{L}{d} 3 \lambda$$

Step 3: Put the values in the formula and calculate.

a) $\Delta x = \frac{120 \times 10^{-2}}{2 \times 10^{-3}} \times 3 \times 6500 \times 10^{-10}$

$\mathbf{\Delta x = 1.17 \times 10^{-3}\ m = 1.17\ mm}$ Answer

b)

Suppose the m^{th} bright fringe due to wavelength 6500 A° coincides with the n^{th} bright fringe due to wavelength 5200 A° then,

$$Y_m = Y_n \rightarrow \frac{L}{d} m \lambda_1 = \frac{L}{d} n \lambda_2 \quad \rightarrow m \lambda_1 = n \lambda_2$$

$$\frac{m}{n} = \frac{5200\ A°}{6500\ A°} = \frac{4}{5}$$

Hence the minimum values of m and n for the two bright fringes coincide are

$m = 4$ and $n = 5$

Therefore; $\mathbf{x = \frac{L}{d} m \lambda_1} \rightarrow x = \frac{120 \times 10^{-2}}{2 \times 10^{-3}} \times 4 \times 6500 \times 10^{-10}$

$\mathbf{x = 1.56 \times 10^{-3}\ m = 1.56\ mm}$

Similarly; $\mathbf{x = \frac{L}{d} n \lambda_2} = \frac{120 \times 10^{-2}}{2 \times 10^{-3}} \times 5 \times 5200 \times 10^{-10}$

$\mathbf{x = 1.56 \times 10^{-3}\ m = 1.56\ mm}$

Worked Example 13.2

A soap bubble in air is of thickness 320 nm. If it is illuminated with white light at near normal incidence, what color will appear to be in reflected light?
(Refractive index of soap bubble n= 1.50).

Solution:

Step 1: Write the known quantities and point out the quantities to be found.

Thickness of film, $t = 320$ nm
Refractive index, $n = 1.50$
Wavelength $\lambda = ?$
Colors $= ?$

Step 2: Write the formula and rearrange if necessary.

$$2nt = (m + \tfrac{1}{2})\lambda$$

$$\lambda = \frac{2nt}{m+\frac{1}{2}}$$

$m = 0, 1, 2, 3, \ldots\ldots$

Step 3: Put the values in the formula and calculate.

$$\lambda = \{\frac{2nt}{m+\frac{1}{2}}\}\ \text{nm}$$

for $m = 0$,

$$\lambda = \{\frac{2 \times 1.50 \times 320}{0+\frac{1}{2}}\}\ \text{nm} = \{\frac{2 \times 1.50 \times 320}{0+\frac{1}{2}}\}\ \text{nm}$$

$\lambda = 1920$ nm , similarly
for $m = 1$, $\lambda = 640$ nm
for $m = 2$, $\lambda = 384$ nm
for $m = 3$, $\lambda = 274$ nm

We note that only the maxima, for m=1 and m=2 lies in the visible region and the colors for 640 nm 384 nm are nearly red and violet.

Worked Example 13.3

In a Newton's ring experiment the diameter of the 16^{th} bright ring was found to be 0.653 cm and that of 5^{th} bright ring is 0.346 cm. if the radius of curvature of the lens is 100 cm, find the wavelength of light.

Solution:

Step 1: Write the known quantities and point out the quantities to be found.

Diameter of Newton's 16^{th} bright ring

$(d_{16}) = 0.653\text{ cm} = 0.653\times10^{-2}\text{ m}$

Radius of Newton's 16^{th} bright ring

$r_{16}^{b} = 3.265\times10^{-3}\text{ m}$

Diameter of Newton's 5^{th} dark ring

$(d_5) = 0.346\text{ cm} = 0.346\times10^{-2}\text{ m}$

Radius of Newton's 5^{th} dark ring

$r_5^d = 1.73\times10^{-3}\text{ m}$

Radius of curvature of lens, R = 100 cm = 1 m

Wave length of light (λ) =?

Step 2: Write the formula and rearrange if necessary.

From equation 13.17 and 3.18,

$r_b^2 = R(N-\frac{1}{2})\lambda$ and $r_d^2 = m\lambda R$

$r_b^2 - r_d^2 = R(N-\frac{1}{2})\lambda - m\lambda R$

$\lambda = \frac{r_b^2 - r_d^2}{(N-\frac{1}{2}-m)}$

Step 3: Put the values in the formula and calculate.

$\lambda = \frac{(3.265\times10^{-3})^2-(1.73\times10^{-3})^2}{(16-\frac{1}{2}-5)}$

$\lambda = \mathbf{7302\times10^{-10}\ m}$

Section (A): Multiple Choice Questions (MCQs)

1. If the wavelength of an electromagnetic wave is about the diameter of a cricket ball, what type of radiation is it.

a) X-ray b) Ultraviolet

c) Radio waves d) Visible light

2. Electromagnetic waves from an unknown source in space are found to be diffracted when passing through gaps of the order of 10 m, which type of the wave are they most likely to be?

a) microwaves
b) Ultraviolet
c) Radio waves
d) infra-red waves

3. Huygens's conception of secondary waves

a) helps us to find the focal length of a thick lens

b) is a geometrical method to find a wavefront

c) is used to determine the velocity of light

d) is used to explain the polarization of light

4. Interference fringes are produced using monochromatic light of same intensity from a double slit screen. If the intensity of light emerging from one of the slit is reduced, the effect on interference pattern will be

a) All the dark and bright fringes become brighter.

b) All the dark and bright fringes become darker.

c) Bright fringes become brighter and dark fringes become darker.

d) Bright fringes become darker and dark fringes become brighter.

5. In Young's experiment when the distance between slits and screen is doubled, while separation of slits is halved, then fringe width will be:

a) 4 times
b) ¼ times
c) doubled
d) unchanged

6. A ray of light passes from air into water. Striking the surface of the water with an angle of incidence 45^{o}. Which of these quantities change as the light enters the water. i) Wavelength ii) frequency iii) speed of propagation iv) direction of propagation

a) i and ii only.
b) iii and iv only.
c) i, iii, and iv only
d) all of them.

7. A hill separates a television (TV) transmitter from a house. The Transmitter cannot be seen from the house but still the TV in the house has good reception. What wave phenomena make it possible?

a) Coherence of waves b) Diffraction of waves

c) Interference of waves d) Refraction of waves

8. Monochromatic light is incident on a diffraction grating and a diffraction pattern is observed. Which effects is observed by replacing the grating with one that has more lines per millimeter?

a) Number of maxima decreases with decrease in angle between first and second order maxima.

b) Number of maxima decreases with increase in angle between first and second order maxima.

c) Number of maxima increases with decrease in angle between first and second order maxima

d) Number of maxima increases with increase in angle between first and second order maxima

9. Optically active substances are those substances which

a) produce polarized light

b) rotate the plane of polarization of polarized light

c) produce double refraction

d) convert a plane polarized light into circularly polarized light

10. Plane polarized light is passed through a Polaroid. On viewing through the Polaroid we find that when Polaroid is given one complete rotation about the direction of light

a) The intensity of light gradually decreases to zero and remains at zero.

b) The intensity of light gradually increases to maximum and remains at maximum

c) There is no change in the intensity of light

d) The intensity of light varies such that it is twice maximum and twice zero.

KEY:

1. c	2. c	3. b	4. d	5. a
6. c	7. b	8. b	9. b	10. d

CRQs:

1. In every day experience, visible light seem to travel in straight lines while radio waves do not, Explain.

Ans) Visible light and radio waves both travel in straight lines through space, but the perception that they behave differently in everyday experience is due to differences in their interactions with objects. Visible light, with its shorter wavelength, interacts with objects at the macroscopic scale, casting shadows and appearing to follow straight paths when obstructed. In contrast, radio waves, with their much longer wavelengths, tend to pass through or around objects at the macroscopic scale without noticeable scattering, creating the impression that they don't travel in straight lines. However, both types of waves fundamentally obey the laws of straight-line propagation when not significantly affected by obstacles or the atmosphere.

2. Explain why two waves of significantly different frequencies cannot be coherent?

Ans) Two waves cannot be considered coherent if they have significantly different frequencies. Coherence requires a constant phase relationship and similar frequencies between waves. Waves with different frequencies will quickly lose their phase relationship over time, making them incoherent. Coherence is a critical concept in fields like optics and communication.

3. In a Young's double slit experiment, how the interference phenomenon is affected by changing the slits separation and the distance between the slits and screen?

