Exercise 1.1

Q. 1: Write the following quadratic equations in the standard form and point out pure quadratic equations.

(i) $(x + 7)(x - 3) = -7$

$x^2 - 3x + 7x - 21 = -7$

$x^2 + 4x - 21 = -7$

$x^2 + 4x - 21 + 7 = 0$

$x^2 + 4x - 14 = 0$

as $b \neq 0$, So the quadratic equation is not pure.

(ii) $\frac{x^2+4}{3} - \frac{x}{7} = 1$

$\frac{7(x^2+4)-3x}{21} = 1$

$\frac{7x^2+28-3x}{21} = 1$

$7x^2 + 28 - 3x = 21$

$7x^2 - 3x + 28 - 21 = 0$

$7x^2 - 3x + 7 = 0$

as $b \neq 0$, So the quadratic equation is not pure.

(iii) $\frac{x}{x+1} - \frac{x+1}{x} = 6$

$\frac{x^2-(x+1)(x+1)}{x(x+1)} = 6$

$\frac{x^2-(x^2+x+x+1)}{x^2+x} = 6$

$\frac{x^2-(x^2+2x+1)}{x^2+x} = 6$

$\frac{x^2-x^2-2x-1}{x^2+x} = 6$

$\frac{-2x-1}{x^2+x} = 6$

$-2x - 1 = 6(x^2 + x)$

$-2x - 1 = 6x^2 + 6x$

$-2x - 1 - 6x^2 - 6x = 0$

$-1 - 6x^2 - 8x = 0$

as $b \neq 0$, So the quadratic equation is not pure.

(iv) $\frac{x+4}{x-2} - \frac{x-2}{x} + 4 = 0$

$\frac{x(x+4)-(x-2)(x-2)}{x(x-2)} = -4$

$\frac{x^2+4x-(x^2-2x-2x+4)}{x^2-2x} = -4$

$\frac{x^2+4x-(x^2-4x+4)}{x^2-2x} = -4$

$\frac{x^2+4x-x^2+4x-4}{x^2-2x} = -4$

$\frac{8x-4}{x^2-2x} = -4$

$8x - 4 = -4(x^2 - 2x)$

$4(2x-1) = -4(x^2-2x)$

$(2x-1) = \frac{-4(x^2-2x)}{4}$

$2x-1 = -x^2+2x$

$2x-1+x^2-2x = 0$

$x^2-1 = 0$

as b = 0, So the quadratic equation is pure.

(v) $\frac{x+3}{x+4} - \frac{x-5}{x} = 1$

$\frac{x(x+3)-(x-5)(x+4)}{x(x+4)} = 1$

$\frac{x^2+3x-(x^2-5x+4x-20)}{x^2+4x} = 1$

$\frac{x^2+3x-(x^2-x-20)}{x^2+4x} = 1$

$\frac{x^2+3x-x^2+x+20}{x^2+4x} = 1$

$\frac{4x+20}{x^2+4x} = 1$

$4x+20 = 1(x^2+4x)$

$4x+20 = x^2+4x$

$4x+20-x^2-4x = 0$

$-x^2+20 = 0$

$x^2-20 = 0$

as b ≠ 0, So the quadratic equation is not pure.

(vi) $\frac{x+1}{x+2} - \frac{x+2}{x+3} = \frac{25}{12}$

$\frac{(x+1)(x+3)-(x+2)(x+2)}{(x+2)(x+3)} = \frac{25}{12}$

$\frac{(x^2+x+3x+3)-(x^2+2x+2x+4)}{x^2+2x+3x+6} = \frac{25}{12}$

$\frac{(x^2 4x+3)-(x^2+4x+4)}{x^2+5x+6} = \frac{25}{12}$

$\frac{x^2+4x+3-x^2-4x-4}{x^2+5x+6} = \frac{25}{12}$

$\frac{-1}{x^2+5x+6} = \frac{25}{12}$

$-12 = 25(x^2+5x+6)$

$-12 = 25x^2+125x+150$

$-25x^2-125x-12-150 = 0$

$-25x^2-125x-162 = 0$

$25x^2+125x+162 = 0$

as b ≠ 0, So the quadratic equation is not pure.

Q. 2: Solve the factorization:

(i) $x^2-x-20 = 0$

$x^2-5x+4x-20 = 0$

$x(x-5)+4(x-5) = 0$

$(x-5)(x+4) = 0$

$x - 5 = 0$ and $x + 4 = 0$

$x = 5$ and $x = -4$

S.S $= \{5, -4\}$

(ii) $3y^2 = y(y - 5)$

$3y^2 = y^2 - 5y$

$3y^2 - y^2 + 5y = 0$

$2y^2 + 5y = 0$

$y(2y + 5) = 0$

$y = 0$ and $2y + 5 = 0$

$y = 0$ and $2y = -5$

$y = 0$ and $y = \frac{-5}{2}$

S.S $= \left\{0, -\frac{5}{2}\right\}$

(iii) $4 - 32x = 17x^2$

$0 = 17x^2 + 32x - 4$

$17x^2 + 32x - 4 = 0$

$17x^2 + 34x - 2x - 4 = 0$

$17x(x + 2) - 2(x + 2) = 0$

$(x + 2)(17x - 2) = 0$

$x + 2 = 0$ and $17x - 2 = 0$

$x = -2$ and $17x = 2$

$x = -2$ and $x = \frac{2}{17}$

S.S $= \left\{-2, \frac{2}{17}\right\}$

(iv) $x^2 - 11x = 152$

$x^2 - 11x - 152 = 0$

$x^2 - 19x + 8x - 152 = 0$

$x(x - 19) + 8(x - 19) = 0$

$(x - 19)(x + 8) = 0$

$x - 19 = 0$ and $x + 8 = 0$

$x = 19$ and $x = -8$

S.S $= \{19, -8\}$

(v) $\frac{x+1}{x} + \frac{x}{x+1} = \frac{25}{12}$

$\frac{(x+1)(x+1)+x^2}{x(x+1)} = \frac{25}{12}$

$\frac{(x^2+x+x+1)+x^2}{x^2+x} = \frac{25}{12}$

$\frac{(x^2+2x+1)+x^2}{x^2+x} = \frac{25}{12}$

$\frac{x^2+2x+1+x^2}{x^2+x} = \frac{25}{12}$

$\frac{2x^2+2x+1}{x^2+x} = \frac{25}{12}$

$12(2x^2 + 2x + 1) = 25(x^2 + x)$
$24x^2 + 24x + 12 = 25x^2 + 25x$
$0 = 25x^2 + 25x - 24x^2 - 24x - 12$
$0 = x^2 + x - 12$
$x^2 + x - 12 = 0$
$x^2 + 4x - 3x - 12 = 0$
$x(x + 4) - 3(x + 4) = 0$
$(x + 4)(x - 3) = 0$
$x + 4 = 0$ and $x - 3 = 0$
$x = -4$ and $x = 3$
S.S $= \{-4, 3\}$

(vi) $\frac{2}{x-9} = \frac{1}{x-3} - \frac{1}{x-4}$
$\frac{2}{x-9} + \frac{1}{x-4} = \frac{1}{x-3}$
$\frac{2(x-4)+x-9}{(x-9)(x-4)} = \frac{1}{x-3}$
$\frac{2x-8+x-9}{x^2-9x-4x+36} = \frac{1}{x-3}$
$\frac{3x-17}{x^2-13x+36} = \frac{1}{x-3}$
$(x - 3)(3x - 17) = 1(x^2 - 13x + 36)$
$3x^2 - 9x - 17x + 51 = x^2 - 13x + 36$
$3x^2 - 26x + 51 = x^2 - 13x + 36$
$3x^2 - 26x + 51 - x^2 + 13x - 36 = 0$
$2x^2 - 13x + 15 = 0$
$2x^2 - 10x - 3x + 15 = 0$
$2x(x - 5) - 3(x - 5) = 0$
$(x - 5)(x - 3) = 0$
$x - 5 = 0$ and $x - 3 = 0$
$x = 5$ and $x = 3$
S.S $= \{5, 3\}$

Q. 3: Solve the following equations by completing square:

(i) $7x^2 + 2x - 1 = 0$
$7x^2 + 2x = 1$

Dividing by 7

$\frac{7}{7}x^2 + \frac{2}{7}x = \frac{1}{7}$
$x^2 + 2(x)\frac{1}{7} = \frac{1}{7}$

Adding $\left(\frac{1}{7}\right)^2$ on both sides

$x^2 + 2(x)\frac{1}{7} + \left(\frac{1}{7}\right)^2 = \frac{1}{7} + \left(\frac{1}{7}\right)^2$
$\left(x + \frac{1}{7}\right)^2 = \frac{1}{7} + \frac{1}{49}$

$$\left(x+\frac{1}{7}\right)^2 = \frac{7+1}{49}$$
$$\left(x+\frac{1}{7}\right)^2 = \frac{8}{49}$$

Taking square on both sides

$$\sqrt{\left(x+\frac{1}{7}\right)^2} = \sqrt{\frac{8}{49}}$$
$$x+\frac{1}{7} = \pm\frac{2\sqrt{2}}{7}$$
$$x = \pm\frac{2\sqrt{2}}{7}-\frac{1}{7}$$
$$x = +\frac{2\sqrt{2}}{7}-\frac{1}{7} \quad \text{and} \quad x = -\frac{2\sqrt{2}}{7}-\frac{1}{7}$$
$$x = \frac{2\sqrt{2}-1}{7} \quad \text{and} \quad x = \frac{-2\sqrt{2}-1}{7}$$
$$\text{S.S} = \left\{\frac{2\sqrt{2}-1}{7}, \frac{-2\sqrt{2}-1}{7}\right\}$$

(ii) $ax^2+4x-a=0$

$$ax^2+4x = a$$

Dividing by a

$$\frac{a}{a}x^2+\frac{4}{a}x = \frac{a}{a}$$
$$x^2+2(x)\frac{2}{a} = 1$$

Adding $\left(\frac{2}{a}\right)^2$ on both sides

$$x^2+2(x)\frac{2}{a}+\left(\frac{2}{a}\right)^2 = 1+\left(\frac{2}{a}\right)^2$$
$$\left(x+\frac{2}{a}\right)^2 = 1+\frac{4}{a^2}$$
$$\left(x+\frac{2}{a}\right)^2 = \frac{a^2+4}{a^2}$$
$$\left(x+\frac{2}{a}\right)^2 = \frac{a^2+4}{a^2}$$

Taking square on both sides

$$\sqrt{\left(x+\frac{2}{a}\right)^2} = \sqrt{\frac{a^2+4}{a^2}}$$
$$x+\frac{2}{a} = \pm\frac{\sqrt{a^2+4}}{a}$$
$$x = \pm\frac{\sqrt{a^2+4}}{a}-\frac{2}{a}$$
$$x = \pm\frac{\sqrt{a^2+4}-2}{a}$$
$$\text{S.S} = \left\{\pm\frac{\sqrt{a^2+4}-2}{a}\right\}$$

(iii) $11x^2-34x+3=0$

$$11x^2-34x = -3$$

Dividing by 11

$$\frac{11}{11}x^2-\frac{34}{11}x = \frac{-3}{11}$$

$$x^2 + 2(x)\frac{17}{11} = \frac{-3}{11}$$

Adding $\left(\frac{17}{11}\right)^2$ on both sides

$$x^2 + 2(x)\frac{17}{11} + \left(\frac{17}{11}\right)^2 = \frac{1}{11} + \left(\frac{17}{11}\right)^2$$

$$\left(x + \frac{17}{11}\right)^2 = \frac{1}{11} + \frac{289}{121}$$

$$\left(x + \frac{17}{11}\right)^2 = \frac{11+289}{121}$$

$$\left(x + \frac{17}{11}\right)^2 = \frac{300}{121}$$

Taking square on both sides

$$\sqrt{\left(x + \frac{17}{11}\right)^2} = \sqrt{\frac{300}{121}}$$

$$x + \frac{17}{11} = \pm\frac{10\sqrt{3}}{11}$$

$$x = \pm\frac{10\sqrt{3}}{11} - \frac{17}{11}$$

$x = +\frac{10\sqrt{3}}{11} - \frac{17}{11}$ and $x = -\frac{10\sqrt{3}}{11} - \frac{17}{11}$

$x = \frac{10\sqrt{3}-17}{11}$ and $x = \frac{-10\sqrt{3}-17}{11}$

S.S $= \left\{\frac{10\sqrt{3}-17}{11}, \frac{-10\sqrt{3}-17}{11}\right\}$

(iv) $lx^2 + mx + n = 0$

$$lx^2 + mx = -n$$

Dividing by l

$$\frac{l}{l}x^2 + \frac{m}{l}x = -\frac{n}{l}$$

$$x^2 + 2(x)\frac{m}{2l} = -\frac{n}{l}$$

Adding $\left(\frac{m}{2l}\right)^2$ on both sides

$$x^2 + 2(x)\frac{m}{2l} + \left(\frac{m}{2l}\right)^2 = -\frac{n}{l} + \left(\frac{m}{2l}\right)^2$$

$$\left(x + \frac{m}{2l}\right)^2 = -\frac{n}{l} + \frac{m^2}{4l^2}$$

$$\left(x + \frac{m}{2l}\right)^2 = \frac{-4ln+m^2}{4l^2}$$

$$\left(x + \frac{m}{2l}\right)^2 = \frac{m^2-4ln}{4l^2}$$

Taking square on both sides

$$\sqrt{\left(x + \frac{m}{2l}\right)^2} = \sqrt{\frac{m^2-4ln}{4l^2}}$$

$$x + \frac{m}{2l} = \pm\frac{\sqrt{m^2-4ln}}{2l}$$

$$x = \pm\frac{\sqrt{m^2-4ln}}{2l} - \frac{m}{2l}$$

S.S $= \left\{\frac{-m\pm\sqrt{m^2-4ln}}{2l}\right\}$

(v) $3x^2 + 7x = 0$

$3x^2 + 7x = 0$

Dividing by l

$\frac{3}{3}x^2 + \frac{7}{3}x = \frac{0}{3}$

$x^2 + 2(x)\frac{7}{6} = 0$

Adding $\left(\frac{7}{6}\right)^2$ on both sides

$x^2 + 2(x)\frac{7}{6} + \left(\frac{7}{6}\right)^2 = \left(\frac{7}{6}\right)^2$

$\left(x + \frac{7}{6}\right)^2 = \frac{49}{36}$

$\left(x + \frac{7}{6}\right)^2 = \frac{49}{36}$

$\left(x + \frac{7}{6}\right)^2 = \frac{49}{36}$

Taking square on both sides

$\sqrt{\left(x + \frac{7}{6}\right)^2} = \sqrt{\frac{49}{36}}$

$x + \frac{7}{6} = \pm\frac{7}{6}$

$x = +\frac{7}{6} - \frac{7}{6}$ and $x = -\frac{7}{6} - \frac{7}{6}$

$x = 0$ and $x = \frac{-7-7}{6}$

$x = 0$ and $x = \frac{-14}{6}$

S.S $= \left\{0, \frac{-7}{3}\right\}$

(vi) $x^2 - 2x - 195 = 0$

$x^2 - 2x = 195$

$x^2 - 2x = 195$

$x^2 - 2(x)(1) = 195$

Adding $(1)^2$ on both sides

$x^2 + 2(x)(1) + (1)^2 = 195 + (1)^2$

$(x + 1)^2 = 195 + 1$

$(x + 1)^2 = 196$

$(x + 1)^2 = 196$

Taking square on both sides

$\sqrt{(x + 1)^2} = \sqrt{196}$

$x + 1 = \pm 14$

$x = +14 - 1$ and $x = -14 - 1$

$x = 13$ and $x = -15$

$x = 13$ and $x = -15$

S.S $= \{13, -15\}$

(vii) $-x^2 + \frac{15}{2} = \frac{7}{2}x$

$$\frac{15}{2} = x^2 + \frac{7}{2}x$$

$$x^2 + \frac{7}{2}x = \frac{15}{2}$$

$$x^2 + 2(x)\frac{7}{4} = \frac{15}{2}$$

Adding $\left(\frac{7}{4}\right)^2$ on both sides

$$x^2 + 2(x)\frac{7}{4} + \left(\frac{7}{4}\right)^2 = \frac{15}{2} + \left(\frac{7}{4}\right)^2$$

$$\left(x + \frac{7}{4}\right)^2 = \frac{15}{2} + \frac{49}{16}$$

$$\left(x + \frac{7}{4}\right)^2 = \frac{120+49}{16}$$

$$\left(x + \frac{7}{4}\right)^2 = \frac{169}{16}$$

Taking square on both sides

$$\sqrt{\left(x + \frac{7}{4}\right)^2} = \sqrt{\frac{169}{16}}$$

$$x + \frac{7}{4} = \pm\frac{13}{4}$$

$x = +\frac{13}{4} - \frac{7}{4}$ and $x = -\frac{13}{4} - \frac{7}{4}$

$x = 0$ and $x = \frac{-7-7}{6}$

$x = 0$ and $x = \frac{-14}{6}$

S.S $= \left\{0, \frac{-7}{3}\right\}$

(viii) $x^2 + 17x + \frac{33}{4} = 0$

$$x^2 + 17x = -\frac{33}{4}$$

$$x^2 + 2(x)\frac{17}{2} = \frac{-33}{4}$$

Adding $\left(\frac{17}{2}\right)^2$ on both sides

$$x^2 + 2(x)\frac{17}{2} + \left(\frac{17}{2}\right)^2 = \frac{-33}{4} + \left(\frac{17}{2}\right)^2$$