Ans)

1. **Slits Separation (d):**
    - Narrower separation leads to a wider interference pattern on the screen.
    - Wider separation results in a narrower interference pattern with fringes closer together.
    - The angular separation between fringes (θ) is inversely proportional to the slit separation (d).

2. **Distance Between Slits and Screen (L):**
    - Increasing L makes the interference pattern more spread out with larger, less intense fringes.
    - Decreasing L makes the interference pattern more compact with narrower and more distinct fringes.
    - The angular separation between fringes (θ) is directly proportional to the distance between the slits and the screen (L).

4. Explain how the double-slit experiment provides evidence for the wave nature of light.

Ans) The double-slit experiment provides evidence for the wave nature of light through its observation of an interference pattern. When light passes through two closely spaced slits and strikes a screen, it creates alternating bright and dark fringes, which can only be explained by the wave-like behavior of light. This experiment demonstrated the principles of superposition and interference, establishing that light exhibits wave-like properties.

5. Discuss the concept of monochromatic light in the context of Newton's rings.

Ans) Monochromatic light, consisting of a single well-defined wavelength or color, is crucial in Newton's rings experiments. When monochromatic light is used, it creates a distinct interference pattern of concentric bright and dark rings when passed through a plano-convex lens and a flat glass plate. The radii of these rings depend on the wavelength of the light, allowing for wavelength determination. Additionally, Newton's rings are used to study thin films and assess the quality of optical surfaces in practical applications, making monochromatic light a fundamental component of this optical phenomenon.

6. Explain why monochromatic light is preferred for obtaining clear and well-defined interference patterns in the experiment.

Ans) Monochromatic light is preferred for obtaining clear and well-defined interference patterns in optical experiments, such as Newton's rings, due to several key advantages. Monochromatic light consists of a single wavelength, ensuring consistent phase relationships and simplifying interference patterns. It produces sharp, high-contrast fringes, allows for accurate wavelength determination, reduces complexity in the analysis, and ensures consistency and reproducibility in experimental results.

7. Discuss how the interference of light waves leads to the formation of bright and dark rings in the experiment.

Ans) The formation of bright and dark rings in experiments like Newton's rings is a consequence of the interference of light waves. In this setup, a monochromatic light source is directed at a plano-convex lens in contact with a flat glass plate. Light waves reflected from the lens's surface and those transmitted through it travel different paths before meeting on a screen below.

Interference occurs when the path length difference between these two sets of waves is a multiple of the wavelength of light (λ). Constructive interference, resulting in bright rings, happens when the path length difference is an integer multiple of λ. Destructive interference, leading to dark rings, occurs when the path length difference is a half-integer multiple of λ.

As a result, an alternating pattern of bright and dark rings forms on the screen, with the central region being bright and subsequent regions exhibiting concentric bright and dark rings. This pattern is a manifestation of wave interference in the experiment.

ERQs:

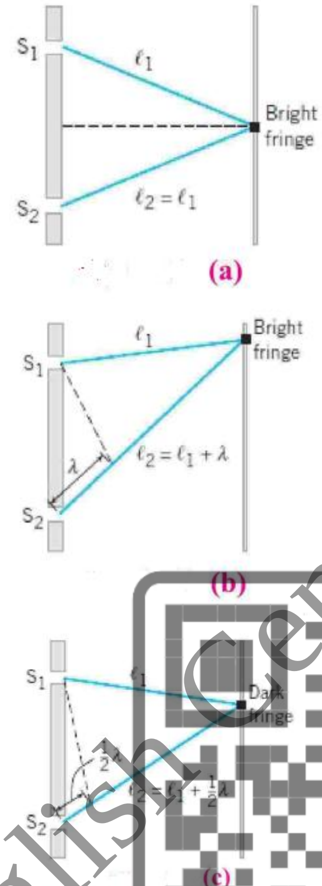
1. Explain the concept of interference in physical optics. Discuss constructive and destructive interference.

Ans)

Definition: Interference in physical optics is a phenomenon that occurs when two or more coherent waves overlap and interact with each other.

Conditions for interference of light: Although any number of waves can in principle interfere, but in order to keep the case simple we consider here the interference of only two waves. To produce observable interference effect, it is necessary to have

- Two coherent sources, i.e, they must have the same frequency.
- The two waves must be monochromatic, i.e, they must have the same color (wavelength).
- Be always in phase with each other or have a constant phase difference.
- For Constructive Interference (Bright Fringes) the path difference must be
  Phase difference = $2m\pi$ rad
  Path difference $\Delta S = m\lambda$
  ($m = 0, \pm1, \pm2, \pm3.....$) Fig. (a,b)



Phase difference and path difference between two coherent sources

- For Destructive Interference (Dark Fringes) the phase difference must be
  Phase difference = $(m + \frac{1}{2})\, 2\pi$ rad
  Path difference $\Delta S = (m + \frac{1}{2})\, \pi$
  ($m = 0, \pm 1, \pm 2, \pm 3.....$) Fig. (c)

Application: Interference concept is applied in following

1. Young's Double Slit Experiment
2. Interference in thin film
3. Newton's rings
4. The Michelson Interferometer

Constructive Interference:

Definition: Constructive interference occurs when waves combine in such a way that their amplitudes add together, resulting in a wave with a larger amplitude at the point of superposition.

In-Phase Waves: For constructive interference to happen, the waves involved must be in phase. In other words, the crests of one wave align with the crests of the other wave, and the troughs align with troughs.

Examples: Examples of constructive interference include the bright regions (fringes) seen in double-slit experiments, where waves from two slits overlap constructively to produce bright lines on a screen.

Destructive Interference:

Definition: Destructive interference occurs when waves combine in such a way that their amplitudes subtract from each other, resulting in a wave with a smaller or even zero amplitude at the point of superposition.

Out-of-Phase Waves: Destructive interference occurs when the waves are out of phase. In this case, the crests of one wave align with the troughs of the other wave, leading to complete or partial cancellation.

Examples: Destructive interference is observed in scenarios like the dark regions (fringes) in double-slit experiments, where waves from two slits overlap destructively to produce dark lines on a screen.

2. Discuss the conditions required for interference and provide examples of interference in daily life.

Ans)

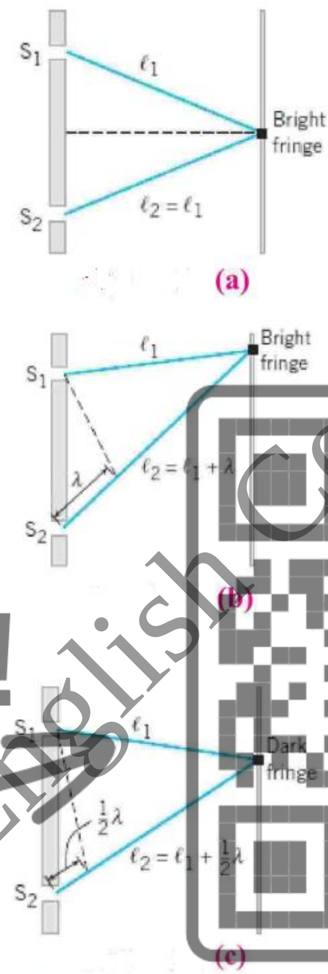
Conditions for interference of light: Although any number of waves can in principle interfere, but in order to keep the case simple we consider here the interference of only two waves. To produce observable interference effect, it is necessary to have

- Two coherent sources, i.e, they must have the same frequency.
- The two waves must be monochromatic, i.e, they must have the same color (wavelength).
- Be always in phase with each other or have a constant phase difference.
- For Constructive Interference (Bright Fringes) the path difference must be

Phase difference = $2m\pi$ rad

Path difference $\Delta S = m\lambda$

($m = 0, \pm1, \pm2, \pm3.....$) Fig. (a,b)



Phase difference and path difference between two coherent sources

- For Destructive Interference (Dark Fringes) the phase difference must be

Phase difference = $(m + \frac{1}{2})\,2\pi$ rad

Path difference $\Delta S = (m + \frac{1}{2})\,\pi$

($m = 0, \pm1, \pm2, \pm3.....$) Fig. (c)

Examples of interference in daily life: Interference is a fundamental wave phenomenon that can be observed in various aspects of daily life. Here are some common examples:

1. **Soap Bubbles:** When you blow soap bubbles, you can observe interference patterns on the surface of the bubble. The thin soap film acts as a diffraction grating, and as light reflects off the inner and outer surfaces of the film, it interferes, creating colorful patterns.
2. **Oil Slicks on Water:** When oil spreads on the surface of water, it forms a thin layer. Light reflecting off the top and bottom surfaces of the oil layer can interfere, producing colorful patterns.
3. **Thin Film Coatings:** Interference is used in the design of camera lenses. By applying thin films with specific thicknesses to the surface, unwanted reflections can be minimized through destructive interference, improving visibility.
4. **Iridescent Materials:** Some materials, like the wings of certain butterflies, feathers of birds, or the surface of CDs and DVDs, exhibit iridescence due to interference. The varying thickness of microscopic structures on these surfaces causes light to interfere and produce shifting colors.
5. **Soap Films in Dishwashing:** When you wash dishes, soap bubbles or thin soap films can form on the surface of water. These films display interference patterns and can be used as a playful way to understand interference.
6. **Newton's Rings:** When a plano-convex lens is placed on a flat glass surface, interference patterns called Newton's rings can be observed. These concentric circular rings result from the interference of light waves reflecting off the two surfaces (the lens and the glass), creating alternating bright and dark rings.
7. **Michelson Interferometer:** The Michelson interferometer is a precision optical instrument that uses interference to measure very small distances, wavelengths, and refractive indices. It has applications in fields like astronomy and metrology.
8. **Holography:** Holograms are created by recording the interference pattern between a reference beam of light and a beam scattered off an object. When this pattern is reconstructed with light, it forms a three-dimensional image of the object.
9. **Anti-Reflective Coatings on Eyeglasses:** Eyeglass lenses are often coated with thin films that are carefully designed to reduce unwanted reflections. These coatings work through destructive interference, where the light reflecting off the outer and inner surfaces of the lens interferes in such a way that certain wavelengths of light cancel each other out, minimizing glare and enhancing visibility.
10. **Rainbow Formation:** Rainbows are natural optical phenomena that result from the dispersion and interference of sunlight by raindrops. When sunlight enters a raindrop, it undergoes both refraction and internal reflection, followed by dispersion as it exits the drop. This dispersion separates the light into its component colors. When the dispersed light waves exit the raindrop and interfere with each other, they create the circular spectrum of colors that we perceive as a rainbow.

3. Describe the setup and working principle of the Michelson interferometer. Also explain how the Michelson interferometer can be used to measure the wavelength of monochromatic light.

Ans) Michelson's Interferometer:

Principle: It works on the principle of interference in which a light beam from a monochromatic source is divided into two coherent beams by partial reflection and refraction (at semi-silvered glass plate) and made to reunite or superimposed after traversing two different optical paths.

Construction: Michelson interferometer consists of two highly polished plane mirrors M_1 & M_2 a semi silvered glass plate & a glass plate D. The mirror M_1 is fixed whereas the mirror M_2 is movable. The semi silvered glass plane is called "beam splitter" & is inclined at an angle of 45^o relative to the incident light beam. The plate D which is identical to the plate is known as “Compensating plate”. Its purpose is to ensure that the light rays pass through the same thickness of glass.

Working: When a beam of monochromatic light (of wavelength "λ ") from a source S falls on semi silvered plate C, it splits into two parts 1 & 2 at the point P. The part 1 of light passes through the semi silver surface of the plate C continues its journey, passes through the compensating plate D & finally falls on the fixed mirror M_1.

It passes through the plate D on its return journey & finally it is incident on the silver surface of the plate C from where it is reflected to the observer's eye.

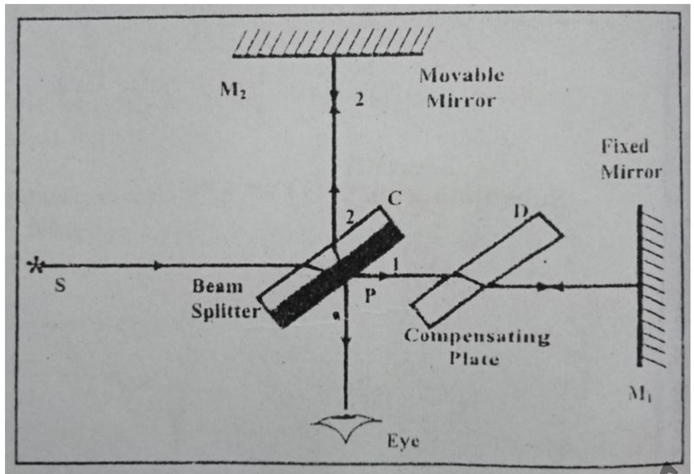
The part 2 of the light is reflected from the silvered surface of the plate C to the movable mirror M_2 . After reflection from M_2, it returns to the observer's eye through the plate C. The two rays recombine between the plate C & eye to produce interference pattern which can be viewed.

Equation for wavelength: If the distance covered by the light beam is zero, the rays will interfere constructively & brightness is seen. If the mirror M_2 is moved away through one quarter the wave length of light, i.e, $\frac{\lambda}{4}$ ($\frac{\lambda}{4} + \frac{\lambda}{4} = \frac{\lambda}{2}$) as compared to ray 1. Both rays will interfere destructively & darkness is seen. If M_2 is further moved away through λ/4,i.e, λ/2 from original position, the path difference between rays 1 & 2 will become (λ/2 + λ/2 = λ). Both rays will interfere constructively & brightness is seen. Thus by moving mirror M_2 through λ/2 each time, alternate bright & dark fringes can be seen. The bright fringe will be obtained if the mirror M_2 is moved through λ/2 each time.

If for the "m" brightness the mirror M_2 moves through displacement “x” then

$$x = m\lambda/2$$

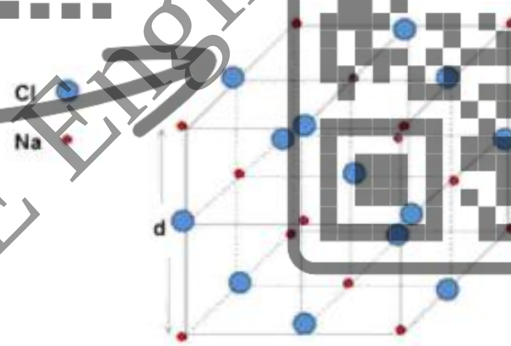
$$\lambda = 2x/m$$



4. Describe the setup and procedure of the diffraction of X-rays through a crystal experiment.

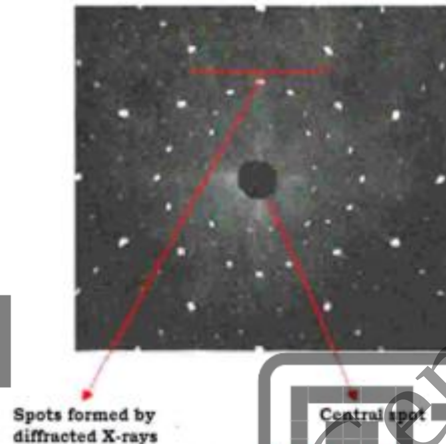
Ans)

Setup: In 1912, Max von Laue realized that the regular arrangements of atoms in a crystal make a perfect grating for X-rays. The regular arrangement and spacing of the atoms is analogous to the regular spacing of the slits in a conventional grating, but a crystal is a three-dimensional grating (as opposed to the two-dimensional gratings we use for visible light). Figure below shows the atomic structure of NaCl.



Crystal structure of NaCl (Rock salt).

Working: When a beam of x-rays passes through the crystal, the x-rays are scattered in all directions by the atoms. The x-rays scattered in a particular direction from different atoms interfere with each other. In certain directions they interfere constructively, giving maximum intensity in those directions Photographic film records those directions as a collection of spots for a single crystal as shown in Fig below.



The X-ray diffraction pattern of NaCl. The central spot created by X – rays that are not scattered by the sample.

Numerical:

1. A monochromatic light of wavelength 6900 Å is used to illuminate two parallel slits. On a screen that is 3.30 m away from the slits, interference fringes are observed. The distance between adjacent bright fringes in the centre of the pattern is 1.80 cm. what is the distance between the slits.

Data:

$\lambda = 6900\ \text{Å} = 6900 \times 10^{-10}$ m

$L = 3.30$ m

$\Delta x = 1.80$ cm $= 1.80 \times 10^{-2}$ m

$d = ?$

Solution:

$1.80 \times 10^{-2} = \frac{3.3}{d} \times 6900 \times 10^{-10}$

$\mathbf{d = 1.265 \times 10^{-4}}$ **m**

2. A Michelson interferometer is adjusted so that a bright fringe is appeared on the screen. As one of the mirrors is moved 25.8 micrometer, 92 bright fringes are counted on the screen. What is the wavelength of light used in the interferometer?

Data:

$x = 25.8$ micrometer $= 25.8 \times 10^{-6}$ m

$m = 92$

$\lambda = ?$

Solution:

$x = m\frac{\lambda}{2}$

$25.8 \times 10^{-6} = 92\left(\frac{\lambda}{2}\right)$

$\lambda = 560$ nm

3. In section 13.4 we studied interference due to thin films. Why must the film be thin? Why don't we see the interference effect when looking through a window or at a poster covered by a plate of glass, even if the glass is optically flat.

Ans) Interference effects in thin films are most pronounced when the film's thickness is on the order of the wavelength of the incident light. In everyday objects like windows and posters covered by glass, the thickness of the glass is much greater than the wavelength of visible light, so interference effects are negligible. This is why we don't typically see interference patterns when looking through such materials, even if the glass is optically flat and smooth. Interference occurs when the film is "thin" relative to the wavelength of light.

4. Newton's rings are formed by the light of 400 nm wavelength. Determine the change in air film thickness between the third and sixth bright fringe. If the radius of curvature of the curved surface is 5.0m, what is the radius of third bright fringe?

Data:

$\lambda = 400$ nm $= 400 \times 10^{-9}$ m

$\Delta t = ?$ (between the third and sixth bright fringe)

$R = 5.0$m

$r_N = ?$

Solution:

$$2t_N = (m + \frac{1}{2})\ \lambda$$

For third bright fringe

$$2t_3 = (2 + \frac{1}{2}) \times 400 \times 10^{-9}$$

$$t_3 = 5 \times 10^{-7}\ m$$

For sixth bright fringe

$$2t_6 = (5 + \frac{1}{2}) \times 400 \times 10^{-9}$$

$$t_6 = 1.1 \times 10^{-6}\ m$$

$$\Delta t = t_6 - t_3$$

$\Delta t = 1.1 \times 10^{-6} - 5 \times 10^{-7} = 6 \times 10^{-7}\ m =$ **600 nm**

$$r = \sqrt{R\left(N - \frac{1}{2}\right)\lambda}$$

$$r = \sqrt{5\left(3 - \frac{1}{2}\right)400\text{x}10^{-9}}$$

$r = 2.236\text{x}10^{-3}$ m = 2.236 mm

5. A soap film has an index of refraction n=1.50. The film is viewed in reflected light.

(a) At a spot where the film thickness is 910.0 nm, which wavelengths are missing in the reflected light?

(b) Which wavelengths are strongest in the visible light?

Data:

n =1.50

$t = 910.0\ nm = 910 \times 10^{-9}\ m$

(a) Wavelengths missing in the reflected light = ?
(b) Wavelengths strongest in the visible light = ?

Solution:

(a)

The expression for the wavelength of light in the film is mentioned below:

$$\lambda_f = \frac{\lambda}{n}$$

The missing wavelength will be calculated by using the condition of destructive interference, and as the wave is going under phase change due to reflection so the equation for path difference in this case of destructive interference will be:

$$2t = m\,\lambda_f \longrightarrow (1)$$

Put $\lambda_f = \frac{\lambda}{n}$ inin equation (1)

$$2t = m\,\frac{\lambda}{n}$$

In order to find the missing wavelengths, we will put m =1,2,3 ……

For the value of m = 1:

$$2\,(910 \times 10^{-9}) = (1)\left(\frac{\lambda}{1.5}\right)$$

$\lambda = 2.73 \times 10^{-6}$ m = 2730 nm

For the value of m = 2:

$$2\,(910 \times 10^{-9}) = (2)\left(\frac{\lambda}{1.5}\right)$$

$\lambda = 1.365 \times 10^{-6}$ m = 1365 nm

For the value of m=3:

$$2\,(910 \times 10^{-9}) = (3)\left(\frac{\lambda}{1.5}\right)$$

$\lambda = 9.1 \times 10^{-7}$ m = 910 nm

For the value of m=4:

$$2\,(910 \times 10^{-9}) = (4)\left(\frac{\lambda}{1.5}\right)$$

$\lambda = 6.825 \times 10^{-7}$ m = 682.5 nm ≈ 683 nm

For the value of m=5:

$2\,(910 \times 10^{-9}) = (5)\,(\frac{\lambda}{1.5})$

$\lambda = 5.46 \times 10^{-7}$ m = 546 nm

For the value of m=6:

$2\,(910 \times 10^{-9}) = (6)\,(\frac{\lambda}{1.5})$

$\lambda = 4.55 \times 10^{-7}$ m = 455 nm

(b)

For strongest wavelengths, we will use condition of constructive interference in this case i.e.

$2t = (m + \frac{1}{2})\lambda_f \longrightarrow (2)$

Put $\lambda_f = \frac{\lambda}{n}$ inin equation (2)

$2t = (m + \frac{1}{2})\,\frac{\lambda}{n}$

For the value of m=1:

$2\,(910 \times 10^{-9}) = (1 + \frac{1}{2})\,\frac{\lambda}{1.5}$

$\lambda = 1.82 \times 10^{-6}$ m = 1820 nm

For the value of m=2:

$2\,(910 \times 10^{-9}) = (2 + \frac{1}{2})\,\frac{\lambda}{1.5}$

$\lambda = 1.092 \times 10^{-6}$ m = 1092 nm

For the value of m=3:

$2\,(910 \times 10^{-9}) = (3 + \frac{1}{2})\,\frac{\lambda}{1.5}$

$\lambda = 7.8 \times 10^{-7}$ m = 780 nm

For the value of m=4:

$2\,(910 \times 10^{-9}) = (4 + \frac{1}{2})\,\frac{\lambda}{1.5}$

$\lambda = 6.07 \times 10^{-7}$ m = 607 nm

For the value of m=5:

$$2\ (910 \times 10^{-9}) = (5 + \tfrac{1}{2})\ \frac{\lambda}{1.5}$$

$$\boldsymbol{\lambda = 4.96 \times 10^{-7}\ m = 496\ nm}$$

For the value of m=6:

$$2\ (910 \times 10^{-9}) = (6 + \tfrac{1}{2})\ \frac{\lambda}{1.5}$$

$$\boldsymbol{\lambda = 4.2 \times 10^{-7}\ m = 420\ nm}$$

6. What is the difference between deviation and diffraction? What do diffraction and interference have in common?

Ans) Difference between deviation and diffraction:

Aspect	Deviation	Diffraction
Definition	Change in the direction of a wave as it passes from one medium into another with a different refractive index.	Spreading and bending of waves when they encounter an obstacle or aperture.
Cause	Caused by changes in the speed and direction of a wave due to differences in refractive indices between two media.	Caused by the interference and spreading of waves as they interact with obstacles or apertures.
Occurrence	Happens when a wave crosses a boundary between two media with different refractive indices.	Occurs when waves encounter obstacles or apertures that are comparable in size to their wavelength.
Examples	Light bending when passing from air into glass, causing refraction.	Diffraction of light waves when passing through a narrow slit, creating a diffraction pattern.
Mathematical Laws	Important in optics, such as in the design of lenses and prisms.	Widely used in various scientific and engineering applications, including the study of wave behavior, microscopy, and the design of diffraction gratings.

Diffraction and interference have in common:

1. **Wave Nature:** Both diffraction and interference are manifestations of the wave nature of light and other types of waves. They cannot be explained by particle-like behavior.
2. **Superposition:** Both phenomena rely on the principle of superposition, which states that when two or more waves overlap in space and time, their amplitudes add together at each point, resulting in a net wave amplitude.
3. **Interference Patterns:** Both diffraction and interference can produce characteristic patterns of alternating bright and dark regions. These patterns result from the constructive and destructive interference of wave amplitudes.
4. **Wavelength Dependency:** Both phenomena depend on the wavelength of the waves involved. The extent of diffraction and the spacing of interference fringes are directly related to the wavelength of the waves.
5. **Wavefront Interaction:** In both cases, it's the interaction of wavefronts that leads to the observed effects.

11. Describe what happens to a single slit diffraction pattern as the width of the slit is slowly decreased.

Ans) As the width of a single slit is gradually reduced in a diffraction experiment:

1. The central maximum becomes narrower and brighter.
2. The secondary maxima (smaller peaks) on either side of the central maximum become wider and less intense.
3. Some secondary maxima may become barely visible or disappear entirely.
4. The angular spread of the diffraction pattern increases, with greater separation between maxima.
5. The ability to distinguish fine details in the pattern decreases as it becomes more spread out.