$$\left(x + \frac{17}{2}\right)^2 = \frac{-33}{4} + \frac{289}{4}$$

$$\left(x + \frac{17}{2}\right)^2 = \frac{-33+289}{4}$$

$$\left(x + \frac{7}{2}\right)^2 = \frac{256}{4}$$

Taking square on both sides

$$\sqrt{\left(x + \frac{7}{2}\right)^2} = \sqrt{\frac{256}{4}}$$

$$x + \frac{7}{2} = \pm\frac{16}{2}$$

$x = +\frac{16}{2} - \frac{7}{2}$ and $x = -\frac{16}{2} - \frac{7}{2}$

$x = \frac{16-7}{2}$ and $x = \frac{-16-7}{2}$

$x = \frac{9}{2}$ and $x = \frac{-23}{2}$

S.S $= \left\{\frac{9}{2}, \frac{-23}{2}\right\}$

(ix) $4 - \frac{8}{3x+1} = \frac{3x^2+5}{3x+1}$

$4 = \frac{3x^2+5}{3x+1} + \frac{8}{3x+1}$

$4 = \frac{3x^2+5+8}{3x+1}$

$4(3x+1) = 3x^2 + 13$

$12x + 4 = 3x^2 + 13$

$0 = 3x^2 + 13 - 12x - 4$

$0 = 3x^2 - 12x - 9$

$3x^2 - 12x - 9 = 0$

$3x^2 - 12x = 9$

Dividing both sides by 3

$\frac{3}{3}x^2 - \frac{12}{3}x = \frac{9}{3}$

$x^2 - 4x = 3$

$x^2 - 2(x)(2) = 3$

Adding $(2)^2$ on both sides

$x^2 + 2(x)(2) + (2)^2 = 3 + (2)^2$

$(x+2)^2 = 3 + 4$

$(x+2)^2 = 7$

$(x+2)^2 = 7$

Taking square on both sides

$\sqrt{(x+2)^2} = \sqrt{7}$

$x + 2 = \pm\sqrt{7}$

$x = -2 \pm \sqrt{7}$

S.S $= \{-2 \pm \sqrt{7}\}$

(x) $7(x+2a)^2 + 3a^2 = 5a(7x + 23a)$

$7(x^2 + 2ax + 4a^2) + 3a^2 = 35ax + 115a^2$

$7x^2 + 14ax + 28a^2 + 3a^2 = 35ax + 115a^2$

$7x^2 + 14ax + 31a^2 = 35ax + 115a^2$

$7x^2 + 14ax + 31a^2 - 35ax - 115a^2 = 0$

$7x^2 - 21ax - 84a^2 = 0$

Dividing both sides by 7

$\frac{7}{7}x^2 - \frac{21}{7}ax - \frac{84}{7}a^2 = 0$

$x^2 - 3ax - 12a^2 = 0$

$x^2 - 3ax = 12a^2$

$x^2 - 2(x)\left(\frac{3a}{2}\right) = 12a^2$

Adding $\left(\frac{3a}{2}\right)^2$ on both sides

$$x^2 + 2(x)\frac{3a}{2} + \left(\frac{3a}{2}\right)^2 = 12a^2 + \left(\frac{3a}{2}\right)^2$$

$$\left(x + \frac{3a}{2}\right)^2 = 12a^2 + \frac{9a^2}{4}$$

$$\left(x + \frac{3a}{2}\right)^2 = \frac{48a^2 + 9a^2}{4}$$

$$\left(x + \frac{3a}{2}\right)^2 = \frac{57a^2}{4}$$

Taking square on both sides

$$\sqrt{\left(x + \frac{3a}{2}\right)^2} = \sqrt{\frac{57a^2}{4}}$$

$$x + \frac{3a}{2} = \pm\frac{\sqrt{57}a}{2}$$

$$x = -\frac{3a}{2} \pm \frac{\sqrt{57}a}{2}$$

$$x = \frac{-3a \pm \sqrt{57}a}{2}$$

$$\text{S.S} = \left\{\frac{-3a \pm \sqrt{57}a}{2}\right\}$$

Exercise 1.2

Q. 1: Solve the following equations using quadratic formula.

(i) $2 - x^2 = 7x$

$-x^2 - 7x + 2 = 0$

a = -1, b = -7, c = 2

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-7) \pm \sqrt{(-7)^2-4(-1)(2)}}{2(-1)}$

$x = \frac{7 \pm \sqrt{49+8}}{-2}$

$x = \frac{7 \pm \sqrt{57}}{-2}$

S.S $= \left\{\frac{7 \pm \sqrt{57}}{-2}\right\}$

(ii) $5x^2 + 8x + 1 = 0$

a = 5, b = 8, c = 1

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-(8) \pm \sqrt{(8)^2-4(5)(1)}}{2(5)}$

$x = \frac{-8 \pm \sqrt{64-20}}{10}$

$x = \frac{-8 \pm \sqrt{44}}{10}$

$x = \frac{-8 \pm \sqrt{4 \times 11}}{10}$

$x = \frac{-8 \pm 2\sqrt{11}}{10}$

$x = \frac{2(-4 \pm \sqrt{11})}{10}$

$x = \frac{-4 \pm \sqrt{11}}{5}$

S.S $= \left\{\frac{-4 \pm \sqrt{11}}{5}\right\}$

(iii) $\sqrt{3}x^2 + x = 4\sqrt{3}$

$\sqrt{3}x^2 + x - 4\sqrt{3} = 0$

a = $\sqrt{3}$, b = 1, c = $-4\sqrt{3}$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-(1) \pm \sqrt{(1)^2-4(\sqrt{3})(-4\sqrt{3})}}{2(\sqrt{3})}$

$x = \frac{-1 \pm \sqrt{1+16(\sqrt{3})^2}}{2\sqrt{3}}$

$x = \frac{-1 \pm \sqrt{1+16(3)}}{2\sqrt{3}}$

$x = \frac{-1\pm\sqrt{1+48}}{2\sqrt{3}}$

$x = \frac{-1\pm\sqrt{49}}{2\sqrt{3}}$

$x = \frac{-1\pm 7}{2\sqrt{3}}$

$x = \frac{-1+7}{2\sqrt{3}}$ and $x = \frac{-1-7}{2\sqrt{3}}$

$x = \frac{6}{2\sqrt{3}}$ and $x = \frac{-8}{2\sqrt{3}}$

$x = \frac{3}{\sqrt{3}}$ and $x = \frac{-4}{\sqrt{3}}$

$x = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ and $x = \frac{-4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$x = \frac{3\sqrt{3}}{(\sqrt{3})^2}$ and $x = \frac{-4\sqrt{3}}{(\sqrt{3})^2}$

$x = \frac{3\sqrt{3}}{3}$ and $x = \frac{-4\sqrt{3}}{3}$

$x = \sqrt{3}$ and $x = \frac{-4\sqrt{3}}{3}$

S.S $= \left\{\sqrt{3}, \frac{-4\sqrt{3}}{3}\right\}$

(iv) $4x^2 - 14 = 3x$

$4x^2 - 3x - 14 = 0$

a = 4, b = -3, c = -14

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-3)\pm\sqrt{(-3)^2-4(4)(-14)}}{2(4)}$

$x = \frac{3\pm\sqrt{9+224}}{8}$

$x = \frac{3\pm\sqrt{233}}{8}$

S.S $= \left\{\frac{3\pm\sqrt{233}}{8}\right\}$

(v) $6x^2 - 3 - 7x = 0$

$6x^2 - 7x - 3 = 0$

a = 6, b = -7, c = -3

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-7)\pm\sqrt{(-7)^2-4(6)(-3)}}{2(6)}$

$x = \frac{7\pm\sqrt{49+72}}{12}$

$x = \frac{7\pm\sqrt{121}}{12}$

$x = \frac{7\pm 11}{12}$

$x = \frac{7+11}{12}$ and $x = \frac{7-11}{12}$

$x = \frac{18}{12}$ and $x = \frac{-4}{12}$

$x = \frac{3}{2}$ and $x = \frac{-1}{3}$

S.S $= \left\{\frac{3}{2}, \frac{-1}{3}\right\}$

(vi) $3x^2 + 8x + 2 = 0$

a = 3, b = 8, c = 2

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(3)(2)}}{2(3)}$

$x = \frac{-8 \pm \sqrt{64 - 24}}{6}$

$x = \frac{-8 \pm \sqrt{40}}{6}$

$x = \frac{-8 \pm \sqrt{4 \times 10}}{6}$

$x = \frac{-8 \pm 2\sqrt{10}}{6}$

$x = \frac{2\left(-4 \pm \sqrt{10}\right)}{6}$

$x = \frac{-4 \pm \sqrt{10}}{3}$

$x = \frac{-4 + \sqrt{10}}{3}$ and $x = \frac{-4 - \sqrt{10}}{3}$

S.S $= \left\{\frac{-4 + \sqrt{10}}{3}, \frac{-4 - \sqrt{10}}{3}\right\}$

(vii) $\frac{3}{x-6} - \frac{4}{x-5} = 1$

$\frac{3(x-5) - 4(x-6)}{(x-6)(x-5)} = 1$

$\frac{3x - 15 - 4x + 24}{x^2 - 6x - 5x + 30} = 1$

$\frac{-x + 9}{x^2 - 11x + 30} = 1$

$-x + 9 = x^2 - 11x + 30$

$0 = x^2 - 11x + 30 + x - 9$

$0 = x^2 - 10x + 21$

a = 1, b = -10, c = 21

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(21)}}{2(1)}$

$x = \frac{10 \pm \sqrt{100 - 84}}{2}$

$x = \frac{10 \pm \sqrt{16}}{2}$

$x = \frac{10 \pm 4}{2}$

$x = \frac{10 + 4}{2}$ and $x = \frac{10 - 4}{2}$

$x = \frac{14}{2}$ and $x = \frac{6}{2}$

S.S $= \{7, 3\}$

(viii) $\frac{x+2}{x-1} - \frac{4-x}{2x} = 2\frac{1}{3}$

$\frac{2x(x+2)-(4-x)(x-1)}{2x(x-1)} = \frac{7}{3}$

$\frac{(2x^2+4x)-(4x-4-x^2+x)}{2x^2-2x} = \frac{7}{3}$

$\frac{(2x^2+4x)-(5x-4-x^2)}{2x^2-2x} = \frac{7}{3}$

$\frac{2x^2+4x-5x+4+x^2}{2x^2-2x} = \frac{7}{3}$

$\frac{3x^2-x+4}{2x^2-2x} = \frac{7}{3}$

$3(3x^2 - x + 4) = 7(2x^2 - 2x)$

$9x^2 - 3x + 12 = 14x^2 - 14x$

$9x^2 - 3x + 12 - 14x^2 + 14x = 0$

$-5x^2 + 11x + 12 = 0$

a = −5, b = 11, c = 12

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-(11)\pm\sqrt{(11)^2-4(-5)(12)}}{2(-5)}$

$x = \frac{-11\pm\sqrt{121+240}}{-10}$

$x = \frac{-11\pm\sqrt{361}}{-10}$

$x = \frac{-11\pm 19}{-10}$

$x = \frac{-11+19}{-10}$ and $x = \frac{-11-19}{-10}$

$x = \frac{8}{-10}$ and $x = \frac{-30}{-10}$

$x = \frac{-4}{5}$ and $x = 3$

S.S $= \left\{\frac{-4}{5}, 3\right\}$

(ix) $\frac{a}{x-b} + \frac{b}{x-a} = 2$

$\frac{a(x-a)+b(x-b)}{(x-a)(x-b)} = 2$

$\frac{ax-a^2+bx-b^2}{x^2-bx-ax+ab} = 2$

$\frac{ax+bx-a^2-b^2}{x^2-ax-bx+ab} = 2$

$ax + bx - a^2 - b^2 = 2(x^2 - ax - bx + ab)$

$ax + bx - a^2 - b^2 = 2x^2 - 2ax - 2bx + 2ab$

$0 = 2x^2 - 2ax - 2bx + 2ab - ax - bx + a^2 + b^2$

$0 = 2x^2 - 3ax - 3bx + 2ab + a^2 + b^2$

$0 = 2x^2 - 3(a + b)x + 2ab + a^2 + b^2$

a = 2, b = -3(a + b), c = $a^2 + b^2 + 2ab$

$$x = \frac{-b \pm \sqrt{(b)^2 - 4ac}}{2a}$$

$$x = \frac{-(-3(a+b)) \pm \sqrt{(-3(a+b))^2 - 4(2)(a^2+b^2+2ab)}}{2(2)}$$

$$x = \frac{3a+3b \pm \sqrt{9(a+b)^2 - 8(a+b)^2}}{4}$$

$$x = \frac{3a+3b \pm \sqrt{(a+b)^2}}{4}$$

$$x = \frac{3a+3b \pm (a+b)}{4}$$

$x = \frac{3a+3b+(a+b)}{4}$ and $x = \frac{3a+3b-(a+b)}{4}$

$x = \frac{3a+3b+a+b}{4}$ and $x = \frac{3a+3b-a-b}{4}$

$x = \frac{4a+4b}{4}$ and $x = \frac{2a+2b}{4}$

$x = \frac{4(a+b)}{4}$ and $x = \frac{2(a+b)}{4}$

$x = a + b$ and $x = \frac{a+b}{2}$

S.S $= \left\{a + b, \frac{a+b}{2}\right\}$

(x) $-(l + m) - lx^2 + (2l + m)x = 0$

$-lx^2 + (2l + m)x - (l + m) = 0$

a = $-l$, b = $(2l + m)$, c = $-(l + m)$

$$x = \frac{-b \pm \sqrt{(b)^2 - 4ac}}{2a}$$

$$x = \frac{-(2l+m) \pm \sqrt{(2l+m)^2 - 4(-l)(-(l+m))}}{2(-l)}$$

$$x = \frac{-(2l+m) \pm \sqrt{4l^2+m^2+4lm-4l^2-4lm}}{-2l}$$

$$x = \frac{-(2l+m) \pm \sqrt{m^2}}{-2l}$$

$$x = \frac{-(2l+m) \pm m}{-2l}$$

$x = \frac{-2l-m+m}{-2l}$ and $x = \frac{-2l-m-m}{-2l}$

$x = \frac{-2l}{-2l}$ and $x = \frac{-2l-2m}{-2l}$

$x = 1$ and $x = \frac{-2(l+m)}{-2l}$

S.S $= \left\{1, \frac{l+m}{l}\right\}$

Exercise 1.3

Solve the following equations.

1. $2x^4 - 11x^2 + 5 = 0$

$2(x^2)^2 - 11(x^2) + 5 = 0$

let $x^2 = y$------------------------(a)

So,

$2y^2 - 11y + 5 = 0$

$2y^2 - 10y - y + 5 = 0$

$2y(y-5) - 1(y-5) = 0$

$(y-5)(2y-1) = 0$

$y - 5 = 0$ and $2y - 1 = 0$

$y = 5$ and $y = \frac{1}{2}$

Put the values y in equ---(a)

$x^2 = 5$ and $x^2 = \frac{1}{2}$

Taking square root of both sides

$\sqrt{x^2} = \pm\sqrt{5}$ and $\sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$

$x = \pm\sqrt{5}$ and $x = \pm\frac{1}{\sqrt{2}}$

S.S $= \left\{\pm\sqrt{5}, \pm\frac{1}{\sqrt{2}}\right\}$

2. $2x^4 = 9x^2 - 4$

$2x^4 - 9x^2 + 4 = 0$

$2(x^2)^2 - 9(x^2) + 4 = 0$

let $x^2 = y$------------------------(a)

So,

$2y^2 - 9y + 4 = 0$

$2y^2 - 8y - y + 4 = 0$

$2y(y-4) - 1(y-4) = 0$

$(y-4)(2y-1) = 0$

$y - 4 = 0$ and $2y - 1 = 0$

$y = 4$ and $y = \frac{1}{2}$

Put the values y in equ---(a)

$x^2 = 4$ and $x^2 = \frac{1}{2}$

Taking square root of both sides

$\sqrt{x^2} = \pm\sqrt{4}$ and $\sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$

$x = \pm 2$ and $x = \pm\frac{1}{\sqrt{2}}$

S.S $= \left\{\pm 2, \pm \frac{1}{\sqrt{2}}\right\}$

3. $5x^{1/2} = 7x^{1/4} - 2$

$5x^{1/2} - 7x^{1/4} + 2 = 0$

$5\left(x^{1/4}\right)^2 - 7\left(x^{1/4}\right) + 2 = 0$

let $x^{1/4} = y$------------------------(a)

So,

$5y^2 - 7y + 2 = 0$

$5y^2 - 5y - 2y + 2 = 0$

$5y(y-1) - 2(y-1) = 0$

$(5y-2)(y-1) = 0$

$5y - 2 = 0$ and $y - 1 = 0$

$y = \frac{2}{5}$ and $y = 1$

Put the values y in equ---(a)

$x^{1/4} = \frac{2}{5}$ and $x^{1/4} = 1$

Taking power 4 on both sides

$\left(x^{1/4}\right)^4 = \left(\frac{2}{5}\right)^4$ and $\left(x^{1/4}\right)^4 = (1)^4$

$x = \frac{16}{625}$ and $x = 1$

S.S $= \left\{1, \frac{16}{625}\right\}$

4. $x^{2/3} + 54 = 15x^{1/3}$

$x^{2/3} - 15x^{1/3} + 54 = 0$

$\left(x^{1/3}\right)^2 - 15\left(x^{1/3}\right) + 54 = 0$

let $x^{1/3} = y$------------------------(a)

So,

$y^2 - 15y + 54 = 0$

$y^2 - 6y - 9y + 54 = 0$

$y(y-6) - 9(y-6) = 0$

$(y-6)(y-9) = 0$

$y - 6 = 0$ and $y - 9 = 0$

$y = 6$ and $y = 9$

Put the values y in equ---(a)

$x^{1/3} = 6$ and $x^{1/3} = 9$

Taking power 4 on both sides

$\left(x^{1/3}\right)^3 = (6)^3$ and $\left(x^{1/3}\right)^3 = (9)^3$

$x = 216$ and $x = 729$

S.S $= \{216,729\}$

5. $3x^{-2} + 5 = 8x^{-1}$

$3x^{-2} - 8x^{-1} + 5 = 0$

$3(x^{-1})^2 - 8(x^{-1}) + 5 = 0$

let $x^{-1} = y$------------------------(a)

So,

$3y^2 - 8y + 5 = 0$

$3y^2 - 5y - 3y + 5 = 0$

$y(3y - 5) - 1(3y - 5) = 0$

$(3y - 5)(y - 1) = 0$

$3y - 5 = 0$ and $y - 1 = 0$

$y = \frac{5}{3}$ and $y = 1$

Put the values y in equ---(a)

$x^{-1} = \frac{5}{3}$ and $x^{-1} = 1$

Taking power 4 on both sides

$x = \frac{3}{5}$ and $x = 1$

S.S $= \left\{\frac{3}{5}, 1\right\}$

6. $(2x^2 + 1) + \frac{3}{(2x^2+1)} = 4$

let $(2x^2 + 1) = y$------------------------(a)

So,

$y + \frac{3}{y} = 4$

$\frac{y^2+3}{y} = 4$

$y^2 + 3 = 4y$

$y^2 - 4y + 3 = 0$

$y^2 - 3y - y + 3 = 0$

$y(y - 3) - 1(y - 3) = 0$

$(y - 3)(y - 1) = 0$

$y - 3 = 0$ and $y - 1 = 0$

$y = 3$ and $y = 1$

Put the values y in equ---(a)

$2x^2 + 1 = 3$ and $2x^2 + 1 = 1$

Taking power 4 on both sides

$2x^2 = 3 - 1$ and $2x^2 = 1 - 1$

$2x^2 = 2$ and $2x^2 = 0$

$x^2 = \frac{2}{2}$ and $x^2 = \frac{0}{2}$

$x^2 = 1$ and $x^2 = 0$

$x = \pm 1$ and $x = 0$

S.S $= \{\pm 1, 0\}$

7. $\frac{x}{x-3} + 4\left(\frac{x-3}{x}\right) = 4$

let $\frac{x}{x-3} = y$------------------------(a)

So,

$y + \frac{4}{y} = 4$

$\frac{y^2+4}{y} = 4$

$y^2 + 4 = 4y$

$y^2 - 4y + 4 = 0$

$(y-2)^2 = 0$

Taking root on both sides

$(y-2) = 0$

$y = 2$

Put the value y in equ---(a)

$\frac{x}{x-3} = 2$

Taking power 4 on both sides

$x = 2(x-3)$

$x = 2x - 6$

$x - 2x = -6$

$-x = -6$

$x = \frac{-6}{-1}$

S.S $= \{6\}$

8. $\frac{4x+1}{4x-1} + \frac{4x-1}{4x+1} = 2\frac{1}{6}$

let $\frac{4x+1}{4x-1} = y$------------------------(a)

So,

$y + \frac{1}{y} = \frac{13}{6}$

$\frac{y^2+1}{y} = \frac{13}{6}$

$6(y^2+1) = 13y$

$6y^2 + 6 = 13y$

$6y^2 - 13y + 6 = 0$

$6y^2 - 9y - 4y + 6 = 0$

$3y(2y-3)-2(2y-3) = 0$

$(2y-3)(3y-2) = 0$

$2y-3=0$ and $3y-2=0$

$2y = 3$ and $3y = 2$

$y = \frac{3}{2}$ and $y = \frac{2}{3}$

Put the values y in equu---(a)

$\frac{4x+1}{4x-1} = \frac{3}{2}$ and $\frac{4x+1}{4x-1} = \frac{2}{3}$

$2(4x+1) = 3(4x-1)$ and $3(4x+1) = 2(4x-1)$

$8x+2 = 12x-3$ and $12x+3 = 8x-2$

$8x-12x = -3-2$ and $12x-8x = -2-3$

$-4x = -5$ and $4x = -5$

$x = \frac{-5}{-4}$ and $x = \frac{-5}{4}$

$x = \frac{5}{4}$ and $x = \frac{-5}{4}$

S.S $= \left\{\frac{5}{4}, \frac{-5}{4}\right\}$

9. $\frac{x-a}{x+a} - \frac{x+a}{x-a} = \frac{7}{12}$

let $\frac{x-a}{x+a} = y$------------------------(a)

So,

$y - \frac{1}{y} = \frac{7}{12}$

$\frac{y^2-1}{y} = \frac{7}{12}$

$12(y^2-1) = 7y$

$12y^2 - 12 = 7y$

$12y^2 - 7y - 12 = 0$

$12y^2 - 16y + 9y - 12 = 0$

$4y(3y-4) - 3(3y-4) = 0$

$(3y-4)(4y-3) = 0$

$3y-4=0$ and $4y-3=0$

$3y = 4$ and $4y = 3$

$y = \frac{4}{3}$ and $y = \frac{3}{4}$

Put the values y in equu---(a)

$\frac{x-a}{x+a} = \frac{4}{3}$ and $\frac{x-a}{x+a} = \frac{3}{4}$

$3(x-a) = 4(x+a)$ and $4(x-a) = 3(x+a)$

$3x-3a = 4x+4a$ and $4x-4a = 3x+3a$

$3x-4x = +3a+4a$ and $4x-3x = 3a+4a$

$-x = 7a$ and $x = 7a$

$x = -7a$ and $x = 7a$

S.S $= \{-7a, 7a\}$

10. $x^4 - 2x^3 - 2x^2 + 2x + 1 = 0$

dividing by x^2

$x^2 - 2x - 2 + \frac{2}{x} + \frac{1}{x^2} = 0$

$x^2 + \frac{1}{x^2} - 2x + \frac{2}{x} - 2 = 0$

Adding and subtracting 2 on L.H.S

$x^2 + \frac{1}{x^2} - 2 + 2 - 2x + \frac{2}{x} - 2 = 0$

$\left(x - \frac{1}{x}\right)^2 + 2 - 2\left(x - \frac{1}{x}\right) - 2 = 0$

$\left(x - \frac{1}{x}\right)^2 - 2\left(x - \frac{1}{x}\right) = 0$

let $x - \frac{1}{x} = y$------------------------(a)

So,

$y^2 - 2y = 0$

$y(y-2) = 0$

$y = 0$ and $y - 2 = 0$

$y = 0$ and $y = 2$

Put the values y in equ---(a)

$x - \frac{1}{x} = 0$ and $x - \frac{1}{x} = 2$

$\frac{x^2-1}{x} = 0$ and $\frac{x^2-1}{x} = 2$

$x^2 - 1 = 0$ and $x^2 - 1 = 2x$

$(x-1)(x+1) = 0$ and $x^2 - 2x - 1 = 0$

$x - 1 = 0, x + 1 = 0$ and $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = 1$, $x = -1$ and $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$

$x = 1$, $x = -1$ and $x = \frac{2 \pm \sqrt{4+4}}{2(1)}$

$x = 1$, $x = -1$ and $x = \frac{2 \pm \sqrt{8}}{2}$

$x = 1$, $x = -1$ and $x = \frac{2 \pm \sqrt{4 \times 2}}{2}$

$x = 1$, $x = -1$ and $x = \frac{2 \pm 2\sqrt{2}}{2}$

$x = 1$, $x = -1$ and $x = \frac{2(1 \pm \sqrt{2})}{2}$

S.S $= \{1 \pm \sqrt{2}, \pm 1\}$

11. $2x^4 + x^3 - 6x^2 + x + 2 = 0$

dividing by x^2

$2x^2 + x - 6 + \frac{1}{x} + \frac{2}{x^2} = 0$

$2x^2 + \frac{2}{x^2} + x + \frac{1}{x} - 6 = 0$

Adding and subtracting 4 on L.H.S

$2x^2 + \frac{2}{x^2} + 4 - 4 + x + \frac{1}{x} - 6 = 0$

$2\left(x^2 + \frac{1}{x^2} + 2\right) + \left(x + \frac{1}{x}\right) - 10 = 0$

$2\left(x + \frac{1}{x}\right)^2 + \left(x + \frac{1}{x}\right) - 10 = 0$

let $x + \frac{1}{x} = y$------------------------(a)

So,

$2y^2 + y - 10 = 0$

$2y^2 + 5y - 4y - 10 = 0$

$y(2y + 5) - 2(2y - 5) = 13$

$2y + 5 = 0$ and $y - 2 = 0$

$2y + 5 = 0$ and $y = 2$

$2y = -5$ and $y = 2$

$y = \frac{-5}{2}$ and $y = 2$

Put the values y in equ---(a)

$x + \frac{1}{x} = \frac{-5}{2}$ and $x + \frac{1}{x} = 2$

$\frac{x^2+1}{x} = \frac{-5}{2}$ and $\frac{x^2+1}{x} = 2$

$2x^2 + 2 = -5x$ and $x^2 + 1 = 2x$

$2x^2 + 5x + 2 = 0$ and $x^2 - 2x + 1 = 0$

$2x^2 + 4x + x + 2 = 0$ and $x^2 - 2x + 1 = 0$

$2x(x + 2) + 1(x + 2) = 0$ and $x^2 - 2x + 1 = 0$

$(x + 2)(2x + 1) = 0$ and $(x - 1) = 0$

$x + 2 = 0, 2x + 1 = 0$ and $(x + 1)^2 = 0$

$x = -2, x = \frac{-1}{2}$ and $x = 1$

S.S $= \left\{-2, \frac{-1}{2}, 1\right\}$

12. $4.2^{2x+1} - 9.2^x + 1 = 0$

$4.2^{2x}.2^1 - 9.2^x + 1 = 0$

$8.2^{2x} - 9.2^x + 1 = 0$

$8(2^x)^2 - 9(2^x) + 1 = 0$

let $2^x = y$------------------------(a)

So,

$8y^2 - 9y + 1 = 0$

$8y^2 - 8y - y + 1 = 0$

$8y(y-1) - 1(y-1) = 0$

$y - 1 = 0$ and $8y - 1 = 0$

$y - 1 = 0$ and $8y = 1$

$y = 1$ and $y = \frac{1}{8}$

Put the values y in equ---(a)

$2^x = 1$ and $2^x = \frac{1}{8}$

$2^x = 2^0$ and $2^x = 2^{-3}$

$x = 0$ and $x = -3$

S.S $= \{0, -3\}$

13. $3^{2x+2} = 12.3^x - 3$

$3^{2x}.3^2 - 12.3^x + 3 = 0$

$9.3^{2x} - 12.3^x + 3 = 0$

$9(3^x)^2 - 12(3^x) + 3 = 0$

let $3^x = y$------------------------(a)

So,

$9y^2 - 12y + 3 = 0$

$9y^2 - 9y - 3y + 3 = 0$

$9y(y-1) - 3(y-1) = 0$

$(y-1)(9y-3) = 0$

$y - 1 = 0$ and $9y - 3 = 0$

$y - 1 = 0$ and $9y = 3$

$y = 1$ and $y = \frac{3}{9}$

$y = 1$ and $y = \frac{1}{3}$

Put the values y in equ---(a)

$3^x = 1$ and $3^x = \frac{1}{3}$

$3^x = 3^0$ and $3^x = 3^{-1}$

$x = 0$ and $x = -1$

S.S $= \{0, -1\}$

14. $2^x + 64.2^{-x} - 20 = 0$

$2^x + \frac{64}{2^x} - 20 = 0$

let $2^x = y$------------------------(a)

So,

$y + \frac{64}{y} - 20 \quad = 0$

$\frac{y^2 - 64 - 20y}{y} \quad = 0$

$y^2 + 64 - 20y = y \times 0$

$y^2 - 20y + 64 = 0$

$y^2 - 16y - 4y + 64 = 0$

$y(y - 16) - 4(y - 16) = 0$

$(y - 16)(y - 4) = 0$

$y - 16 = 0$ and $y - 4 = 0$

$y = 16$ and $y = 4$

Put the values y in equ---(a)

$2^x = 16$ and $2^x = 4$

$2^x = 2^4$ and $2^x = 2^2$

$x = 4$ and $x = 2$

S.S $= \{4,2\}$

15. $(x + 1)(x + 3)(x - 5)(x - 7) = 192$

$(x + 1)(x - 5)(x + 3)(x - 7) = 192$

$(x^2 - 5x + x - 5)(x^2 - 7x + 3x - 21) = 192$

$(x^2 - 4x - 5)(x^2 - 4x - 21) = 192$

let $x^2 - 4x = y$------------------------(a)

So,

$(y - 5)(y - 21) \quad = 192$

$y^2 - 21y - 5y + 105 = 192$

$y^2 - 26y + 105 - 192 = 0$

$y^2 - 26y - 87 = 0$

$y^2 - 29y + 3y - 87 = 0$

$y(y - 29) + 3(y - 29) = 0$

$(y - 29)(y + 3) = 0$

$y - 29 = 0$ and $y + 3 = 0$

$y = 29$ and $y = -3$

Put the values y in equ---(a)

$x^2 - 4x = 29$ and $x^2 - 4x = -3$

$x^2 - 4x - 29 = 0$ and $x^2 - 4x + 3 = 0$

$x^2 - 4x - 29 = 0$ and $x^2 - 4x + 3 = 0$

$a = 1, b = -4, c = -29$ and $x^2 - 3x - x + 3 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and $x(x-3) - 1(x-3) = 0$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-29)}}{2(-1)}$ and $(x-3)(x-1) = 0$

$x = \frac{4 \pm \sqrt{16+116}}{-2}$ and $x - 3 = 0$, $x - 1 = 0$

$x = \frac{4 \pm \sqrt{132}}{-2}$ and $x = 3$, $x = 1$

$x = \frac{4 \pm 2\sqrt{33}}{-2}$ and $x = 3$, $x = 1$

$x = \frac{2(2 \pm \sqrt{33})}{-2}$ and $x = 3$, $x = 1$

$x = -(2 \pm \sqrt{33})$ and $x = 3$, $x = 1$

S.S $= \{2 \pm \sqrt{33}, 3, 1\}$

16. $(x-1)(x-2)(x-8)(x+5) + 360 = 0$

$(x-1)(x-2)(x-8)(x+5) = -360$

$(x-1)(x-2)(x-8)(x+5) = -360$

$(x^2 - 2x - x + 2)(x^2 + 5x - 8x - 40) = -360$

$(x^2 - 3x + 2)(x^2 - 3x - 40) = -360$

let $x^2 - 3x = y$------------------------------(a)

So,

$(y+2)(y-40) = -360$

$y^2 - 40y + 2y - 80 = -360$

$y^2 - 38y - 80 + 360 = 0$

$y^2 - 38y + 280 = 0$

$y^2 - 28y - 10y + 280 = 0$

$y(y-28) - 10(y-28) = 0$

$(y-28)(y-10) = 0$

$y - 28 = 0$ and $y - 10 = 0$

$y = 28$ and $y = 10$

Put the values y in equ---(a)

$x^2 - 3x = 28$ and $x^2 - 3x = 10$

$x^2 - 3x - 28 = 0$ and $x^2 - 3x - 10 = 0$

$x^2 + 4x - 7x - 28 = 0$ and $x^2 - 5x + 2x - 10 = 0$

$x(x+4) - 7(x+4) = 0$ and $x(x-5) + 2(x-5) = 0$

$(x+4)(x-7) = 0$ and $(x-5)(x+2) = 0$

$x + 4 = 0$, $x - 7 = 0$ and $x - 5 = 0$, $x + 2 = 0$

$x = -4$, $x = 7$ and $x = 5$, $x = -2$

S.S $= \{-4, 7, 5, -2\}$

Exercise 1.4

Solve the following equations.

1. $2x + 5 = \sqrt{7x + 16}$

squaring both sides

$$(2x+5)^2 = \left(\sqrt{7x+16}\right)^2$$
$$4x^2 + 20x + 25 = 7x + 16$$
$$4x^2 + 20x + 25 - 7x - 16 = 0$$
$$4x^2 + 13x + 9 = 0$$
$$4x^2 + 9x + 4x + 9 = 0$$
$$x(4x+9) + 1(4x+9) = 0$$
$$(4x+9)(x+1) = 0$$

$4x + 9 = 0$ and $x + 1 = 0$

$4x = -9$ and $x = -1$

$x = -\frac{9}{4}$ and $x = -1$

Checking:

$2\left(\frac{-9}{4}\right) + 5 = \sqrt{7\left(\frac{-9}{4}\right) + 16}$ and $2(-1) + 5 = \sqrt{7(-1) + 16}$

$\frac{-9}{2} + 5 = \sqrt{\frac{-63}{4} + 16}$ and $-2 + 5 = \sqrt{-7 + 16}$

$\frac{-9+10}{2} = \sqrt{\frac{-63+64}{4}}$ and $3 = \sqrt{9}$

$\frac{1}{2} = \sqrt{\frac{1}{4}}$ and $3 = 3$

True and True

So,

S.S $= \left\{-1, -\frac{9}{4}\right\}$

2. $\sqrt{x+3} = 3x - 1$

squaring both sides

$$\left(\sqrt{x+3}\right)^2 = (3x-1)^2$$
$$x + 3 = 9x^2 - 6x + 1$$
$$0 = 9x^2 - 6x + 1 - x - 3$$
$$0 = 9x^2 - 7x - 2$$
$$9x^2 - 7x - 2 = 0$$
$$9x^2 - 9x + 2x - 2 = 0$$
$$9x(x-1) + 2(x-1) = 0$$
$$(x-1)(9x+2) = 0$$

$9x + 2 = 0$ and $x - 1 = 0$

$9x = -2$ and $x = 1$

$x = -\frac{2}{9}$ and $x = 1$

Checking:

$\sqrt{x+3} = 3x - 1$

$\sqrt{\left(\frac{-2}{9}\right)+3} = 3\left(\frac{-2}{9}\right) - 1$ and $\sqrt{(1)+3} = \sqrt{3(1)-1}$

$\sqrt{\frac{-2}{9}+3} = \frac{-2}{3} - 1$ and $\sqrt{4} = 3 - 1$

$\sqrt{\frac{-2+27}{9}} = \frac{-2-3}{3}$ and $\sqrt{4} = 2$

$\sqrt{\frac{25}{9}} = \frac{-5}{3}$ and $2 = 2$

False and True

So,

S.S $= \{1\}$

3. $4x = \sqrt{13x+14} - 3$

$4x + 3 = \sqrt{13x+14}$

squaring both sides

$(4x+3)^2 = \left(\sqrt{13x+14}\right)^2$

$16x^2 + 24x + 9 = 13x + 14$

$16x^2 + 24x + 9 - 13x - 14 = 0$

$16x^2 + 11x - 5 = 0$

$16x^2 + 16x - 5x - 5 = 0$

$16x(x+1) - 5(x+1) = 0$

$(x+1)(16x-5) = 0$

$16x - 5 = 0$ and $x + 1 = 0$

$16x = 5$ and $x = -1$

$x = \frac{5}{16}$ and $x = -1$

Checking:

$4x = \sqrt{13x+14} - 3$

$4\left(\frac{5}{16}\right) = \sqrt{13\left(\frac{5}{16}\right)+14} - 3$ and $4(-1) = \sqrt{13(-1)+14} - 3$

$\frac{5}{4} = \sqrt{\frac{65}{16}+14} - 3$ and $-4 = \sqrt{-13+14} - 3$

$\frac{5}{4} = \sqrt{\frac{65+224}{16}} - 3$ and $-4 = \sqrt{1} - 3$

$\frac{5}{4} = \sqrt{\frac{289}{16}} - 3$ and $-4 = \sqrt{1} - 3$

$\frac{5}{4} = \frac{17}{4} - 3$ and $-4 = 1 - 3$

$\frac{5}{4} = \frac{17-12}{4}$ and $-4 = -2$

$\frac{5}{4} = \frac{5}{4}$ and $-4 = -2$

True and False

So,

S.S $= \left\{\frac{5}{16}\right\}$

4. $\sqrt{3x+100} - x = 4$

$\sqrt{3x+100} = x + 4$

squaring both sides

$$\left(\sqrt{3x+100}\right)^2 = (x+4)^2$$
$$3x + 100 = x^2 + 8x + 16$$
$$0 = x^2 + 8x + 16 - 3x - 100$$
$$0 = x^2 + 5x - 84$$
$$x^2 + 5x - 84 = 0$$
$$x^2 + 12x - 7x - 84 = 0$$
$$x(x+12) - 7(x+12) = 0$$
$$(x+12)(x-7) = 0$$