12. The diffraction pattern from a single slit of width 0.020 mm is viewed on a screen. If the screen is 1.20 m from the slit and light of wavelength 430 nm is used. What is the width of the central maximum?

Data:

$d = 0.020\ mm = 0.020 \times 10^{-3}\ m$

$D = 1.20\ m$

$\lambda = 430\ nm = 430 \times 10^{-9}\ m$

Width of central maximum = ?

Solution:

Width of central maximum = $\frac{2\lambda D}{d}$

Width of central maximum = $\frac{2(430 \times 10^{-9})(1.2)}{0.020 \times 10^{-3}}$

Width of central maximum = 0.0516 m = 5.16 cm

13. A grating has exactly 8000 lines uniformly spaced over 2.54 cm and is illuminated by light from a mercury vapor discharge lamp. What is the expected angle for the third order maximum of the light of wavelength 546 nm.

Data:

N = $\frac{8000}{2.54 \times 10^{-2}}$ = 314960.63 lines/m

m = 3

λ = 546 nm = 546×10^{-9} m

θ = ?

Solution:

d = $\frac{1}{N} = \frac{1}{314960.63}$ = 3.175×10^{-6} m/lines

m λ = d sinθ

(3) (546×10^{-9}) = (3.175×10^{-6}) sinθ

θ = 31.06^o

OR

θ = $31^o 3' 28.8''$

14. How many lines per centimeter are there in a grating which gives 1st order spectra at an angle of 30^o when the wavelength of light is 6×10^{-5} cm?

Data:

N = ?

m = 1

θ = 30^o

λ = 6×10^{-5} cm = 6×10^{-7} m

Solution:

$m\lambda = d \sin\theta$

$(1)(6 \times 10^{-7}) = d \sin 30^o$

$d = 1.2 \times 10^{-6}$ m/lines

$$\mathbf{N = \frac{1}{d} = \frac{1}{1.2 \times 10^{-6}} = 833333.3333 \text{ lines/m} = \frac{833333.3333}{100} = 8333.333 \text{ lines/cm}}$$

15. Light of wavelength 450 nm is incident on a diffraction grating on which 5000 lines/cm have been ruled. Determine

(i) How many orders of spectra can be observed on either side of spectra?

(ii) Determine the angle corresponding to each order.

Data:

$\lambda = 450 \text{ nm} = 450 \times 10^{-9}$ m

N = 5000 lines/cm = 500000 lines/m

(i) m = ?
(ii) $\theta_1 = ?$, $\theta_2 = ?$, $\theta_3 = ?$, $\theta_4 = ?$

Solution:

$d = \frac{1}{N} = \frac{1}{500000} = 2 \times 10^{-6}$ m/lines

$m\lambda = d \sin\theta$

$m(450 \times 10^{-9}) = (2 \times 10^{-6}) \sin 90^o$

$m = 4.444 \approx 4$

$m\lambda = d \sin\theta$

For first order:

$(1)(450 \times 10^{-9}) = (2 \times 10^{-6}) \sin\theta$

$\theta = 13^o$

For second order:

$(2)(450 \times 10^{-9}) = (2 \times 10^{-6}) \sin\theta$

$\theta = 26.7^o$

For third order:

$(3)\ (450 \times 10^{-9}) = (2 \times 10^{-6}) \sin\theta$

$\boldsymbol{\theta = 42.5^{o}}$

For fourth order:

$(4)\ (450 \times 10^{-9}) = (2 \times 10^{-6}) \sin\theta$

$\boldsymbol{\theta = 64.2^{o}}$

16. Why does a crystal act as a three dimensional grating for X-rays but not for visible light?

Ans) A crystal acts as a three-dimensional grating for X-rays but not for visible light because of the vast difference in their wavelengths. X-rays have much shorter wavelengths and interact with the crystal lattice through Bragg diffraction, producing distinct diffraction patterns due to the crystal's periodic structure. Visible light, with longer wavelengths, interacts differently with the lattice, causing refraction and dispersion but not diffraction patterns.

17. A beam of X-rays of wavelength 0.071 nm is diffracted by a diffracting plane of rock salt with distance between the atomic planes are 1.98Å. Find the glancing angle for the second-order diffraction.

Data:

$\lambda = 0.071 \text{ nm} = 0.071 \times 10^{-9} \text{ m}$

$d = 1.98\text{Å} = 1.98 \times 10^{-10} \text{ m}$

$\theta = ?$

$m = 2$

Solution:

$2d \sin\theta = m\lambda$

$2\ (1.98 \times 10^{-10}) \sin\theta = (2)\ (0.071 \times 10^{-9})$

$\boldsymbol{\theta = 21^{o}}$

18. Unpolarized light I_o passes through two polarizers in turn with polarization axes at 45^o to each other. What is the fraction of the incident light intensity that is transmitted?

Data:

$\theta = 45^o$

Fraction of the incident light intensity that is transmitted = ?

Solution:

Since through first polarizer, intensity will get halved. Hence, light incident on second polarizer will have intensity $\frac{I_0}{2}$. Through the second polarizer, intensity will decrease by a factor of $\cos^2\theta$. Hence intensity after passing through second polarizer is

$$\frac{I_o}{2}\cos^2\theta = \frac{I_o}{2}\cos^2 45^o = \frac{I_o}{4}$$

Unit #14: Communication

Section (A): Multiple Choice Questions (MCQ)

1. In Radio and Television broadcast, the information signal is in the form of:

a) analog signal b) digital signal

c) Both analog & digital signals d) neither analog nor digital signal

2. A communication channel consists of:

a) transmission line only b) optical fibre only

c) free space only d) All of them

3. Voltage signal generated by a microphone is:

a) digital in nature b) analog in nature

c) hybrid in nature d) consists of bits & bytes

4. As compared to sound waves frequency of radio waves is:

a) higher b) equal

c) lower d) may be higher or lower

5. The process of superimposing signal frequency on the carrier wave is known as

a) transmission b) detection

c) reception d) modulation

6. What is the frequency range of signals that can be transmitted in case of twisted pair of wires?

a) 10MHz to 15MHz b) 5MHz to 10MHz

c) 100Hz to 5MHz d) 10Hz to 100Hz

7. The maintenance of a satellite's orbital position is called:

a) maintenance keeping b) station keeping

c) station maintenance d) attitude control

8. Process of mapping the sampled analog voltage values to discrete voltage levels is called:

a) sampling b) sampling frequency

c) quantizing d) encoding

9. AM is used for broadcasting because:

a) It requires less transmitting power compare with other systems

b) It is more noise immune than other modulation system

c) No other modulation can provide the necessary hand width faithful transmission

d) Its use avoids receiver complexity

10. Data in compact disc is stored in the form of

a) analog signal b) digital signal

c) noise d) both (a) &(b)

KEY:

1. a	2. d	3. b	4. a	5. d
6. c	7. b	8. c	9. d	10. b

Section (B): Structured Questions

CRQs:

1. A male voice after modulation-transmission sounds like that of a female to the receiver. Give reason.

Ans) A male voice may sound like a female voice after modulation and transmission due to various factors. These include frequency shifting, bandwidth limitations, equalization and filtering, signal loss, noise, and receiver characteristics. These factors can alter the voice's pitch and spectral characteristics during the transmission process, leading to a perceived change in gender.

2. Why is an AM signal likely to be more noisy than a FM signal upon transmission through a channel?

Ans) An AM (Amplitude Modulation) signal is more likely to be noisy than an FM (Frequency Modulation) signal during transmission due to AM's sensitivity to amplitude variations and noise. AM encodes information in signal strength, making it vulnerable to noise-induced amplitude changes, leading to audible static. FM encodes information in signal frequency, which is less affected by amplitude variations, resulting in less noticeable noise and better signal quality.

3. Write 5 differences between analog and digital transmission.

Ans)

Aspect	Analog Transmission	Digital Transmission
Representation	Uses continuous signals, varying in amplitude and frequency.	Uses discrete signals represented by binary digits (0s and 1s).
Signal Quality	Prone to signal degradation and noise interference during transmission.	Resistant to signal degradation and noise; can be perfectly reconstructed if errors occur.
Bandwidth Efficiency	Less efficient in terms of bandwidth utilization. Requires more bandwidth for the same information.	Highly bandwidth-efficient, as it can transmit more information in the same bandwidth.
Ease of Processing	Analog signals are more challenging to process, manipulate, and transmit without quality loss.	Digital signals are easier to process, manipulate, and transmit without quality loss.
Error Correction	Difficult to correct errors in analog signals once they occur.	Allows for error detection and correction through various coding techniques.