$x + 12 = 0$ and $x - 7 = 0$

$x = -12$ and $x = 7$

Checking:

$\sqrt{3x+100} - x = 4$

$\sqrt{3(-12)+100} - (-12) = 4$	and	$\sqrt{3(7)+100} - (7) = 4$
$\sqrt{-36+100} + 12 = 4$	and	$\sqrt{21+100} - 7 = 4$
$\sqrt{64} + 12 = 4$	and	$\sqrt{121} - 7 = 4$
$8 + 12 = 4$	and	$11 - 7 = 4$
$20 = 4$	and	$4 = 4$
False	and	True

So,

S.S $= \{7\}$

5. $\sqrt{x+5} + \sqrt{x+21} = \sqrt{x+60}$

Squaring both sides

$$\left(\sqrt{x+5} + \sqrt{x+21}\right)^2 = \left(\sqrt{x+60}\right)^2$$
$$\left(\sqrt{x+5}\right)^2 + \left(\sqrt{x+21}\right)^2 + 2\left(\sqrt{x+5}\right)\left(\sqrt{x+21}\right) = x + 60$$
$$x + 5 + x + 21 + 2\sqrt{(x+5)(x+21)} = x + 60$$
$$2x + 26 + 2\sqrt{x^2 + 21x + 5x + 105} = x + 60$$
$$2x + 26 + 2\sqrt{x^2 + 26x + 105} = x + 60$$
$$2x + 26 - x - 60 = -2\sqrt{x^2 + 26x + 105}$$
$$x - 34 = -2\sqrt{x^2 + 26x + 105}$$

squaring both sides

$$\left(-2\sqrt{x^2 + 26x + 105}\right)^2 = (x - 34)^2$$
$$4(x^2 + 26x + 105) = x^2 - 68x + 1156$$

$$4x^2+104x+420 = x^2-68x+1156$$
$$4x^2+104x+420-x^2+68x-1156 = 0$$
$$3x^2+172x-736 = 0$$
$$3x^2+184x-12x-736 = 0$$
$$x(3x+184)-4(3x+184) = 0$$
$$(3x+184)(x-4) = 0$$
$3x+184=0$ and $x-4=0$

$3x=-184$ and $x=4$

$x=-\frac{184}{3}$ and $x=4$

Checking:

$$\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$$

$\sqrt{\frac{-184}{3}+5}+\sqrt{\frac{-184}{3}+21}=\sqrt{\frac{-184}{3}+60}$ and $\sqrt{4+5}+\sqrt{4+21}=\sqrt{4+60}$

$\sqrt{\frac{-184+15}{3}}+\sqrt{\frac{-184+63}{3}}=\sqrt{\frac{-184+180}{3}}$ and $\sqrt{9}+\sqrt{25}=\sqrt{64}$

$\sqrt{\frac{-169}{3}}+\sqrt{\frac{-121}{3}}=\sqrt{\frac{-4}{3}}$ and $3+5=8$

False and True

So,

S.S $=\{4\}$

6. $\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$

Squaring both sides

$$\left(\sqrt{x+1}+\sqrt{x-2}\right)^2 = \left(\sqrt{x+6}\right)^2$$
$$\left(\sqrt{x+1}\right)^2+\left(\sqrt{x-2}\right)^2+2\left(\sqrt{x+1}\right)\left(\sqrt{x-2}\right)=x+6$$
$$x+1+x-2+2\sqrt{(x+1)(x-2)} = x+6$$
$$2x-1+2\sqrt{x^2-x-2} = x+6$$
$$2x-1-x-6 = -2\sqrt{x^2-x-2}$$
$$x-7 = -2\sqrt{x^2-x-2}$$

squaring both sides

$$\left(-2\sqrt{x^2-x-2}\right)^2 = (x-7)^2$$
$$4(x^2-x-2) = x^2-14x+49$$
$$4x^2-4x-8 = x^2-14x+49$$
$$4x^2-4x-8-x^2+14x-49 = 0$$
$$3x^2+10x-57 = 0$$
$$3x^2+19x-9x-57 = 0$$
$$x(3x+19)-3(3x+19) = 0$$
$$(3x+19)(x-3) = 0$$

$3x + 19 = 0$ and $x - 3 = 0$

$3x = -19$ and $x = 3$

$x = -\frac{19}{3}$ and $x = 3$

Checking:

$\sqrt{x+1} + \sqrt{x-2} = \sqrt{x+6}$

$\sqrt{\frac{-19}{3}+1} + \sqrt{\frac{-19}{3}-2} = \sqrt{\frac{-19}{3}+6}$ and $\sqrt{3+1} + \sqrt{3-2} = \sqrt{3+6}$

$\sqrt{\frac{-19+3}{3}} + \sqrt{\frac{-19-6}{3}} = \sqrt{\frac{-19+18}{3}}$ and $\sqrt{4} + \sqrt{1} = \sqrt{9}$

$\sqrt{\frac{-16}{3}} + \sqrt{\frac{-13}{3}} = \sqrt{\frac{-1}{3}}$ and $2 + 1 = 3$

False and True

So,

S.S $= \{3\}$

7. $\sqrt{11-x} - \sqrt{6-x} = \sqrt{27-x}$

Squaring both sides

$$\left(\sqrt{11-x} - \sqrt{6-x}\right)^2 = \left(\sqrt{27-x}\right)^2$$

$$\left(\sqrt{11-x}\right)^2 + \left(\sqrt{6-x}\right)^2 - 2\left(\sqrt{11-x}\right)\left(\sqrt{6-x}\right) = 27 - x$$

$$11 - x + 6 - x - 2\sqrt{(11-x)(6-x)} = 27 - x$$

$$-2x + 17 - 2\sqrt{66 - 11x - 6x + x^2} = 27 - x$$

$$-2x + 17 - 2\sqrt{66 - 17x + x^2} = 27 - x$$

$$-2x + 17 + x - 27 = 2\sqrt{66 - 17x + x^2}$$

$$-x - 10 = 2\sqrt{66 - 17x + x^2}$$

$$-(x + 10) = 2\sqrt{66 - 17x + x^2}$$

squaring both sides

$$\left(2\sqrt{66 - 17x + x^2}\right)^2 = \left(-(x+10)\right)^2$$

$$4(66 - 17x + x^2) = x^2 + 20x + 100$$

$$264 - 68x + 4x^2 = x^2 + 20x + 100$$

$$4x^2 - 68x + 264 - x^2 - 20x - 100 = 0$$

$$3x^2 - 88x + 164 = 0$$

$$3x^2 - 82x - 6x + 164 = 0$$

$$x(3x - 82) - 2(3x - 82) = 0$$

$$(3x - 82)(x - 2) = 0$$

$3x - 82 = 0$ and $x - 2 = 0$

$3x = 82$ and $x = 2$

$x = \frac{82}{3}$ and $x = 2$

Checking:

$\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$

$\sqrt{11-\frac{82}{3}}+\sqrt{6-\frac{82}{3}} = \sqrt{27-\frac{82}{3}}$ and $\sqrt{11-2}+\sqrt{6-2} = \sqrt{27-2}$

$\sqrt{\frac{33-82}{3}}+\sqrt{\frac{18-82}{3}} = \sqrt{\frac{81-82}{3}}$ and $\sqrt{9}+\sqrt{4} = \sqrt{25}$

$\sqrt{\frac{-49}{3}}+\sqrt{\frac{-64}{3}} = \sqrt{\frac{-1}{3}}$ and $3+2 = 5$

False and True

So,

S.S $=\{2\}$

8. $\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$

Squaring both sides

$$\left(\sqrt{4a+x}-\sqrt{a-x}\right)^2 = \left(\sqrt{a}\right)^2$$

$$\left(\sqrt{4a+x}\right)^2+\left(\sqrt{a-x}\right)^2-2\left(\sqrt{4a+x}\right)\left(\sqrt{a-x}\right)=a$$

$$4a+x+a-x-2\sqrt{(4a+x)(a-x)} = a$$

$$5a-2\sqrt{4a^2-4ax+ax-x^2} = a$$

$$5a-2\sqrt{4a^2-3ax-x^2} = a$$

$$5a-a = 2\sqrt{4a^2-3ax-x^2}$$

$$4a = 2\sqrt{4a^2-3ax-x^2}$$

squaring both sides

$$\left(2\sqrt{4a^2-3ax-x^2}\right)^2=(4a)^2$$

$$4(4a^2-3ax-x^2) = 16a^2$$

$$4a^2-3ax-x^2 = \frac{16a^2}{4}$$

$$4a^2-3ax-x^2 = 4a^2$$

$$-3ax-x^2 = 4a^2-4a^2$$

$$-3ax-x^2 = 0$$

$$-x(3a+x) = 0$$

$x = 0$ and $3a+x = 0$

$x = 0$ and $x = -3a$

Checking:

$\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$

$\sqrt{4a+0}-\sqrt{a-0} = \sqrt{a}$ and $\sqrt{4a-3a}-\sqrt{a-(-3a)} = \sqrt{a}$

$\sqrt{4a}-\sqrt{a} = \sqrt{a}$ and $\sqrt{4a-3a}-\sqrt{a+3a} = \sqrt{a}$

$2\sqrt{a}-\sqrt{a} = \sqrt{a}$ and $\sqrt{a}-\sqrt{4a} = \sqrt{a}$

$2\sqrt{a}-\sqrt{a} = \sqrt{a}$ and $\sqrt{a}-2\sqrt{a} = \sqrt{a}$

$\sqrt{a} = \sqrt{a}$ and $-\sqrt{a} = \sqrt{a}$

True and False

So,

S.S $= \{0\}$

9. $\sqrt{x^2+x+1} - \sqrt{x^2+x-1} = 1$

let $x^2 + x = y$ -- (a)

So

$$\sqrt{y+1} - \sqrt{y-1} = 1$$

Squaring both sides

$$\left(\sqrt{y+1} - \sqrt{y-1}\right)^2 = (1)^2$$

$$\left(\sqrt{y+1}\right)^2 + \left(\sqrt{y-1}\right)^2 - 2\left(\sqrt{y+1}\right)\left(\sqrt{y-1}\right) = 1$$

$$y + 1 + y - 1 - 2\sqrt{(y+1)(y-1)} = 1$$

$$2y - 2\sqrt{y^2-1} = 1$$

$$2y - 2\sqrt{y^2-1} = 1$$

$$2y - 1 = 2\sqrt{y^2-1}$$

$$2y - 1 = 2\sqrt{y^2-1}$$

squaring both sides

$$\left(2\sqrt{y^2-1}\right)^2 = (2y-1)^2$$

$$4(y^2-1) = 4y^2 - 4y + 1$$

$$4y^2 - 4 = 4y^2 - 4y + 1$$

$$4y^2 - 4y^2 + 4y = 1 + 4$$

$$4y = 5$$

$$y = \frac{5}{4}$$

put value of y in equation (a)

$$x^2 + x = \frac{5}{4}$$

$$4x^2 + 4x = 5$$

$$4x^2 + 4x - 5 = 0$$

a = 4, b = 4, c = -5

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(4)(-5)}}{2(4)}$$

$$x = \frac{-4 \pm \sqrt{16+80}}{8}$$

$$x = \frac{-4 \pm \sqrt{96}}{8}$$

$$x = \frac{-4 \pm \sqrt{16\times 6}}{8}$$

$$x = \frac{-4 \pm 4\sqrt{6}}{8}$$

$x = \frac{4(-1\pm\sqrt{6})}{8}$

S.S $= \left\{\frac{-1\pm\sqrt{6}}{2}\right\}$

10. $\sqrt{x^2+3x+8} + \sqrt{x^2+3x+2} = 3$

let $x^2 + 3x = y$ -- (a)

So

$$\sqrt{y+8} + \sqrt{y+2} = 3$$

Squaring both sides

$$\left(\sqrt{y+8} + \sqrt{y+2}\right)^2 = (3)^2$$

$$\left(\sqrt{y+8}\right)^2 + \left(\sqrt{y+2}\right)^2 + 2\left(\sqrt{y+8}\right)\left(\sqrt{y+2}\right) = 9$$

$$y + 8 + y + 2 + 2\sqrt{(y+8)(y+2)} = 9$$

$$2y + 10 + 2\sqrt{y^2 + 2y + 8y + 16} = 9$$

$$2y + 10 + 2\sqrt{y^2 + 10y + 16} = 9$$

$$2y + 10 - 9 = -2\sqrt{y^2 + 10y + 16}$$

$$2y + 1 = -2\sqrt{y^2 + 10y + 16}$$

squaring both sides

$$\left(-2\sqrt{y^2+10y+16}\right)^2 = (2y+1)^2$$

$$4(y^2 + 10y + 16) = 4y^2 + 4y + 1$$

$$4y^2 + 40y + 64 = 4y^2 + 4y + 1$$

$$4y^2 + 40y + 64 - 4y^2 - 4y - 1 = 0$$

$$36y + 63 = 0$$

$$36y = -63$$

$$y = \frac{-63}{36}$$

$$y = \frac{-7}{4}$$

put value of y in equation (a)

$$x^2 + 3x = -\frac{7}{4}$$

$$4x^2 + 12x = -7$$

$$4x^2 + 12x + 7 = 0$$

a = 4, b = 12, c = 7

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(12)\pm\sqrt{(12)^2-4(4)(7)}}{2(4)}$$

$$x = \frac{-12\pm\sqrt{144-112}}{8}$$

$$x = \frac{-12\pm\sqrt{32}}{8}$$

$$x = \frac{-12\pm\sqrt{16\times2}}{8}$$

$$x = \frac{-12\pm4\sqrt{2}}{8}$$

$x \quad = \frac{4(-3\pm\sqrt{2})}{8}$

S.S $\quad = \left\{\frac{-3\pm\sqrt{2}}{2}\right\}$

11. $\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4} = 5$

let $x^2+3x = y$ -- (a)

So

$$\sqrt{y+9}+\sqrt{y+4} = 5$$

Squaring both sides

$$\left(\sqrt{y+9}+\sqrt{y+4}\right)^2 = (5)^2$$

$$\left(\sqrt{y+9}\right)^2+\left(\sqrt{y+4}\right)^2+2\left(\sqrt{y+9}\right)\left(\sqrt{y+4}\right) = 25$$

$$y+9+y+4+2\sqrt{(y+9)(y+4)} = 25$$

$$2y+13+2\sqrt{y^2+4y+9y+36} = 25$$

$$2y+13+2\sqrt{y^2+13y+36} = 25$$

$$2y+13-25 = -2\sqrt{y^2+13y+36}$$

$$2y-12 = -2\sqrt{y^2+13y+36}$$

$$2(y-6) = -2\sqrt{y^2+13y+36}$$

squaring both sides

$$\left(-\sqrt{y^2+13y+36}\right)^2 = (y-6)^2$$

$$y^2+13y+36 = y^2-12y+36$$

$$y^2+13y+36 = y^2-12y+36$$

$$y^2+13y+36-y^2+12y-36 = 0$$

$$25y = 0$$

put value of y in equation (a)

$$x^2+3x = 0$$

$$x(x+3) = 0$$

$x = 0$ and $x+3 = 0$

$x = 0$ and $x = -3$

S.S $= \{0, -3\}$

Exercise 2.1

Q. 1: Find the discriminant of the following given quadratic equations:

(i) $2x^2 + 3x - 1 = 0$

a = 2, b = 3, c = -1

$Disc = b^2 - 4ac$
$= (3)^2 - 4(2)(-1)$
$= 9 + 8$
$= 17$

(ii) $6x^2 - 8x + 3 = 0$

a = 6, b = -8, c = 3

$Disc = b^2 - 4ac$
$= (-8)^2 - 4(6)(3)$
$= 64 - 72$
$= -8$

(iii) $9x^2 - 30x + 25 = 0$

a = 9, b = -30, c = 25

$Disc = b^2 - 4ac$
$= (-30)^2 - 4(9)(25)$
$= 900 - 900$
$= 0$

(iv) $4x^2 - 7x - 2 = 0$

a = 4, b = -7, c = -2

$Disc = b^2 - 4ac$
$= (-7)^2 - 4(4)(-2)$
$= 49 + 32$
$= 81$

Q. 2: Find the nature of the roots of the following given quadratic equations and verify the result by solving the equations:

(i) $x^2 - 23x + 120 = 0$

a = 1, b = -23, c = 120

$Disc = b^2 - 4ac$
$= (-23)^2 - 4(1)(120)$
$= 529 - 480$
$= 49$

Disc > 0 and a perfect square, So the roots are rational (real) and unequal.

Now, Verify by using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(-23) \pm \sqrt{(-23)^2-4(1)(120)}}{2(1)}$$

$$= \frac{23 \pm \sqrt{529-480}}{2}$$

$$= \frac{23 \pm \sqrt{49}}{2}$$

$$= \frac{23 \pm 7}{2}$$

$x = \frac{23+7}{2}$ and $x = \frac{23-7}{2}$

$x = 15$ and $x = 8$

(ii) $2x^2 + 3x + 7 = 0$

a = 2, b = 3, c = 7

$$Disc = b^2 - 4ac$$

$$= (3)^2 - 4(2)(7)$$

$$= 9 - 56$$

$$= -47$$

Disc < 0, So the roots are imaginary (complex conjugates).

Now, Verify by using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(3) \pm \sqrt{-47}}{2(2)}$$

$$= \frac{-3 \pm \sqrt{-47}}{4}$$

(iii) $16x^2 - 24x + 9 = 0$

a = 16, b = -24, c = 9

$$Disc = b^2 - 4ac$$

$$= (-24)^2 - 4(16)(9)$$

$$= 576 - 576$$

$$= 0$$

Disc = 0, So the roots are rational (real) and equal.

Now, Verify by competing square method

$$16x^2 - 24x + 9 = 0$$

$$(4x)^2 - 2(4x)(3) + (3)^2 = 0$$

$$(4x-3)^2 = 0$$

$$(4x-3)(4x-3) = 0$$

$x = \frac{3}{4}$ and $x = \frac{3}{4}$

(iv) $3x^2 + 7x - 13 = 0$

a = 3, b = 7, c = -13

$$Disc = b^2 - 4ac$$
$$= (7)^2 - 4(3)(-13)$$
$$= 49 + 156$$
$$= 205$$

Disc > 0, So the roots are Irrational (real) and unequal.