4. Define core and cladding. On what principle does optical fibre work?

Ans) Core: The core is the inner part of the optical fibre. It is the medium, which allows the light to pass through it. The core region has the highest refractive index in the optical fibre. The core is made up of Glass or Plastic.

Cladding: Cladding is the substance that is covered over the core of the optical fibre. It has a lower refractive index while compared to the core of the fibre. The cladding of the Optical fibre is made up of Glass / Plastic.

Principle of optical fibre: The fibre optic networks mainly work on the principle of total internal reflection. Here, the incident angle should be more than the critical angle.

5. What is a Digital Communication System? Why it is advantageous to use digital communication over analog communication?

Ans)

Digital Communication System: A Digital Communication System is a system that transmits and receives information or data using discrete, quantized values, typically represented as binary digits (0s and 1s).

Advantages of digital communication over analog communication:
Digital communication offers advantages over analog communication due to its resistance to noise, error correction capabilities, bandwidth efficiency, ease of processing, versatility, compatibility with modern technology, and enhanced security. It enables reliable, high-quality data transmission and supports a wide range of applications, making it the preferred choice in modern telecommunications and data transmission.

6. Give 5 advantages of satellite communication.

Ans)

1. **Global Coverage:** Satellites can provide communication coverage to almost any location on Earth, including remote and underserved areas that may not have access to other forms of communication infrastructure.
2. **High Bandwidth:** Satellites can transmit large amounts of data, including voice, video, and internet traffic, at high speeds.
3. **Reliability:** Satellite communication is known for its reliability. Satellites are situated in geostationary orbits or constellations, providing consistent and uninterrupted service.
4. **Scalability:** Satellite communication systems are highly scalable.
5. **Quick Deployment:** Satellite communication systems can be quickly deployed in areas affected by natural disasters or emergencies, providing instant communication and connectivity.

7. Write advantages and disadvantages of frequency modulation.

Ans)

Advantages of Frequency Modulation (FM):

1. **Better Signal Quality:** FM offers higher signal quality compared to amplitude modulation (AM).
2. **Resilience to Amplitude Variations:** FM is resistant to amplitude variations in the carrier wave, making it less affected by interference and changes in signal strength during transmission.

3. **Bandwidth Efficiency:** FM signals are more bandwidth-efficient than AM signals.
4. **Low Noise Reception:** FM receivers can provide low-noise reception, enhancing the overall listening experience, especially in FM radio broadcasts.
5. **Applications in High-Frequency Communication:** FM is commonly used in applications like VHF (Very High Frequency) and UHF (Ultra High Frequency) radio, aviation communication, and high-quality audio broadcasting.

Disadvantages of Frequency Modulation (FM):

1. **Large Bandwidth Requirements:** FM signals require a relatively wide bandwidth to transmit compared to some digital modulation techniques.
2. **Complexity of Receivers:** FM receivers are more complex and costly to manufacture compared to AM receivers.
3. **Susceptibility to Multipath Interference:** In certain environments, FM signals can experience multipath interference, where signals bounce off objects and arrive at the receiver through multiple paths. This can cause signal distortion.
4. **Limited Range:** FM signals, especially in VHF and UHF bands, have a relatively limited range compared to lower-frequency AM signals.
5. **Less Suitable for Digital Data Transmission:** While FM is excellent for analog audio transmission, it is not well-suited for transmitting digital data without additional modulation schemes.

8. What are limitations of amplitude modulation?

Ans)

1. **Susceptibility to Noise and Interference:** AM signals are highly susceptible to noise and interference.
2. **Inefficient Bandwidth Usage:** AM uses a relatively wide bandwidth to transmit information.
3. **Limited Frequency Response:** AM has a limited frequency response, which means it can transmit only a certain range of audio frequencies.
4. **Low Power Efficiency:** In AM transmission, a significant portion of the transmitted power is used to transmit the carrier wave, which carries no useful information.
5. **Cross-Modulation and Intermodulation:** In AM broadcasting, strong signals from nearby stations can interfere with each other through cross-modulation and intermodulation effects. This can result in unwanted signal mixing and distortion.

ERQs:

1. What is modulation? Why it is needed for the transmission of signals?

Ans) Modulation: Modulation is a fundamental technique used in telecommunications and signal processing to transmit information or data over a communication channel.

Need of modulation for transmission of signals: Modulation is essential for the transmission of signals in communication systems for several important reasons:

1. **Efficient Use of Spectrum:** The electromagnetic spectrum (the range of frequencies used for communication) is a finite and valuable resource. Modulation allows multiple signals to share the same spectrum without interfering with each other. Different modulation techniques can be used to allocate and utilize available frequency bands efficiently.
2. **Compatibility with Transmission Media:** Different transmission media (e.g., air, cables, optical fibers) have different characteristics and limitations. Modulation allows signals to be adapted to the specific properties of the transmission medium, optimizing signal transmission and reception.
3. **Signal Propagation:** Modulation can influence how a signal propagates through a transmission medium. For example, different modulation techniques can improve a signal's resistance to interference, noise, and distortion, enhancing its propagation characteristics and overall reliability.
4. **Information Encoding:** Modulation encodes information onto a carrier signal, allowing it to carry data, voice, or other forms of information. Without modulation, the carrier signal would be a constant waveform with no capacity to convey information.
5. **Compatibility with Receivers:** Modulation ensures that the received signal can be effectively processed by receivers. It provides a standardized format for signal demodulation, which allows receivers to extract and interpret the original information accurately.
6. **Signal Multiplexing:** Modulation techniques enable multiplexing, where multiple signals can be combined and transmitted simultaneously over the same channel. This is crucial for applications like broadcasting and network communication, where multiple users or services share the same transmission medium.
7. **Signal Security:** Modulation can be used to encrypt or secure information during transmission, providing privacy and protection against unauthorized access or tampering.
8. **Noise Tolerance:** Different modulation techniques offer varying degrees of tolerance to noise and interference. Some modulation schemes are more robust in noisy environments, ensuring reliable communication even when the signal is degraded.

9. **Adaptability:** Modulation techniques can be adapted to suit different communication needs, ranging from simple analog voice transmission to complex digital data transfer, making them versatile for various applications.

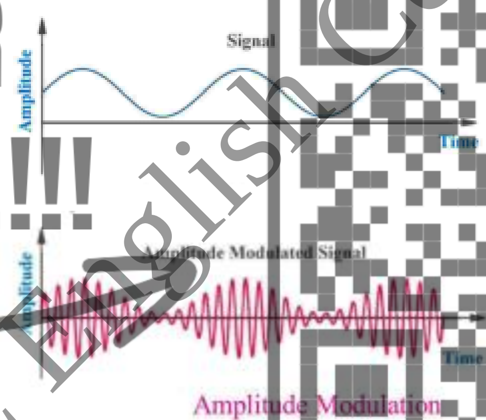
2. How many types of modulation are there? Explain each type of modulation in detail.

Ans) Types of modulation: The Analogue Modulation which is largely divided into two major Types

1. Amplitude Modulation
2. Angle Modulation

1. Amplitude Modulation: It is the type of modulation in which the amplitude of the carrier signal is varied in proportion to the message signal whereas the frequency and phase of the carrier are constant. Television

Example: Broadcast is an example of AM.



2. Angle Modulation: Angle modulation is a modulation technique used in telecommunications to encode information onto a carrier wave by varying the phase or frequency of the carrier signal.

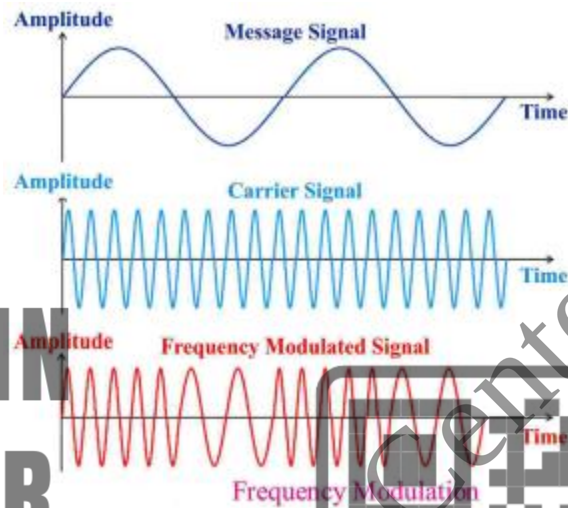
Example: FM radio broadcasting is an example of angle modulation.