Now, Verify by using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(7) \pm \sqrt{205}}{2(3)}$$
$$= \frac{-7 \pm \sqrt{205}}{6}$$

$x = \frac{-7+\sqrt{205}}{6}$ and $x = \frac{-7-\sqrt{205}}{6}$

Q. 3: For what value of k, the expression

$k^2x^2 + 2(k+1)x + 4$ is perfect square.

The given equation is perfect square if Disc = 0.

So,

$$Disc = 0$$
$$b^2 - 4ac = 0$$
$$[2(k+1)]^2 - 4(k^2)(4) = 0$$
$$4(k^2 + 2k + 1) - 16k^2 = 0$$
$$4k^2 + 8k + 4 - 16k^2 = 0$$
$$-12k^2 + 8k + 4 = 0$$

a = -12, b = 8, c = 4

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(8) \pm \sqrt{(8)^2 - 4(-12)(4)}}{2(-12)}$$
$$= \frac{-(8) \pm \sqrt{64 + 192}}{2(-12)}$$
$$= \frac{-8 \pm \sqrt{256}}{-24}$$

$k = \frac{-8+16}{-24}$ and $k = \frac{-8-16}{-24}$

$k = \frac{8}{-24}$ and $k = \frac{-24}{-24}$

$k = \frac{-1}{3}$ and $k = 1$

Q. 4: Find the value of k, if the roots of the following equations are equal.

(i) $(2k-1)x^2 + 3kx + 3 = 0$

The given equation have equal roots if Disc = 0.

So,

$$\text{Disc} = 0$$

$$b^2 - 4ac = 0$$

$$[3k]^2 - 4(2k-1)(3) = 0$$

$$9k^2 - 24k + 12 = 0$$

a = 9, b = -24, c = 12

$$k = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(-24) \pm \sqrt{(-24)^2 - 4(9)(12)}}{2(9)}$$

$$= \frac{24 \pm \sqrt{576-432}}{2(9)}$$

$$= \frac{24 \pm \sqrt{144}}{18}$$

$$= \frac{24 \pm 12}{18}$$

$k = \frac{24+12}{18}$ and $k = \frac{24-12}{18}$

$k = \frac{36}{18}$ and $k = \frac{12}{18}$

$k = 2$ and $k = \frac{2}{3}$

(ii) $x^2 + 2(k+2)x + (3k+4) = 0$

The given equation have equal roots if Disc = 0.

So,

$$\text{Disc} = 0$$

$$b^2 - 4ac = 0$$

$$[2(k+2)]^2 - 4(1)(3k+4) = 0$$

$$4(k^2 + 4k + 4) - 12k - 16 = 0$$

$$4k^2 + 16k + 16 - 12k - 16 = 0$$

$$4k^2 + 4k = 0$$

a = 4, b = 4, c = 0

$$k = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(4) \pm \sqrt{(4)^2 - 4(4)(0)}}{2(4)}$$

$$= \frac{-4 \pm \sqrt{16-0}}{8}$$

$$= \frac{-4 \pm \sqrt{16}}{8}$$

$k = \frac{-4+4}{8}$ and $k = \frac{-4-4}{8}$

$k = 0$ and $k = -1$

(iii) $(3k+2)x^2 - 5(k+1)x + (2k+3) = 0$

The given equation have equal roots if Disc = 0.

So,

$$\text{Disc} = 0$$
$$b^2 - 4ac = 0$$
$$[-5(k+1)]^2 - 4(3k+2)(2k+3) = 0$$
$$25(k^2+2k+1) - 4(6k^2+9k+4k+6) = 0$$
$$25k^2 + 50k + 25 - 4(6k^2+13k+6) = 0$$
$$25k^2 + 50k + 25 - 24k^2 - 52k - 24 = 0$$
$$k^2 - 2k + 1 = 0$$
$$(k-1)^2 = 0$$
$$(k-1)(k-1) = 0$$

$k = 1$ and $k = 1$

Q. 5: Show that the equation $x^2 + (mx+c)^2 = a^2$ has equal roots,
if $c^2 = a^2(1+m^2)$

$$x^2 + (mx+c)^2 = a^2$$
$$x^2 + m^2x^2 + 2mcx + c^2 = a^2$$
$$(m^2+1)x^2 + (2mc)x + (c^2 - a^2) = 0$$

The given equation have equal roots if Disc = 0.

So,

$$\text{Disc} = 0$$
$$b^2 - 4ac = 0$$
$$(2mc)^2 - 4(m^2+1)(c^2-a^2) = 0$$
$$4m^2c^2 - 4(m^2c^2 - m^2a^2 + c^2 - a^2) = 0$$
$$4m^2c^2 - 4m^2c^2 - 4(-m^2a^2 + c^2 - a^2) = 0$$
$$-4(-m^2a^2 + c^2 - a^2) = 0$$
$$-m^2a^2 + c^2 - a^2 = 0$$
$$c^2 = m^2a^2 + a^2$$
$$c^2 = a^2(1+m^2)$$

Hence proved.

Q. 6: Find the condition that the roots of the equation $(mx+c)^2 - 4ax = 0$ are equal.

$$(mx+c)^2 - 4ax = 0$$
$$m^2x^2 + 2mcx + c^2 - 4ax = 0$$
$$(m^2)x^2 + (2mc - 4a)x + (c^2) = 0$$

The given equation have equal roots if Disc = 0.

So,

$$\text{Disc} = 0$$
$$b^2 - 4ac = 0$$
$$(2mc - 4a)^2 - 4(m^2)(c^2) = 0$$
$$4m^2c^2 - 16mac + 16a^2 - 4m^2c^2 = 0$$

$$16a^2 = 16mac$$
$$a = mc$$

Q. 7: If the roots of the equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$ are equal, then $a = 0$ and $a^3 + b^3 + c^3 = 3abc$

$(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$

The given equation have equal roots if Disc = 0.

So,

$$\text{Disc} = 0$$
$$b^2 - 4ac = 0$$
$$\left(2(a^2 - bc)\right)^2 - 4(c^2 - ab)(b^2 - ac) = 0$$
$$4(a^4 - 2a^2bc + b^2c^2) - 4(b^2c^2 - ac^3 - ab^3 + a^2bc) = 0$$
$$4[(a^4 - 2a^2bc + b^2c^2) - (b^2c^2 - ac^3 - ab^3 + a^2bc)] = 0$$
$$a^4 - 2a^2bc + b^2c^2 - b^2c^2 + ac^3 + ab^3 - a^2bc = 0$$
$$a^4 - 2a^2bc + ac^3 + ab^3 - a^2bc = 0$$
$$a(a^3 - 2abc + c^3 + b^3 - abc) = 0$$
$$a(a^3 - 3abc + c^3 + b^3) = 0$$

$a = 0$ and $a^3 - 3abc + c^3 + b^3 = 0$

$a = 0$ and $a^3 + b^3 + c^3 = 3abc$

Q. 8: Show that the roots of the following equations are rational.

(i) $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$

$$\begin{aligned}
\text{Disc} &= b^2 - 4ac \\
&= [b(c-a)]^2 - 4\big(a(b-c)\big)\big(c(a-b)\big) \\
&= (bc - ab)^2 - 4ac(b-c)(a-b) \\
&= b^2c^2 - 2ab^2c + a^2b^2 - 4ac(ab - b^2 - ac + bc) \\
&= b^2c^2 - 2ab^2c + a^2b^2 - 4a^2bc + 4ab^2c + 4a^2c^2 - 4abc^2 \\
&= b^2c^2 + a^2b^2 - 4a^2bc + 2ab^2c + 4a^2c^2 - 4abc^2 \\
&= b^2c^2 + a^2b^2 + 2ab^2c - 4abc^2 - 4a^2bc + 4a^2c^2 \\
&= (bc + ab)^2 - 4ac(bc + ab) + 4a^2c^2 \\
&= (bc + ab)^2 - 2(bc + ab)(2ac) + (2ac)^2 \\
&= (bc + ab - 2ac)^2
\end{aligned}$$

as Disc is a perfect square so the roots are rational.

(ii) $(a + 2b)x^2 + 2(a + b + c)x + (a + 2c) = 0$

$$\begin{aligned}
\text{Disc} &= b^2 - 4ac \\
&= [2(a + b + c)]^2 - 4(a + 2b)(a + 2c) \\
&= 4(a + b + c)^2 - 4(a + 2b)(a + 2c)
\end{aligned}$$

$= 4(a+b+c)^2 - 4(a^2 + 2ac + 2ab + 4bc)$
$= 4(a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - a^2 - 2ca - 2ab - 4bc)$
$= 4(b^2 + c^2 - 2bc)$
$= 4(b-c)^2$
$= [2(b-c)]^2$

as Disc is a perfect square so the roots are rational.

Q. 9: For all values of k, prove that the roots of the equation
$x^2 - 2\left(k + \frac{1}{k}\right)x + 3 = 0$, $k \neq 0$ are real.

Disc $= b^2 - 4ac$
$= \left[-2\left(k + \frac{1}{k}\right)\right]^2 - 4(1)(3)$
$= 4\left(k + \frac{1}{k}\right)^2 - 4(3)$
$= 4\left(\left(k + \frac{1}{k}\right)^2 - 3\right)$

For all values of k the Disc > 0, so the roots of the equation are real.

Q. 10: Show that the roots of the equation
$(b-c)x^2 + (c-a)x + (a-b) = 0$ are real.

Disc $= b^2 - 4ac$
$= (c-a)^2 - 4(b-c)(a-b)$
$= (c-a)^2 - 4(ab - b^2 - ac + bc)$
$= (c^2 - 2ac + a^2 - 4ab + 4b^2 + 4ac - 4bc)$
$= (c^2 + 2ac + a^2 - 4ab - 4bc + 4b^2)$
$= ((c+a)^2 - 4b(c+a) + 4b^2)$
$= (c+a)^2 - 2(c+a)(2b) + (2b)^2$
$= (c+a-2b)^2$

as Disc is greater than zero and perfect square so the roots are rational and real.

Exercise 2.2

Q. 1: Find the cube roots of -1, 8, -27, 64.

cube root of -1:

let $x^3 = -1$

$x^3 + 1 = 0$

$x^3 + 1^3 = 0$

$(x+1)(x^2 - x + 1^2) = 0$

$x + 1 = 0$ and $x^2 - x + 1 = 0$

$x = -1$ and $x^2 - x + 1 = 0$

applying quadratic formula a = 1, b = -1, c = 1

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1-4}}{2}$$

$$= -\frac{-1 \pm \sqrt{-3}}{2}$$

$x = -\frac{-1+\sqrt{-3}}{2}$ and $x = -\frac{-1-\sqrt{-3}}{2}$

$x = -\omega$ and $x = -\omega^2$

so the cube roots of -1 are -1, $-\omega$, $-\omega^2$

cube root of 8:

let $x^3 = 8$

$x^3 - 8 = 0$

$x^3 - 2^3 = 0$

$(x-2)(x^2 + 2x + 2^2) = 0$

$x - 2 = 0$ and $x^2 + 2x + 4 = 0$

$x = 2$ and $x^2 + 2x + 4 = 0$

applying quadratic formula a = 1, b = 2, c = 4

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4-16}}{2}$$

$$= \frac{-2 \pm \sqrt{-12}}{2}$$

$$= \frac{-2 \pm \sqrt{4 \times -3}}{2}$$

$$= \frac{-2 \pm 2\sqrt{-3}}{2}$$

$$= 2\left(\frac{-1\pm\sqrt{-3}}{2}\right)$$

$x = 2\left(\frac{-1+\sqrt{-3}}{2}\right)$ and $x = 2\left(\frac{-1-\sqrt{-3}}{2}\right)$

$x = 2\omega$ and $x = 2\omega^2$

so the cube roots of 8 are 2, 2ω, $2\omega^2$

cube root of -27:

$$\text{let} \quad x^3 = -27$$
$$x^3 + 27 = 0$$
$$x^3 + 3^3 = 0$$
$$(x+3)(x^2 - 3x + 3^2) = 0$$

$x + 3 = 0$ and $x^2 - 3x + 9 = 0$

$x = -3$ and $x^2 - 3x + 9 = 0$

applying quadratic formula a = 1, b = -3, c = 9

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$= \frac{-(-3)\pm\sqrt{(-3)^2-4(1)(9)}}{2(1)}$$
$$= \frac{3\pm\sqrt{9-36}}{2}$$
$$= \frac{3\pm\sqrt{-27}}{2}$$
$$= \frac{3\pm\sqrt{9\times-3}}{2}$$
$$= \frac{3\pm3\sqrt{-3}}{2}$$
$$= -3\left(\frac{-1\pm\sqrt{-3}}{2}\right)$$

$x = -3\left(\frac{-1+\sqrt{-3}}{2}\right)$ and $x = -3\left(\frac{-1-\sqrt{-3}}{2}\right)$

$x = -3\omega$ and $x = -3\omega^2$

so the cube roots of -27 are -3, -3ω, $-3\omega^2$

cube root of 64:

$$\text{let} \quad x^3 = 64$$
$$x^3 - 64 = 0$$
$$x^3 - 4^3 = 0$$
$$(x-4)(x^2 + 4x + 4^2) = 0$$

$x - 4 = 0$ and $x^2 + 4x + 16 = 0$

$x = 4$ and $x^2 + 4x + 16 = 0$

applying quadratic formula a = 1, b = 4, c = 16

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$= \frac{-(4)\pm\sqrt{(4)^2-4(1)(16)}}{2(1)}$$

$= \frac{-4 \pm \sqrt{16-64}}{2}$

$= \frac{-4 \pm \sqrt{-48}}{2}$

$= \frac{-4 \pm \sqrt{16 \times -3}}{2}$

$= \frac{-4 \pm 4\sqrt{-3}}{2}$

$= 4\left(\frac{-1 \pm \sqrt{-3}}{2}\right)$

$x = 4\left(\frac{-1+\sqrt{-3}}{2}\right)$ and $x = 4\left(\frac{1-\sqrt{-3}}{2}\right)$

$x = 4\omega$ and $x = 4\omega^2$

so the cube roots of 64 are 4, 4ω, $4\omega^2$

Q. 2: Evaluate:

(i) $(1-\omega-\omega^2)^7$ $= \left(1-(\omega+\omega^2)\right)^7$

as $1+\omega+\omega^2 = 0$ and $\omega+\omega^2 = -1$

$= \left(1-(-1)\right)^7$

$= (1+1)^7$

$= (2)^7$

$= 128$

(ii) $(1-3\omega-3\omega^2)^5$ $= \left(1-3(\omega+\omega^2)\right)^5$

as $1+\omega+\omega^2 = 0$ and $\omega+\omega^2 = -1$

$= \left(1-3(-1)\right)^5$

$= (1+3)^5$

$= (4)^5$

$= 1024$

(iii) $(9+4\omega+4\omega^2)^3$ $= \left(9+4(\omega+\omega^2)\right)^3$

as $1+\omega+\omega^2 = 0$ and $\omega+\omega^2 = -1$

$= \left(9+4(-1)\right)^3$

$= (9-4)^3$

$= (5)^3$

$= 125$

(iv) $(2+2\omega-2\omega^2)(3-3\omega+3\omega^2)$ $= (2(1+\omega)-2\omega^2)\left(-3\omega+3(1+\omega^2)\right)$

as $1+\omega+\omega^2 = 0$ and $1+\omega^2 = -\omega$ and $1+\omega = -\omega^2$

$= (2(-\omega^2)-2\omega^2)\left(-3\omega+3(-\omega)\right)$

$= (-2\omega^2-2\omega^2)(-3\omega-3\omega)$

$= (-4\omega^2)(-6\omega)$

$= 24\omega^3$

as $\omega^3 = 1$

$= 24(1)$

$= 24$

(v) $\left(-1+\sqrt{-3}\right)^6 + \left(-1-\sqrt{-3}\right)^6$ $= \left(2 \times \frac{-1+\sqrt{-3}}{2}\right)^6 + \left(2 \times \frac{-1-\sqrt{-3}}{2}\right)^6$

$= 64(\omega)^6 + 64(\omega^2)^6$

$= 64(\omega)^6 + 64(\omega)^{12}$

$= 64(\omega^3)^2 + 64(\omega^3)^4$

as $\omega^3 = 1$

$= 64(1)^2 + 64(1)^4$

$= 64 + 64$

$= 128$

(vi) $\left(\frac{-1+\sqrt{-3}}{2}\right)^9 + \left(\frac{-1-\sqrt{-3}}{2}\right)^9$ $= (\omega)^9 + (\omega^2)^9$

$= (\omega)^9 + (\omega)^{18}$

$= (\omega^3)^3 + (\omega^3)^6$

as $\omega^3 = 1$

$= (1)^3 + (1)^6$

$= 1 + 1$

$= 2$

(vii) $\omega^{37} + \omega^{38} - 5$ $= \omega^{36}.\omega + \omega^{36}.\omega^2 - 5$

$= (\omega^3)^{12}.\omega + (\omega^3)^{12}.\omega^2 - 5$

$= (\omega^3)^{12}.\omega + (\omega^3)^{12}.\omega^2 - 5$

$= 1.\omega + 1.\omega^2 - 5$

$= \omega + \omega^2 - 5$

as $1 + \omega + \omega^2 = 0$ and $\omega + \omega^2 = -1$

$= -1 - 5$

$= -6$

(viii) $\omega^{-13} + \omega^{-17}$ $= \frac{1}{\omega^{13}} + \frac{1}{\omega^{17}}$

$= \frac{1}{\omega^{12}.\omega} + \frac{1}{\omega^{15}.\omega^2}$

$= \frac{1}{(\omega^3)^4.\omega} + \frac{1}{(\omega^3)^5.\omega^2}$

as $\omega^3 = 1$

$= \frac{1}{(1)^4.\omega} + \frac{1}{(1)^5.\omega^2}$

$= \frac{1}{\omega} + \frac{1}{\omega^2}$

$= \frac{\omega + \omega^2}{\omega^3}$

as $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$ and $\omega + \omega^2 = -1$

$= \frac{-1}{1}$

$= -1$

Q. 3: Prove that $(x^3 + y^3) = (x + y)(x + \omega y)(x + \omega^2 y)$

R.H.S $= (x + y)(x + \omega y)(x + \omega^2 y)$

$= (x + y)(x^2 + \omega^2 xy + \omega xy + \omega^3 y^2)$

$= (x + y)(x^2 + (\omega^2 + \omega)xy + (\omega^3)y^2)$

as $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$ and $\omega + \omega^2 = -1$

$= (x + y)(x^2 + (-1)xy + (1)y^2)$

$= (x + y)(x^2 - xy + y^2)$

as $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$= x^3 + y^3$ = L.H.S

Q. 4: Prove that $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x + \omega y + \omega^2 z)(x + \omega^2 y + \omega z)$

R.H.S $= (x + y + z)(x + \omega y + \omega^2 z)(x + \omega^2 y + \omega z)$

$= (x + y + z)(x^2 + \omega^2 xy + \omega xz + \omega xy + \omega^3 y^2 + \omega^2 yz + \omega^2 xz + \omega^4 yz + \omega^3 z^2)$

$= (x + y + z)(x^2 + \omega^2 xy + \omega xz + \omega xy + \omega^3 y^2 + \omega^2 yz + \omega^2 xz + \omega^3 . \omega yz + \omega^3 z^2)$

as $\omega^3 = 1$

$= (x + y + z)(x^2 + \omega^2 xy + \omega xz + \omega xy + y^2 + \omega^2 yz + \omega^2 xz + \omega yz + z^2)$

$= (x + y + z)(x^2 + y^2 + z^2 + (\omega + \omega^2)xy + (\omega + \omega^2)yz + (\omega + \omega^2)xz)$

as $1 + \omega + \omega^2 = 0$ and $\omega + \omega^2 = -1$

$= (x + y + z)(x^2 + y^2 + z^2 + (-1)xy + (-1)yz + (-1)xz)$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

as $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$, So

$= x^3 + y^3 + z^3 - 3xyz$ = L.H.S

Q. 5: Prove that $(1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8) \ldots\ldots\ldots 2n\ factors = 1$

L.H.S $= (1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8) \ldots\ldots\ldots 2n\ factors$

$= (1 + \omega)(1 + \omega^2)(1 + \omega^3 . \omega)(1 + \omega^6 . \omega^2) \ldots\ldots\ldots 2n\ factors$

$= (1 + \omega)(1 + \omega^2)(1 + (\omega^3) . \omega)(1 + (\omega^3)^2 . \omega^2) \ldots\ldots\ldots 2n\ factors$

as $\omega^3 = 1$

$= (1 + \omega)(1 + \omega^2)(1 + (1) . \omega)(1 + (1)^2 . \omega^2) \ldots\ldots\ldots 2n\ factors$

$= (1 + \omega)(1 + \omega^2)(1 + \omega)(1 + \omega^2)(1 + \omega)(1 + \omega^2) \ldots\ldots\ldots 2n\ factors$

$= (1 + \omega^2 + \omega + \omega^3)(1 + \omega^2 + \omega + \omega^3)(1 + \omega^2 + \omega + \omega^3) \ldots\ldots\ldots 2n\ factors$

$= (1 + \omega + \omega^2 + \omega^3)(1 + \omega + \omega^2 + \omega^3)(1 + \omega + \omega^2 + \omega^3) \ldots\ldots\ldots 2n\ factors$

as $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$

$= (0 + 1)(0 + 1)(0 + 1) \ldots\ldots\ldots 2n\ factors$

$= (1)(1)(1) \ldots\ldots\ldots 2n\ factors$

multiplying 1 up to 2n times we get

= 1 = R.H.S

Exercise 2.3

Q. 1: Without solving, find the sum and the product of the roots of the following quadratic equations.

(i) $x^2 - 5x + 3 \quad = 0$

sum of roots $= \alpha + \beta$ $= -\frac{b}{a}$ $= -\frac{-5}{1}$ $= 5$

Product of roots $= \alpha\beta$ $= \frac{c}{a}$ $= \frac{3}{1}$ $= 3$

(ii) $3x^2 + 7x - 11 = 0$

sum of roots $= \alpha + \beta$ $= -\frac{b}{a}$ $= -\frac{7}{3}$ $= \frac{-7}{3}$

Product of roots $= \alpha\beta$ $= \frac{c}{a}$ $= \frac{-11}{3}$ $= \frac{-11}{3}$

(iii) $px^2 - qx + r = 0$

sum of roots $= \alpha + \beta$ $= -\frac{b}{a}$ $= -\frac{-q}{p}$ $= \frac{q}{p}$

Product of roots $= \alpha\beta$ $= \frac{c}{a}$ $= \frac{r}{p}$ $= \frac{r}{p}$

(iv) $(a + b)x^2 - ax + b = 0$

sum of roots $= \alpha + \beta$ $= -\frac{b}{a}$ $= -\frac{-a}{a+b}$ $= \frac{a}{a+b}$

Product of roots $= \alpha\beta$ $= \frac{c}{a}$ $= \frac{b}{a+b}$ $= \frac{b}{a+b}$

(v) $(l + m)x^2 + (m + n)x + n - l = 0$

sum of roots $= \alpha + \beta$ $= -\frac{b}{a}$ $= -\frac{m+n}{l+m}$ $= -\frac{m+n}{l+m}$

Product of roots $= \alpha\beta$ $= \frac{c}{a}$ $= \frac{n-l}{l+m}$ $= \frac{n-l}{l+m}$

(vi) $7x^2 - 5mx + 9n = 0$

sum of roots $= \alpha + \beta$ $= -\frac{b}{a}$ $= -\frac{-5}{7}$ $= \frac{5m}{7}$

Product of roots $= \alpha\beta$ $= \frac{c}{a}$ $= \frac{9}{7}$ $= \frac{9n}{7}$

Q. 2: Find the value of k, if

(i) Sum of the roots of the equation $2kx^2 - 3x + 4k = 0$ is twice the product of the roots.

sum of roots $= \alpha + \beta$ $= -\frac{b}{a}$ $= -\frac{-3}{2k}$ $= \frac{3}{2k}$

Product of roots $= \alpha\beta$ $= \frac{c}{a}$ $= \frac{4k}{2k}$ $= 2$

According to given condition

$\alpha + \beta = 2\alpha\beta$

$\frac{3}{2k} = 2 \times 2$

$\frac{3}{2k} = 4$

$\frac{3}{8} = k$

$k = \frac{3}{8}$

(ii) Sum of the roots of the equation $x^2 + (3k - 7)x + 5k = 0$ is $\frac{3}{2}$ times the product of the roots.

sum of roots $= \alpha + \beta$ $= -\frac{b}{a}$ $= -\frac{3k-7}{1}$ $= -3k + 7$

Product of roots $= \alpha\beta$ $= \frac{c}{a}$ $= \frac{5k}{1}$ $= 5k$

According to given condition

$$\alpha + \beta = \frac{3}{2}\alpha\beta$$
$$-3k + 7 = \frac{3}{2} \times 5k$$
$$-6k + 14 = 15k$$
$$-6k - 15k = -14$$
$$-21k = -14$$
$$k = \frac{2}{3}$$

Q. 3: Find k, if

(i) sum of the squares of the roots of the equation

$4kx^2 + 3kx - 8 = 0$ is 2.

sum of roots $= \alpha + \beta \quad = -\frac{b}{a} \quad = -\frac{3k}{4k} \quad = \frac{-3}{4}$

Product of roots $= \alpha\beta \quad = \frac{c}{a} \quad = \frac{-8}{4k} \quad = \frac{-2}{k}$

According to given condition

$$\alpha^2 + \beta^2 = 2$$

Adding $2\alpha\beta$ on both sides

$$\alpha^2 + \beta^2 + 2\alpha\beta = 2 + 2\alpha\beta$$
$$(\alpha + \beta)^2 = 2 + 2\alpha\beta$$
$$\left(\frac{-3}{4}\right)^2 = 2 + 2 \times \frac{-2}{k}$$
$$\frac{9}{16} = 2 + \frac{-4}{k}$$
$$\frac{9}{16} = \frac{2k-4}{k}$$
$$9k = 32k - 64$$
$$64 = 32k - 9k$$
$$64 = 23k$$
$$\frac{64}{23} = k$$
$$k = \frac{64}{23}$$

(ii) sum of the squares of the roots of the equation

$x^2 - 2kx + 2k + 1 = 0$ is 6.

sum of roots $= \alpha + \beta \quad = -\frac{b}{a} \quad = -\frac{-2k}{1} \quad = 2k$

Product of roots $= \alpha\beta \quad = \frac{c}{a} \quad = \frac{2k+1}{1} \quad = 2k + 1$

According to given condition

$$\alpha^2 + \beta^2 = 6$$

Adding $2\alpha\beta$ on both sides

$$\alpha^2 + \beta^2 + 2\alpha\beta = 6 + 2\alpha\beta$$
$$(\alpha + \beta)^2 = 6 + 2\alpha\beta$$
$$(2k)^2 = 6 + 2\,(2k + 1)$$

$$4k^2 = 6 + 4k + 2$$
$$4k^2 = 8 + 4k$$
$$4k^2 - 4k - 8 = 0$$
$$4k^2 - 8k + 4k - 8 = 0$$
$$4k(k-2) + 4(k-2) = 0$$
$$(k-2)(4k+4) = 0$$
$k - 2 = 0$ and $4k + 4 = 0$
$k = 2$ and $k = -1$

Q. 4: Find p, if

(i) the roots of the equation $x^2 - x + p^2 = 0$ differ by unity.

let 1st root $= \alpha$

then according to given condition

2nd root $= \alpha - 1$

sum of roots $= \alpha + \alpha - 1 = -\frac{b}{a} = -\frac{-1}{1} = 1$

so, $2\alpha - 1 = 1$
$$2\alpha = 2$$
$$\alpha = 1 \text{ ------------------------- (i)}$$

Product of roots $= \alpha(\alpha - 1) = \frac{c}{a} = \frac{p^2}{1} = p^2$

so, $\alpha^2 - \alpha = p^2$ ------------------------- (ii)

putting the value from equ. (i) in equ. (ii)
$$\alpha^2 - \alpha = p^2$$
$$(1)^2 - 1 = p^2$$
$$1 - 1 = p^2$$
$$0 = p^2$$
$$p = 0$$

(ii) the roots of the equation $x^2 + 3x + p - 2 = 0$ differ by 2.

let 1st root $= \alpha$

then according to given condition

2nd root $= \alpha - 2$

sum of roots $= \alpha + \alpha - 2 = -\frac{b}{a} = -\frac{3}{1} = -3$

so, $2\alpha - 2 = -3$
$$2\alpha = -1$$
$$\alpha = \frac{-1}{2} \text{ ------------------------- (i)}$$

Product of roots $= \alpha(\alpha - 2) = \frac{c}{a} = \frac{p-2}{1} = p - 2$

so, $\alpha^2 - 2\alpha = p - 2$ ------------------------- (ii)

putting the value from equ. (i) in equ. (ii)
$$\alpha^2 - 2\alpha = p - 2$$

$\left(\frac{-1}{2}\right)^2 - 2\left(\frac{-1}{2}\right) = p - 2$

$\frac{1}{4} + 1 = p - 2$

$\frac{1+4}{4} = p - 2$

$\frac{5}{4} = p - 2$

$5 = 4p - 8$

$4p - 8 = 5$

$4p = 5 + 8$

$4p = 13$

$p = \frac{13}{4}$

Q. 5: Find m, if

(i) the roots of the equation $x^2 - 7x + 3m - 5 = 0$ satisfy the relation $3\alpha + 2\beta = 4$.

If α, β are the roots of the given equation.

sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{-7}{1} = 7$

so, $\alpha + \beta = 7$

$\beta = 7 - \alpha$ ---------------------- (i)

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{3m-5}{1} = 3m - 5$

so, $\alpha\beta = 3m - 5$ ---------------------- (ii)

putting the value of β in $3\alpha + 2\beta = 4$.

$3\alpha + 2\beta = 4$

$3\alpha + 2(7 - \alpha) = 4$

$3\alpha + 14 - 2\alpha = 4$

$\alpha = 4 - 14$

$\alpha = -10$

putting the value of α in equ (i)

$\beta = 7 - \alpha$

$\beta = 7 - (-10)$

$\beta = 7 + 10$

$\beta = 17$

putting the values of α, β in equ (ii)

$\alpha\beta = 3m - 5$

$(-10)(17) = 3m - 5$

$-170 = 3m - 5$

$3m - 5 = -170$

$3m = -170 + 5$

$3m = -165$

$m = -\frac{165}{3}$

$m = -55$

(ii) the roots of the equation $x^2 + 7x + 3m - 5 = 0$ satisfy the relation $3\alpha - 2\beta = 4$.

If α, β are the roots of the given equation.

sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{7}{1} = -7$

so, $\alpha + \beta = -7$

$\beta = -7 - \alpha$ ---------------------- (i)

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{3m-5}{1} = 3m - 5$

so, $\alpha\beta = 3m - 5$ ------------------- (ii)

putting the value of β in $3\alpha - 2\beta = 4$.

$3\alpha - 2\beta = 4$

$3\alpha - 2(-7 - \alpha) = 4$

$3\alpha + 14 + 2\alpha = 4$

$5\alpha = 4 - 14$

$\alpha = \frac{-10}{5}$

$\alpha = -2$

putting the value of α in equ (i)

$\beta = -7 - \alpha$

$\beta = -7 - (-2)$

$\beta = -7 + 2$

$\beta = -5$

putting the values of α, β in equ (ii)

$\alpha\beta = 3m - 5$

$(-2)(-5) = 3m - 5$

$10 = 3m - 5$

$10 = 3m - 5$

$3m = -15$

$m = \frac{-15}{3}$

$m = -5$

(iii) the roots of the equation $3x^2 - 2x + 7m + 2 = 0$ satisfy the relation $7\alpha - 3\beta = 18$.

If α, β are the roots of the given equation.

sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{-2}{3} = \frac{2}{3}$

so, $\alpha + \beta = \frac{2}{3}$

$\beta = \frac{2}{3} - \alpha$ ---------------------- (i)

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{7m+2}{3} = \frac{7m+2}{3}$

so, $\alpha\beta = \frac{7m+2}{3}$ ------------------- (ii)

putting the value of β in $7\alpha - 3\beta = 18$.

$7\alpha - 3\beta = 18$

$7\alpha - 3\left(\frac{2}{3} - \alpha\right) = 18$

$7\alpha - 2 + 3\alpha = 18$

$$7\alpha + 3\alpha = 18 + 2$$
$$10\alpha = 20$$
$$\alpha = 2$$

putting the value of α in equ (i)

$$\beta = \frac{2}{3} - \alpha$$
$$\beta = \frac{2}{3} - 2$$
$$\beta = \frac{2-6}{3}$$
$$\beta = \frac{-4}{3}$$

putting the values of α, β in equ (ii)

$$\alpha\beta = \frac{7m+2}{3}$$
$$(2)\left(\frac{-4}{3}\right) = \frac{7m+2}{3}$$
$$\frac{-8}{3} = \frac{7m+2}{3}$$
$$-8 = 7m + 2$$
$$7m + 2 = -8$$
$$7m = -8 - 2$$
$$7m = -10$$
$$m = \frac{-10}{7}$$

Q. 6: Find m, if sum and product of the roots of the following equations is equal to a given number λ.

(i) $(2m + 3)x^2 + (7m - 5)x + (3m - 10) = 0$

If α, β are the roots of the given equation.

sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{7m-5}{2m+3} = -\frac{7m-5}{2m+3}$

according to given condition, $\alpha + \beta = \lambda$

$$\lambda = -\frac{7m-5}{2m+3} \text{ ----------------------- (i)}$$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{3m-10}{2m+3} = \frac{3m-10}{2m+3}$

according to given condition, $\alpha\beta = \lambda$

so, $\lambda = \frac{3m-10}{2m+3}$ ------------------- (ii)

comparing equ (i) and (ii)

$$-\frac{7m-5}{2m+3} = \frac{3m-10}{2m+3}$$
$$-7m + 5 = 3m - 10$$
$$-7m - 3m = -10 - 5$$
$$-10m = -15$$
$$m = \frac{-15}{-10}$$
$$m = \frac{3}{2}$$

(ii) $4x^2 - (3 + 5m)x + (9m - 17) = 0$

If α, β are the roots of the given equation.

sum of roots $= \alpha + \beta$ $= -\frac{b}{a}$ $= -\frac{-(3+5m)}{4}$ $= \frac{3+5m}{4}$

according to given condition, $\alpha + \beta = \boldsymbol{\lambda}$

$\boldsymbol{\lambda} = \frac{3+5m}{4}$---------------------- (i)

Product of roots $= \alpha\beta$ $= \frac{c}{a}$ $= -\frac{9m-17}{4}$ $= -\frac{9m-17}{4}$

according to given condition, $\alpha\beta = \boldsymbol{\lambda}$

so, $\boldsymbol{\lambda} = \frac{9m-17}{4}$------------------- (ii)

comparing equ (i) and (ii)

$$\frac{3+5m}{4} = -\frac{9m-17}{4}$$

$$3 + 5m = -9m + 17$$

$$5m + 9m = 17 - 3$$

$$14m = 14$$

$$m = \frac{14}{14}$$

$$m = 1$$

Exercise 2.4

Q. 1: If α, β are the roots of the equation $x^2 + px + q = 0$, then evaluate.

(i) $\alpha^2 + \beta^2$

sum of roots $= \alpha + \beta \quad = -\frac{b}{a} \quad = -\frac{p}{1} \quad = -p$

Product of roots $= \alpha\beta \quad = \frac{c}{a} \quad = \frac{q}{1} \quad = q$

So,

$\alpha^2 + \beta^2$

adding and subtracting $2\alpha\beta$

$= \alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta$

$= (\alpha + \beta)^2 - 2\alpha\beta$

$= (-p)^2 - 2(q)$

$= p^2 - 2q$

(ii) $\alpha^3\beta + \alpha\beta^3$

sum of roots $= \alpha + \beta \quad = -\frac{b}{a} \quad = -\frac{p}{1} \quad = -p$

Product of roots $= \alpha\beta \quad = \frac{c}{a} \quad = \frac{q}{1} \quad = q$

So,

$\alpha^3\beta + \alpha\beta^3 = \alpha\beta(\alpha^2 + \beta^2)$

adding and subtracting $2\alpha\beta$

$= \alpha\beta(\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta)$

$= \alpha\beta[(\alpha + \beta)^2 - 2\alpha\beta]$

$= q[(-p)^2 - 2(q)]$

$= q[p^2 - 2q]$

$= qp^2 - 2q^2$

(iii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

sum of roots $= \alpha + \beta \quad = -\frac{b}{a} \quad = -\frac{p}{1} \quad = -p$

Product of roots $= \alpha\beta \quad = \frac{c}{a} \quad = \frac{q}{1} \quad = q$