The Angle modulation is further divided into two main types:

1. Frequency Modulation
2. Phase Modulation

1. Frequency Modulation: It is the type of modulation in which the frequency of the carrier signal varies in proportion to the message signal and the amplitude of a carrier wave remains constant.

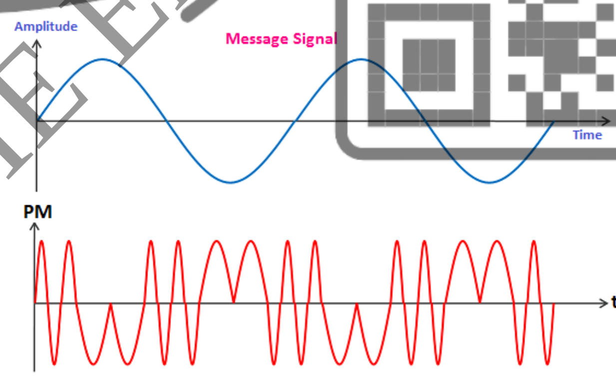
Example: Cellular communication is an example of FM.



Frequency Modulation

2. Phase Modulation: Phase Modulation (PM) is a modulation technique used in telecommunications where information is transmitted by altering the phase of a carrier wave in response to the amplitude variations of a modulating signal.

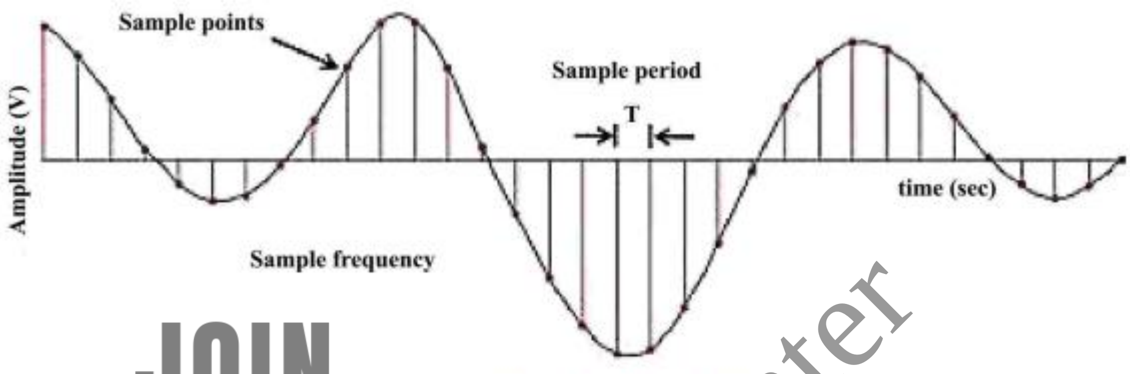
Example: In Binary Phase-Shift Keying (BPSK), there are two possible phase shifts: 0 degrees and 180 degrees.



3. What is A/D conversion? Explain sampling, quantizing and encoding involved in A/D conversion in detail.

Ans) A/D conversion: A/D conversion, or Analog-to-Digital conversion, is the process of converting continuous analog signals into discrete digital values. There are three processes involved in conversion of Analog to Digital signals:

1. Sampling: This is a process of inspecting the value (voltage) of an analog signal at regular time intervals. The time between samples is referred to as the sample period (T, in seconds), and the number of samples taken per second is referred to as the sample frequency (fs, in samples/second or Hz).



Sampling Waveform

The receiver must convert the bits it receives into sample values, and then recreate what it thinks the analog signal looks like from the samples. One must sample faster than the Nyquist sampling rate (also called the Nyquist rate), f_N, given by the formula $f_N = 2f_{max}$, where f_{max} is the highest frequency component of the analog signal. To avoid distortion of your signal $F_s > f_N$ .Some examples of common sample rates are given in the table shown.

Signal	Signal frequency range	Standard Sample Rate
Voice	300 Hz-3 kHz	8KHz
Music	0-20 kHz	44.1 kHz CD Quality
Music	0-20 kHz	192 kHz DVD Quality

2. Quantizing: Quantizing is the process of mapping the sampled analog voltage values to discrete voltage levels, which are then represented by binary numbers (bits). For example, if a sine wave of amplitude 1V is being sampled, the sample values could be between -1V and +1V... an infinite number of possibilities. However, only a finite number of values can be used to represent the samples. Converting a sample value. From infinite possibilities to one of a finite set of values is called quantization. These values are referred to as quantization levels.

An N-bit A/D converter has 2^N quantization levels and outputs binary words of length N. For example, a 3-bit A/D system has $2^3 = 8$ quantization levels, so all samples of 1V analog signal will be quantized into one of only 8 possible quantization levels and each sample will be represented by a 3-bit digital word. In general, the A/D converter will partition a range of voltage from some V_{min} to some V_{max} into 2^N voltage intervals, each of size q volts, where

$$q = \frac{V_{max} - V_{min}}{2^N}$$

3. Encoding: After quantization samples are converted to N-bit binary code words. For the first sample point at time 0, the voltage is 0.613 V, which means that sample is assigned a binary value of 110.

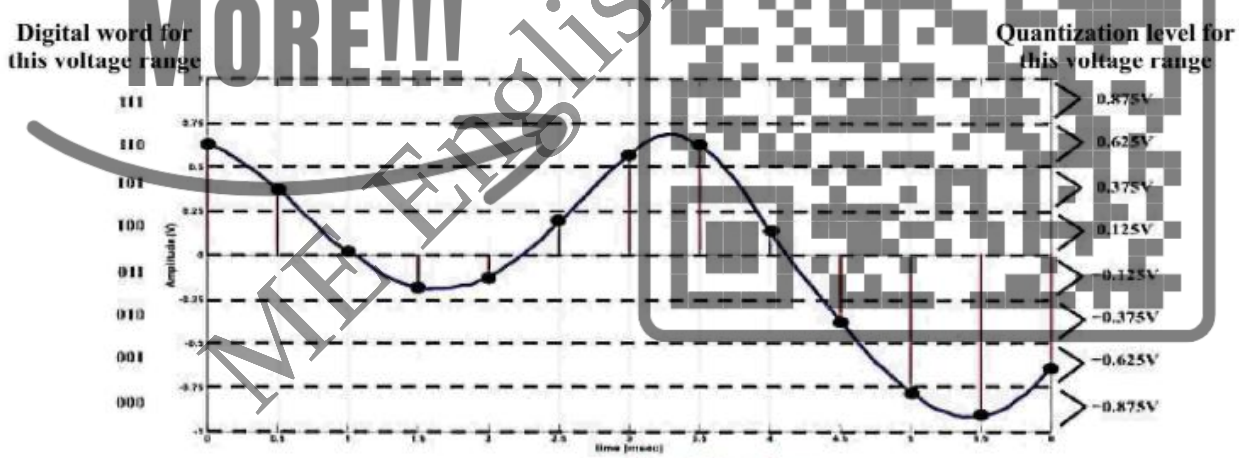
The A/D then creates a voltage signal that represents these bits. The binary representation of the above signal is:

110 101 100 011 011 100 110 110 100 010 000 000 001.

In this example, every sample produces 3 bits (3 bits/sample). The sample rate was 2000 samples/sec. The bit rate (Ra) produced from this:

$$R_b = \frac{3\text{ bits}}{\text{sample}} \times \frac{2000\text{ samples}}{\text{sec}} = 6000\text{ bits/sec (bps)}$$

Bit rate is the speed of transfer of data given in number of bits per second.



Coding Waveform

4. Define communication channel. How information carried through UTP, STP and Coaxial cable? Write their advantages and disadvantages.

Ans) Communication channel: A communication channel is a medium or pathway through which data or information is transmitted from a sender or source to a receiver or destination.

Information carried through UTP: Information is transmitted through Unshielded Twisted Pair (UTP) cables by encoding data into electrical voltage levels (usually binary

0s and 1s). These voltage signals are sent over the twisted pairs of copper wires within the UTP cable. At the receiving end, the signals are detected, decoded, and processed to recover the original data. UTP cables are commonly used for Ethernet networking and other communication applications.