So,

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$

adding and subtracting $2\alpha\beta$

$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta}{\alpha\beta}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{(-p)^2 - 2(q)}{q}$

$= \frac{p^2}{q} - \frac{2q}{q}$

$= \frac{p^2}{q} - 2$

Q. 2: If α, β are the roots of the equation $4x^2 - 5x + 6 = 0$, then find the values of

(i) $\frac{1}{\alpha} + \frac{1}{\beta}$

sum of roots $= \alpha + \beta \quad = -\frac{b}{a} \quad = -\frac{-5}{4} \quad = \frac{5}{4}$

Product of roots $= \alpha\beta \quad = \frac{c}{a} \quad = \frac{6}{4} \quad = \frac{3}{2}$

So,

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$$
$$= \frac{5/4}{3/2}$$
$$= \frac{5}{4} \times \frac{2}{3}$$
$$= \frac{5}{6}$$

(ii) $\alpha^2\beta^2$

sum of roots $= \alpha + \beta \quad = -\frac{b}{a} \quad = -\frac{-5}{4} \quad = \frac{5}{4}$

Product of roots $= \alpha\beta \quad = \frac{c}{a} \quad = \frac{6}{4} \quad = \frac{3}{2}$

So,

$$\alpha^2\beta^2 = (\alpha\beta)^2$$
$$= \left(\frac{3}{2}\right)^2$$
$$= \frac{9}{4}$$

(iii) $\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2}$

sum of roots $= \alpha + \beta \quad = -\frac{b}{a} \quad = -\frac{-5}{4} \quad = \frac{5}{4}$

Product of roots $= \alpha\beta \quad = \frac{c}{a} \quad = \frac{6}{4} \quad = \frac{3}{2}$

So,

$$\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2} = \frac{\alpha + \beta}{\alpha^2\beta^2}$$
$$= \frac{\alpha + \beta}{(\alpha\beta)^2}$$
$$= \frac{5/4}{(3/2)^2}$$
$$= \frac{5/4}{9/4}$$
$$= \frac{5}{4} \times \frac{4}{9}$$
$$= \frac{5}{9}$$

(iv) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

sum of roots $= \alpha + \beta \quad = -\frac{b}{a} \quad = -\frac{-5}{4} \quad = \frac{5}{4}$

Product of roots $= \alpha\beta \quad = \frac{c}{a} \quad = \frac{6}{4} \quad = \frac{3}{2}$

So,

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)}{(\alpha\beta)^2}$$

adding and subtracting $2\alpha\beta$

$$= \frac{(\alpha+\beta)(\alpha^2-\alpha\beta-2\alpha\beta+2\alpha\beta+\beta^2)}{(\alpha\beta)^2}$$

$$= \frac{(\alpha+\beta)(\alpha^2+2\alpha\beta+\beta^2-3\alpha\beta)}{(\alpha\beta)^2}$$

$$= \frac{(\alpha+\beta)((\alpha+\beta)^2-3\alpha\beta)}{(\alpha\beta)^2}$$

$$= \frac{(5/4)\left((5/4)^2-3(3/2)\right)}{(3/2)^2}$$

$$= \frac{(5/4)(25/16-9/2)}{9/4}$$

$$= \frac{(5/4)(25-72/16)}{9/4}$$

$$= \frac{(5/4)(-47/16)}{9/4}$$

$$= \frac{(-235/64)}{9/4}$$

$$= \frac{-235}{64}\times\frac{4}{9}$$

$$= \frac{-235}{144}$$

Q. 3: If α, β are the roots of the equation $lx^2+mx+n=0$, then find the values of

(i) $\alpha^3\beta^2+\alpha^2\beta^3$

sum of roots $= \alpha+\beta \quad = -\frac{b}{a} \quad = -\frac{m}{l} \quad = -\frac{m}{l}$

Product of roots $= \alpha\beta \quad = \frac{c}{a} \quad = \frac{n}{l} \quad = \frac{n}{l}$

So,

$$\alpha^3\beta^2+\alpha^2\beta^3 = \alpha^2\beta^2(\alpha+\beta)$$

$$= (\alpha\beta)^2(\alpha+\beta)$$

$$= \left(\frac{n}{l}\right)^2\left(-\frac{m}{l}\right)$$

$$= \frac{n^2}{l^2}\times\left(-\frac{m}{l}\right)$$

$$= -\frac{n^2m}{l^3}$$

(ii) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

sum of roots $= \alpha+\beta \quad = -\frac{b}{a} \quad = -\frac{m}{l} \quad = -\frac{m}{l}$

Product of roots $= \alpha\beta \quad = \frac{c}{a} \quad = \frac{n}{l} \quad = \frac{n}{l}$

So,

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{\alpha^2\beta^2}$$

Adding and subtracting $2\alpha\beta$

$$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta}{(\alpha\beta)^2}$$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$= \frac{\left(-m/_l\right)^2 - 2\left(n/_l\right)}{\left(n/_l\right)^2}$$

$$= \frac{m^2/_{l^2} - 2n/_l}{n^2/_{l^2}}$$

$$= \frac{m^2 - 2nl/_{l^2}}{n^2/_{l^2}}$$

$$= \frac{m^2 - 2nl}{l^2} \times \frac{l^2}{n^2}$$

$$= \frac{m^2 - 2nl}{n^2}$$

Exercise 2.5

Q. 1: Write the quadratic equations having following roots.

(a) $1, 5$

sum of roots $= S = \alpha + \beta = 1 + 5 = 6$

Product of roots $= P = \alpha\beta = 1 \times 5 = 5$

So,

$$x^2 - Sx + P = 0$$
$$x^2 - 6x + 5 = 0$$

(b) $4, 9$

sum of roots $= S = \alpha + \beta = 4 + 9 = 13$

Product of roots $= P = \alpha\beta = 4 \times 9 = 36$

So,

$$x^2 - Sx + P = 0$$
$$x^2 - 13x + 36 = 0$$

(c) $-2, 3$

sum of roots $= S = \alpha + \beta = -2 + 3 = 1$

Product of roots $= P = \alpha\beta = -2 \times 3 = -6$

So,

$$x^2 - Sx + P = 0$$
$$x^2 - x - 6 = 0$$

(d) $0, -3$

sum of roots $= S = \alpha + \beta = 0 - 3 = -3$

Product of roots $= P = \alpha\beta = 0 \times -3 = 0$

So,

$$x^2 - Sx + P = 0$$
$$x^2 + 3x + 0 = 0$$
$$x^2 + 3x = 0$$

(e) $2, -6$

sum of roots $= S = \alpha + \beta = 2 - 6 = -4$

Product of roots $= P = \alpha\beta = 2 \times -6 = -12$

So,

$$x^2 - Sx + P = 0$$
$$x^2 + 4x - 12 = 0$$

(f) $-1, -7$

sum of roots $= S = \alpha + \beta = -1 - 7 = -8$

Product of roots = P = $\alpha\beta$ = -1×-7 = 7

So,

$x^2 - Sx + P$ = 0

$x^2 + 8x + 7$ = 0

(g) $1 + i, 1 - i$

sum of roots = S = $\alpha + \beta$ = $1 + i + 1 - i$ = 2

Product of roots = P = $\alpha\beta$ = $(1 + i)(1 - i)$ = $1 - i^2$ = 1+1 =2

So,

$x^2 - Sx + P$ = 0

$x^2 - 2x + 2$ = 0

(h) $3 + \sqrt{2}, 3 - \sqrt{2}$

sum of roots = S = $\alpha + \beta$ = $3 + \sqrt{2} + 3 - \sqrt{2}$ = 6

Product of roots = P = $\alpha\beta$ = $(3 + \sqrt{2})(3 - \sqrt{2})$ = $9 - (\sqrt{2})^2$ = 9 - 2 = 7

So,

$x^2 - Sx + P$ = 0

$x^2 - 6x + 7$ = 0

Q. 2: If α, β are the roots of the equation $x^2 - 3x + 6 = 0$, Form equations whose roots are

(a) $2\alpha + 1,\ 2\beta + 1$

For given equation

sum of roots = $\alpha + \beta$ = $-\frac{b}{a}$ = $-\frac{-3}{1}$ = 3

Product of roots = $\alpha\beta$ = $\frac{c}{a}$ = $\frac{6}{1}$ = 6

So the equation for the given roots will be driven as follows,

S = $2\alpha + 1 + 2\beta + 1$ = $2\alpha + 2\beta + 2$

= $2(\alpha + \beta) + 2$

= $2(3) + 2$

= $6 + 2$

= 8

P = $(2\alpha + 1)(2\beta + 1)$ = $4\alpha\beta + 2\alpha + 2\beta + 1$

= $4\alpha\beta + 2(\alpha + \beta) + 1$

= $4(6) + 2(3) + 1$

= $24 + 6 + 1$

= 31

So,

$x^2 - Sx + P$ = 0

$x^2 - 8x + 31$ = 0

(b) $\alpha^2,\ \beta^2$

For given equation

sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{-3}{1} = 3$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{6}{1} = 6$

So the equation for the given roots will be driven as follows,

$S = \alpha^2 + \beta^2 = \alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta$

$= (\alpha + \beta)^2 - 2\alpha\beta$

$= (3)^2 - 2(6)$

$= 9 - 12$

$= -3$

$P = \alpha^2\beta^2 = (\alpha\beta)^2$

$= (6)^2$

$= 36$

So,

$x^2 - Sx + P = 0$

$x^2 + 3x + 36 = 0$

(c) $\frac{1}{\alpha}, \frac{1}{\beta}$

For given equation

sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{-3}{1} = 3$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{6}{1} = 6$

So the equation for the given roots will be driven as follows,

$S = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$

$= \frac{3}{6}$

$= \frac{1}{2}$

$P = \frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{\alpha\beta}$

$= \frac{1}{6}$

So,

$x^2 - Sx + P = 0$

$x^2 - \frac{1}{2}x + \frac{1}{6} = 0$

multiplying by 6 on b.s.

$6x^2 - 3x + 1 = 0$

(d) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

For given equation

sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{-3}{1} = 3$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{6}{1} = 6$

So the equation for the given roots will be driven as follows,

$S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$

$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta}{\alpha\beta}$

$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{3^2 - 2(6)}{6}$

$= \frac{9-12}{6}$

$= \frac{-3}{6}$

$= \frac{-1}{2}$

$P = \frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$

So,

$x^2 - Sx + P = 0$

$x^2 - \frac{-1}{2}x + 1 = 0$

$x^2 + \frac{1}{2}x + 1 = 0$

multiplying by 2 on b.s.

$2x^2 + x + 2 = 0$

(e) $\alpha + \beta, \frac{1}{\alpha} + \frac{1}{\beta}$

For given equation

sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{-3}{1} = 3$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{6}{1} = 6$

So the equation for the given roots will be driven as follows,

$S = \alpha + \beta + \frac{1}{\alpha} + \frac{1}{\beta} = (\alpha + \beta) + \frac{\alpha + \beta}{\alpha\beta}$

$= (3) + \frac{3}{6}$

$= 3 + \frac{1}{2}$

$= \frac{6+1}{2}$

$= \frac{6+1}{2}$

$= \frac{7}{2}$

$P = (\alpha + \beta).\frac{\alpha + \beta}{\alpha\beta} = (3) \times \frac{3}{6}$

$= \frac{3}{2}$

So,

$x^2 - Sx + P \quad = 0$

$x^2 - \frac{7}{2}x + \frac{3}{2} \quad = 0$

multiplying by 2 on b.s.

$2x^2 - 7x + 3 = 0$

Q. 3: If α, β are the roots of the equation $x^2 + px + q = 0$, Form equations whose roots are

(a) $\alpha^2,\ \beta^2$

For given equation

sum of roots $= \alpha + \beta \quad = -\frac{b}{a} \quad = -\frac{p}{1} \quad = -p$

Product of roots $= \alpha\beta \quad = \frac{c}{a} \quad = \frac{q}{1} \quad = q$

So the equation for the given roots will be driven as follows,

S = $\alpha^2 + \beta^2 \quad = \alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta$

$= (\alpha + \beta)^2 - 2\alpha\beta$

$= (-p)^2 - 2(q)$

$= p^2 - 2q$

P = $\alpha^2\beta^2 \quad = (\alpha\beta)^2$

$= (q)^2$

$= q^2$

So,

$x^2 - Sx + P \quad = 0$

$x^2 - (p^2 - 2q)x + q^2 = 0$

(b) $\frac{\alpha}{\beta},\ \frac{\beta}{\alpha}$

For given equation

sum of roots $= \alpha + \beta \quad = -\frac{b}{a} \quad = -\frac{p}{1} \quad = -p$

Product of roots $= \alpha\beta \quad = \frac{c}{a} \quad = \frac{q}{1} \quad = q$

So the equation for the given roots will be driven as follows,

S = $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} \quad = \frac{\alpha^2 + \beta^2}{\alpha\beta}$

$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta}{\alpha\beta}$

$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{p^2 - 2q}{q}$

$= \frac{p^2 - 2q}{q}$

P = $\frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} \quad = 1$

So,

$$x^2 - Sx + P = 0$$

$$x^2 - \frac{p^2 - 2q}{q}x + 1 = 0$$

multiplying by q on b.s.

$$qx^2 - (p^2 - 2q)x + q = 0$$

Exercise 2.6

Q. 1: Use synthetic division to find the quotient and the remainder, when

(i) $(x^2 + 7x - 1) \div (x + 1)$

From divisor, $x + a$, here $a = -1$

	1	7	-1
-1	↓	-1	-6
	1	6	-7

So the remainder is -7.

(ii) $(4x^3 - 5x + 15) \div (x + 3)$

From divisor, $x + a$, here $a = -3$

	4	0	-5	15
-3	↓	-12	36	-93
	4	-12	31	-78

So the remainder is -78.

(iii) $(x^3 + x^2 - 3x + 2) \div (x - 2)$

From divisor, $x - a$, here $a = 2$

	1	1	-3	2
2	↓	2	6	6
	1	3	3	8

So the remainder is 8.

Q. 2: Find the value of h using synthetic division, if

(i) 3 is the zero of the polynomial $2x^3 - 3hx^2 + 9$

	2	-3h	0	9
3	↓	6	18-9h	54-27h
	2	6-3h	18-9h	63-27h

According to given condition

$$63 - 27h = 0$$
$$-27h = -63$$
$$h = \frac{-63}{-27}$$
$$h = \frac{7}{3}$$

(ii) 1 is the zero of the polynomial $x^3 - 2hx^2 + 11$

	1	-2h	0	11
1	↓	1	1-2h	1-2h
	1	1-2h	1-2h	12-2h

According to given condition

$$12 - 2h = 0$$
$$-2h = -12$$
$$h = \frac{-12}{-2}$$
$$h = 6$$

(iii) -1 is the zero of the polynomial $2x^3 + 5hx - 23$

	2	0	5h	-23
-1	↓	-2	2	-5h-2
	2	-2	5h+2	-5h-25

According to given condition

$$-5h - 25 = 0$$
$$-5h = 25$$
$$h = \frac{25}{-5}$$
$$h = -5$$

Q. 3: Use synthetic division to find the values of l and m, if

(i) $(x+3)$ and $(x-2)$ are the factors of the polynomial $x^3 + 4x^2 + 2lx + m$

As $x + 3$ is factor of given polynomial

So,

From divisor, $x + a$, here $a = -3$

	1	4	$2l$	m
-3	↓	-3	-3	$-6l + 9$
	1	1	$2l - 3$	$m - 6l + 9$

So,

$$m - 6l + 9 = 0$$
$$m - 6l = -9 \text{ ------------------------ (i)}$$

As $x - 2$ is factor of given polynomial

From divisor, $x - a$, here $a = 2$

	1	4	$2l$	m
2	↓	2	12	$4l + 24$
	1	6	$2l + 12$	$m + 4l + 24$

So,

$m + 4l + 24 = 0$

$m + 4l = -24$ ------------------------ (ii)

Subtracting equation (i) from equation (ii)

$m + 4l = -24$

$-m + 6l = +9$

$10l = -15$

$l = -\frac{3}{2}$

Putting the value in equation (i)

$m - 6l = -9$

$m - 6\left(-\frac{3}{2}\right) = -9$

$m + 9 = -9$

$m = -9 - 9$

$m = -18$

(ii) $(x - 1)$ and $(x + 1)$ are the factors of the polynomial $x^3 - 3lx^2 + 2mx + 6$

As $x - 1$ is factor of given polynomial

So,

From divisor, $x - a$, here $a = 1$

1	1	$-3l$	$2m$	6
	↓	1	$-3l + 1$	$2m - 3l + 1$
	1	$-3l + 1$	$2m - 3l + 1$	$2m - 3l + 7$

So,

$2m - 3l + 7 = 0$

$2m - 3l = -7$ ------------------------ (i)

As $x + 1$ is factor of given polynomial

From divisor, $x + a$, here $a = -1$

-1	1	$-3l$	$2m$	6
	↓	-1	$3l + 1$	$-2m - 3l - 1$
	1	$-3l - 1$	$2m + 3l + 1$	$-2m - 3l + 5$

So,

$-2m - 3l + 5 = 0$

$-2m - 3l = -5$ ------------------------ (ii)

Adding equation (i) and equation (ii)

$2m - 3l = -7$

$-2m - 3l = -5$

$-6l = -12$

$l = 2$

Putting the value in equation (i)

$$2m - 3l = -7$$
$$2m - 3(2) = -7$$
$$2m - 6 = -7$$
$$2m = -7 + 6$$
$$m = -\frac{1}{2}$$

Q. 4: Solve by using synthetic division, if

(i) 2 is the root of the equation $x^3 - 28x + 48 = 0$

as 2 is the root of given equation

	1	0	-28	48
2	↓	2	4	−48
	1	2	−24	0

So,

$$x^2 + 2x - 24 = 0$$
$$x^2 + 6x - 4x - 24 = 0$$
$$x(x+6) - 4(x+6) = 0$$
$$(x+6)(x-4) = 0$$

$x + 6 = 0$ and $x - 4 = 0$

$x = -6$ and $x = 4$

So the roots are 2, -6, 4

(ii) 3 is the root of the equation $2x^3 - 3x^2 - 11x + 6 = 0$

as 3 is the root of given equation

	2	-3	-11	6
3	↓	6	9	−6
	2	3	−2	0

So,

$$2x^2 + 3x - 2 = 0$$
$$2x^2 + 4x - x - 2 = 0$$
$$2x(x+2) - 1(x+2) = 0$$
$$(x+2)(2x-1) = 0$$

$x + 2 = 0$ and $2x - 1 = 0$

$x = -2$ and $2x = 1$

$x = -2$ and $x = \frac{1}{2}$

So the roots are 3, -2, $\frac{1}{2}$

(iii) -1 is the root of the equation $4x^3 - x^2 - 11x - 6 = 0$

as -1 is the root of given equation

	4	-1	-11	-6
-1	↓	-4	5	$+6$
	4	-5	-6	0

So,

$$4x^2 - 5x - 6 = 0$$
$$4x^2 - 8x + 3x - 6 = 0$$
$$4x(x-2) + 3(x-2) = 0$$
$$(x-2)(4x+3) = 0$$

$x - 2 = 0$ and $4x + 3 = 0$

$x = 2$ and $4x = -3$

$x = 2$ and $x = \frac{-3}{4}$

So the roots are -1, 2, $\frac{-3}{4}$

Q. 5: Solve by using synthetic division, if

(i) 1 and 3 are the roots of the equation $x^4 - 10x^2 + 9 = 0$

	1	0	-10	0	9
1	↓	1	1	-9	-9
	1	1	-9	-9	0
3	↓	3	12	9	
	1	4	3	0	

So,

$$x^2 + 4x + 3 = 0$$
$$x^2 + 3x + x + 3 = 0$$
$$x(x+3) + 1(x+3) = 0$$
$$(x+3)(x+1) = 0$$

$x + 3 = 0$ and $x + 1 = 0$

$x = -3$ and $x = -1$

So the roots are 1, 3, -3, -1

(ii) 3 and -4 are the roots of the equation $x^4 + 2x^3 - 13x^2 - 14x + 24 = 0$

	1	2	-13	-14	24
3	↓	3	15	6	-24
	1	5	2	-8	0
-4	↓	-4	-4	8	
	1	1	-2	0	

So,

$$x^2 + x - 2 = 0$$
$$x^2 + 2x - x - 2 = 0$$

$$x(x+2) - 1(x+2) = 0$$

$$(x+2)(x-1) = 0$$

$x + 2 = 0$ and $x - 1 = 0$

$x = -2$ and $x = 1$

So the roots are 3, -4, -2, 1

Exercise 2.7

Solve the following simultaneous equations.