Advantages of UTP Cabling:

1. Cost-effective.
2. Flexible and easy to install.
3. Widely available and compatible.
4. Simple termination with RJ-45 connectors.
5. Suitable for short to medium distances.

Disadvantages of UTP Cabling:

1. Susceptible to electromagnetic and radio frequency interference.
2. Limited distance and signal attenuation.
3. Reduced performance in high-interference environments.
4. Vulnerable to eavesdropping in certain scenarios.
5. Not ideal for harsh industrial conditions.

Information carried through STP: Information is transmitted through Shielded Twisted Pair (STP) cables by encoding digital data into electrical voltage levels, which are sent over pairs of twisted copper wires within the cable. The unique feature of STP cables is the presence of shielding that protects against electromagnetic interference (EMI) and minimizes crosstalk. At the receiving end, the voltage signals are detected, decoded, and processed to recover the original data. STP cables are commonly used in networking applications, providing enhanced protection against external interference compared to Unshielded Twisted Pair (UTP) cables.

Advantages of STP (Shielded Twisted Pair) Cabling:

1. Excellent resistance to interference (EMI/RFI).
2. Enhanced signal integrity and reliability.
3. Better security against eavesdropping.
4. Ideal for high-interference environments.

Disadvantages of STP (Shielded Twisted Pair) Cabling:

1. Higher cost than UTP cables.
2. More complex installation with grounding requirements.

3. Less flexibility and bulkier.
4. Limited bend radius may affect installation.
5. Compatibility issues with some equipment.

Information carried through Coaxial Cable: Information is transmitted through a coaxial cable by encoding it into electrical signals, which are sent along the cable's inner conductor. Coaxial cables have multiple layers, including insulation and a metallic shield, which protect the signal from interference and loss. At the receiving end, the signals are processed and converted back to their original form. Coaxial cables are commonly used for cable television, broadband internet, and telecommunications due to their ability to efficiently transmit high-frequency signals and support two-way communication.

Advantages of Coaxial Cable:

1. High bandwidth capacity.
2. Effective shielding against interference.
3. Suitable for longer distances.
4. Durable for outdoor use.
5. Enhanced security against eavesdropping.

Disadvantages of Coaxial Cable:

1. Bulkier and less flexible.
2. Complex connectors and termination.
3. Limited bend radius.
4. Potentially higher cost.
5. Primarily suited for specific applications.

5. What is optical Fibre? How information carried out through optical fibre? Give advantages of using optical fibre over other communication channels.

Ans) Optical Fibre: Optical fiber, often referred to simply as "fiber optic" or "optical fiber," is a technology used for transmitting data, including voice, video, and digital signals, as pulses of light through strands of very pure glass or plastic fibers.

Information carried out through optical fibre: Information is carried through optical fiber by modulating light signals to represent digital data (0s and 1s). These light signals are generated by a light source, pass through the optical fiber, and are received at the other end by light-sensitive detectors. The detectors convert the light signals back into electrical signals, which are then processed to retrieve the original information. Optical fiber offers high-speed, low-loss, and secure data transmission over long distances, making it a vital technology in modern telecommunications.

Advantages of using optical fibre over other communication channels:

1. **Twisted-Pair Cable (UTP):** Optical fiber offers significantly higher bandwidth, longer-distance transmission, and immunity to electromagnetic interference, whereas twisted-pair cables are susceptible to interference, have limited distance, and lower bandwidth.
2. **Coaxial Cable (e.g., for Cable TV):** Optical fiber provides higher bandwidth, lower signal loss over distance, immunity to EMI/RFI, and enhanced security. Coaxial cables are bulkier, less flexible, and more susceptible to interference.
3. **Radio Waves (Wireless Communication):** Optical fiber has lower latency, higher bandwidth, and superior security compared to wireless communication. Radio waves are subject to interference, signal attenuation, and security risks.
4. **Microwaves (Microwave Transmission):** Optical fiber offers lower signal loss, better security, and greater data capacity than microwave transmission. Microwaves may experience interference from weather conditions.
5. **Satellite Communication:** Optical fiber provides lower latency, higher bandwidth, and immunity to atmospheric interference. Satellite communication may suffer from higher latency, limited bandwidth, and susceptibility to weather-related disruptions.

6. What are the main components of a Satellite? How communication takes place in a satellite. Explain three orbits of a satellite.

Ans) Main components of a Satellite:

1. **Payload:** The payload of a communication satellite consists of transponders and antennas. Transponders receive signals from Earth-based sources, amplify them, and retransmit them to different locations on Earth. Antennas are used to transmit and receive signals.
2. **Bus or Spacecraft Bus:** The spacecraft bus provides support and infrastructure for the satellite's operation. It includes subsystems such as:
   - Power System
   - Thermal Control System
   - Communication System
   - Attitude Control System
   - Onboard Computer
3. **Solar Panels:** Solar panels capture sunlight and convert it into electrical power to operate the satellite's systems and payload.
4. **Antennas:** Communication satellites have specialized antennas for transmitting and receiving signals to and from Earth-based stations.

5. **Telemetry and Control (T&C) Subsystem:** T&C subsystem includes sensors, transmitters, and receivers to collect data about the satellite's status and performance and to communicate this data with ground control.
6. **Command and Data Handling (C&DH) Subsystem:** C&DH subsystem includes the onboard computer, memory, and data interfaces for processing and storing data, as well as executing commands received from ground control.

Communication through Satellite:

1. **Signal Reception:** Communication begins with the reception of signals from Earth-based sources. These signals can include television broadcasts, internet data, voice communication, or other types of data.
2. **Signal Processing:** Within the payload, the received signals are processed to improve their quality and to ensure they are at the appropriate frequency and power levels for transmission. Data compression and error correction techniques may be applied to optimize signal quality.
3. **Frequency Conversion:** The satellite may convert the incoming signals to a different frequency band (uplink frequency) for transmission back to Earth. This is done to avoid interference with the original signal.
4. **Transmission:** The satellite's transponders retransmit the processed signals using the onboard antennas. These signals are sent in the downlink direction from the satellite to Earth.
5. **Communication with Ground Stations:** Earth-based ground stations, such as satellite dishes or ground control stations, receive the downlinked signals from the satellite. Ground stations are equipped with their own antennas and transceivers to communicate with the satellite.
6. **Uploading and Downloading Data:** Ground stations send commands and data to the satellite through the uplink (transmitting to the satellite). The satellite receives the uplinked data and executes commands as necessary. The satellite also uploads data to the ground stations, such as telemetry and payload data.
7. **Telemetry and Control (T&C):** Telemetry and control systems on the satellite collect data about the satellite's status, position, and performance. This data is transmitted to ground control stations through the communication link. Ground control stations use the data to monitor and control the satellite's operations.

Orbits of a satellite: Satellites can be placed in different orbits depending on their intended mission and purpose. Here are explanations of three common satellite orbits:

1. **Low Earth Orbit (LEO):**
   - **Altitude:** LEO satellites orbit at altitudes ranging from approximately 160 kilometers (100 miles) to 2,000 kilometers (1,240 miles) above the Earth's surface.
   - **Characteristics:** LEO is the closest orbit to Earth. Satellites in LEO typically travel at high speeds and complete an orbit around the Earth in roughly 90 minutes. Due to their proximity to Earth, LEO satellites have

low latency, making them suitable for applications requiring real-time communication and observation.
- **Applications:** LEO orbits are commonly used for Earth observation satellites, including those for weather forecasting, environmental monitoring, and remote sensing.

2. **Medium Earth Orbit (MEO):**
   - **Altitude:** MEO satellites orbit at altitudes between approximately 2,000 kilometers (1,240 miles) and 35,786 kilometers (22,236 miles) above the Earth's surface.
   - **Characteristics:** MEO is situated between LEO and geostationary orbits. Satellites in MEO have longer orbital periods compared to LEO satellites but shorter than those in geostationary orbits. They take several hours to complete an orbit.
   - **Applications:** MEO orbits are often used for navigation and global positioning system (GPS) satellites.
3. **Geostationary Orbit (GEO):**
   - **Altitude:** GEO satellites orbit at an altitude of approximately 35,786 kilometers (22,236 miles) above the Earth's equator.
   - **Characteristics:** GEO satellites orbit at the same rotational speed as the Earth, which makes them appear stationary relative to a specific point on the Earth's surface. This geostationary position allows continuous coverage of the same geographic area.
   - **Applications:** GEO orbits are commonly used for communication satellites, including those for television broadcasting and long-distance communication.