1. $x + y = 5$; $x^2 - 2y - 14 = 0$

$x + y = 5$ -------------------------- (i)

$x^2 - 2y - 14 = 0$ -------------------------- (ii)

From equation (i) we have:

$y = 5 - x$ ----------------------(iii)

Putting in equation (ii)

$x^2 - 2y - 14 = 0$

$x^2 - 2(5 - x) - 14 = 0$

$x^2 - 10 + 2x - 14 = 0$

$x^2 + 2x - 24 = 0$

$x^2 + 6x - 4x - 24 = 0$

$x(x + 6) - 4(x + 6) = 0$

$(x + 6)(x - 4) = 0$

$x + 6 = 0$; $x - 4 = 0$

$x = -6$; $x = 4$

Putting the values of x in equation (iii)

$y = 5 - x$; $y = 5 - x$

$y = 5 - (-6)$; $y = 5 - (4)$

$y = 5 + 6$; $y = 5 - 4$

$y = 11$; $y = 1$

So,

S.S = $\{(-6,\ 11),\ (4,\ 1)\}$

2. $3x - 2y = 1$; $x^2 + xy - y^2 = 1$

$3x - 2y = 1$ -------------------------- (i)

$x^2 + xy - y^2 = 1$ -------------------------- (ii)

From equation (i) we have:

$-2y = 1 - 3x$

$y = \frac{1-3x}{-2}$

$y = \frac{3x-1}{2}$ ---------------------- (iii)

Putting in equation (ii)

$x^2 + xy - y^2 = 1$

$x^2 + x\left(\frac{3x-1}{2}\right) - \left(\frac{3x-1}{2}\right)^2 = 1$

$x^2 + \frac{3x^2-x}{2} - \frac{9x^2-6x+1}{4} = 1$

$\frac{4x^2+2(3x^2-x)-(9x^2-6x+1)}{4} = 1$

$$\frac{4x^2+6x^2-2x-9x^2+6x-1}{4} = 1$$

$$\frac{x^2+4x-1}{4} = 1$$

$$x^2 + 4x - 1 = 4$$

$$x^2 + 4x - 1 - 4 = 0$$

$$x^2 + 4x - 5 = 0$$

$$x^2 + 4x - 5 = 0$$

$$x^2 + 5x - x - 5 = 0$$

$$x(x+5) - 1(x+5) = 0$$

$$(x+5)(x-1) = 0$$

$x + 5 = 0$; $x - 1 = 0$

$x = -5$; $x = 1$

Putting the values of x in equation (iii)

$y = \frac{3x-1}{2}$; $y = \frac{3x-1}{2}$

$y = \frac{3(-5)-1}{2}$; $y = \frac{3(1)-1}{2}$

$y = \frac{-16}{2}$; $y = \frac{2}{2}$

$y = -8$; $y = 1$

So,

S.S = $\{(-5, -8), (1, 1)\}$

3. $x - y = 7$; $\frac{2}{x} - \frac{5}{y} = 2$

$x - y = 7$ -------------------------- (i)

$$\frac{2}{x} - \frac{5}{y} = 2$$

$$\frac{2y-5x}{xy} = 2$$

$$2y - 5x = 2xy$$

$2y - 5x - 2xy = 0$ -------------------------- (ii)

From equation (i) we have:

$$x - y = 7$$

$$-y = 7 - x$$

$y = x - 7$---------------------- (iii)

Putting in equation (ii)

$$2y - 5x - 2xy = 0$$

$$2(x-7) - 5x - 2x(x-7) = 0$$

$$2x - 14 - 5x - 2x^2 + 14x = 0$$

$$-2x^2 + 11x - 14 = 0$$

$$2x^2 - 11x + 14 = 0$$

$$2x^2 - 7x - 4x + 14 = 0$$

$$x(2x-7) - 2(2x-7) = 0$$

$(2x-7)(x-2) = 0$

$2x - 7 = 0 \quad ; \quad x - 2 = 0$

$2x = 7 \quad ; \quad x = 2$

$x = \frac{7}{2} \quad ; \quad x = 2$

Putting the values of x in equation (iii)

$y = x - 7 \quad ; \quad y = x - 7$

$y = \frac{7}{2} - 7 \quad ; \quad y = 2 - 7$

$y = \frac{7-14}{2} \quad ; \quad y = -5$

$y = -\frac{7}{2} \quad ; \quad y = -5$

So,

$$\text{S.S} = \left\{\left(\frac{7}{2}, -\frac{7}{2}\right),\ (2, -5)\right\}$$

4. $x + y = a - b \quad ; \quad \frac{a}{x} - \frac{b}{y} = 2$

$x + y = a - b$ ----------------------- (i)

$\frac{a}{x} - \frac{b}{y} = 2$

$\frac{ay-bx}{xy} = 2$

$ay - bx = 2xy$

$ay - bx - 2xy = 0$ ------------------------ (ii)

From equation (i) we have:

$x + y = a - b$

$y = a - b - x$

$y = a - b - x$ ---------------- (iii)

Putting in equation (ii)

$ay - bx - 2xy = 0$

$a(a-b-x) - bx - 2x(a-b-x) = 0$

$a^2 - ab - ax - bx - 2ax + 2bx + 2x^2 = 0$

$2x^2 - 3ax + bx + a^2 - ab = 0$

$2x^2 - (3a-b)x + a^2 - ab = 0$

using quadratic formula

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-(3a-b)) \pm \sqrt{\left(-(3a-b)\right)^2 - 4(2)(a^2-ab)}}{2(-2)}$

$x = \frac{3a-b \pm \sqrt{(3a-b)^2 - 8(a^2-ab)}}{2(2)}$

$x = \frac{3a-b \pm \sqrt{9a^2+b^2-6ab-8a^2+8ab}}{4}$

$x = \frac{3a-b\pm\sqrt{a^2+b^2+2ab}}{4}$

$x = \frac{3a-b\pm\sqrt{(a+b)^2}}{4}$

$x = \frac{3a-b\pm(a+b)}{4}$

$x = \frac{3a-b+(a+b)}{4}$; $x = \frac{3a-b-(a+b)}{4}$

$x = \frac{3a-b+a+b}{4}$; $x = \frac{3a-b-a-b}{4}$

$x = \frac{4a}{4}$; $x = \frac{2a-2b}{4}$

$x = a$; $x = \frac{a-b}{2}$

Putting the values of x in equation (iii)

$y = a-b-x$; $y = a-b-x$

$y = a-b-a$; $y = a-b-\left(\frac{a-b}{2}\right)$

$y = a-b-a$; $y = \frac{2a-2b-(a-b)}{2}$

$y = -b$; $y = \frac{2a-2b-a+b}{2}$

$y = -b$; $y = \frac{a-b}{2}$

So,

$S.S = \left\{(a,-b), \left(\frac{a-b}{2}, \frac{a-b}{2}\right)\right\}$

5. $x^2+(y-1)^2 = 10$; $x^2+y^2+4x = 1$

$x^2+y^2-2y+1 = 10$

$x^2+y^2-2y = 9$ ---------------------- (i)

$x^2+y^2+4x = 1$ ---------------------- (ii)

Subtracting equation (i) from (ii)

$x^2+y^2+4x = 1$

$-x^2-y^2+2y = -9$

$4x+2y = -8$

$2x+y = -4$

$y = -4-2x$ ----------------- (iii)

Putting in equation (ii)

$x^2+y^2+4x = 1$

$x^2+(-4-2x)^2+4x = 1$

$x^2+\left(-(4+2x)\right)^2+4x = 1$

$x^2+(2x+4)^2+4x = 1$

$x^2+4x^2+16+16x+4x = 1$

$5x^2+20x+15 = 0$

$x^2+4x+3 = 0$

$x^2 + 3x + x + 3 = 0$

$x(x+3) + 1(x+3) = 0$

$(x+3)(x+1) = 0$

$x + 3 = 0 \quad ; \quad x + 1 = 0$

$x = -3 \quad ; \quad x = -1$

Putting the values of x in equation (iii)

$y = -4 - 2x \quad ; \quad y = -4 - 2x$

$y = -4 - 2(-3) \quad ; \quad y = -4 - 2(-1)$

$y = -4 + 6 \quad ; \quad y = -4 + 2$

$y = 2 \quad ; \quad y = -2$

So,

S.S = $\{(-3, 2), (-1, -2)\}$

6. $(x+1)^2 + (y+1)^2 = 5 \quad ; \quad (x+2)^2 + y^2 = 5$

$x^2 + 2x + 1 + y^2 + 2y + 1 = 5$

$x^2 + y^2 + 2x + 2y + 2 = 5$

$x^2 + y^2 + 2x + 2y - 3 = 0$ ---------- (i)

$x^2 + 4x + 4 + y^2 = 5$

$x^2 + y^2 + 4x - 1 = 0$ ---------------- (ii)

Subtracting equation (i) from (ii)

$x^2 + y^2 + 4x \qquad - 1 = 0$

$-x^2 - y^2 - 2x - 2y + 3 = 0$

$2x - 2y + 2 = 0$

$2x - 2y = -2$

$2(x - y) = -2$

$x - y = -1$

$-y = -1 - x$

$y = x + 1$ --------------- (iii)

Putting in equation (ii)

$x^2 + y^2 + 4x = 1$

$x^2 + (x+1)^2 + 4x = 1$

$x^2 + x^2 + 2x + 1 + 4x = 1$

$2x^2 + 6x = 0$

$2x(x+3) = 0$

$2x = 0 \quad ; \quad x + 3 = 0$

$x = 0 \quad ; \quad x = -3$

Putting the values of x in equation (iii)

$y = x + 1 \quad ; \quad y = x + 1$

$y = 0 + 1 \quad ; \quad y = -3 + 1$

$y = 1 \quad ; \quad y = -2$

So,

$$S.S = \{(0,\ 1),\ (-3, -2)\}$$

7. $x^2 + 2y^2 = 22$; $5x^2 + y^2 = 29$

$x^2 + 2y^2 = 22$ --------------------------- (i)

$5x^2 + y^2 = 29$ --------------------------- (ii)

multiplying equation (i) by 5

$5x^2 + 10y^2 = 110$ --------------------- (iii)

Subtracting equation (ii) from (iii)

$5x^2 + 10y^2 = 110$

$-5x^2 - y^2 = -29$

$9y^2 = 81$

$y^2 = 9$

$y = \pm 3$

$y = 3$;	$y = -3$

Putting the values of x in equation (i)

$x^2 + 2y^2 = 22$;	$x^2 + 2y^2 = 22$
$x^2 + 2(3)^2 = 22$;	$x^2 + 2(-3)^2 = 22$
$x^2 + 2(9) = 22$;	$x^2 + 2(9) = 22$
$x^2 + 18 = 22$;	$x^2 + 18 = 22$
$x^2 = 4$;	$x^2 = 4$
$x = \pm 2$;	$x = \pm 2$

So,

$$S.S = \{(\pm 2, \pm 3)\}$$

8. $4x^2 - 5y^2 = 6$; $3x^2 + y^2 = 14$

$4x^2 - 5y^2 = 6$ --------------------------- (i)

$3x^2 + y^2 = 14$ --------------------------- (ii)

multiplying equation (ii) by 5

$15x^2 + 5y^2 = 70$ --------------------- (iii)

Adding equation (i) and (iii)

$4x^2 - 5y^2 = 6$

$15x^2 + 5y^2 = 70$

$19x^2 = 76$

$x^2 = 4$

$x = \pm 2$

$x = 2$; $x = -2$

Putting the values of x in equation (i)

$4x^2 - 5y^2 = 6$; $4x^2 - 5y^2 = 6$

$4(2)^2 - 5y^2 = 6$; $4(-2)^2 - 5y^2 = 6$

$4(4) - 5y^2 = 6$; $4(4) - 5y^2 = 6$

$16 - 5y^2 = 6$; $16 - 5y^2 = 6$

$-5y^2 = -10$; $-5y^2 = -10$

$y^2 = 2$; $y^2 = 2$

$y = \pm\sqrt{2}$; $y = \pm\sqrt{2}$

So,

S.S = $\{(\pm 2, \pm\sqrt{2})\}$

9. $7x^2 - 3y^2 = 4$; $2x^2 + 5y^2 = 7$

$7x^2 - 3y^2 = 4$ -------------------------- (i)

$2x^2 + 5y^2 = 7$ -------------------------- (ii)

multiplying equation (i) by 2

$14x^2 - 6y^2 = 8$ ------------------------ (iii)

multiplying equation (ii) by 7

$14x^2 + 35y^2 = 49$ -------------------- (iv)

Subtracting equation (iii) from (iv)

$14x^2 + 35y^2 = 49$

$-14x^2 + 6y^2 = -8$

$41y^2 = 41$

$y^2 = 1$

$y = \pm 1$

$y = 1$; $y = -1$

Putting the values of x in equation (i)

$7x^2 - 3y^2 = 4$; $7x^2 - 3y^2 = 4$

$7x^2 - 3(1)^2 = 4$; $7x^2 - 3(-1)^2 = 4$

$7x^2 - 3 = 4$; $7x^2 - 3 = 4$

$7x^2 = 4 + 3$; $7x^2 = 4 + 3$

$7x^2 = 7$; $7x^2 = 7$

$x^2 = 1$; $x^2 = 1$

$x = \pm 1$; $x = \pm 1$

So,

S.S = $\{(\pm 1,\ \pm 1)\}$

10. $x^2 + 2y^2 = 3$; $x^2 + 4xy - 5y^2 = 0$

$x^2 + 2y^2 = 3$ -------------------------- (i)

$x^2 + 4xy - 5y^2 = 0$ -------------------------- (ii)

from equation (ii)

$x^2 + 4xy - 5y^2 = 0$

$x^2 + 4yx - 5y^2 = 0$

so, a=1, b = 4y, c = $-5y^2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4y \pm \sqrt{(4y)^2 - 4(1)(-5y^2)}}{2(1)}$$

$$= \frac{-4y \pm \sqrt{16y^2 + 20y^2}}{2}$$

$$= \frac{-4y \pm \sqrt{36y^2}}{2}$$

$$= \frac{-4y \pm 6y}{2}$$

$x = \frac{-4y+6y}{2}$; $x = \frac{-4y-6y}{2}$

$x = \frac{2y}{2}$; $x = \frac{-10y}{2}$

$x = y$---(iii) ; $x = -5y$ -------- (iv)

putting the values of y in equation (i)

$y^2 + 2y^2 = 3$; $25y^2 + 2y^2 = 3$

$3y^2 = 3$; $27y^2 = 3$

$y^2 = 1$; $y^2 = \frac{1}{9}$

$y = \pm 1$; $y = \pm \frac{1}{9}$

Putting the values of y in equation (iii) and (iv)

$x = y$; $x = -5y$

$x = \pm 1$; $x = -5 \times \pm \frac{1}{9}$

$x = \pm 1$; $x = \mp \frac{5}{9}$

So,

S.S = $\left\{(1,\ 1),\ (-1, -1),\ \left(\frac{5}{9}, -\frac{1}{9}\right), \left(-\frac{5}{9},\ \frac{1}{9}\right)\right\}$

11. $3x^2 - y^2 = 26$; $3x^2 - 5xy - 12y^2 = 0$

$3x^2 - y^2 = 26$ -------------------------- (i)

$3x^2 - 5xy - 12y^2 = 0$ --------------------------- (ii)

from equation (ii)

$3x^2 - 5xy - 12y^2 = 0$

$3x^2 - 5yx - 12y^2 = 0$

so, a=3, b = -5y, c = $-12y^2$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{+5y \pm \sqrt{(-5y)^2-4(3)(-12y^2)}}{2(3)}$

$= \frac{5y \pm \sqrt{25y^2+144y^2}}{6}$

$= \frac{5y \pm \sqrt{169y^2}}{6}$

$= \frac{5y \pm 13y}{6}$

$x = \frac{5y+13y}{6}$; $x = \frac{5y-13y}{6}$

$x = \frac{18y}{6}$; $x = \frac{-8y}{6}$

$x = 3y$---(iii) ; $x = -\frac{4y}{3}$ -------- (iv)

putting the values of x in equation (i)

$3x^2 - y^2 = 26$; $3x^2 - y^2 = 26$

$3(3y)^2 - y^2 = 26$; $3\left(-\frac{4y}{3}\right)^2 - y^2 = 26$

$27y^2 - y^2 = 26$; $3\left(\frac{16y^2}{9}\right) - y^2 = 26$

$26y^2 = 26$; $\frac{16y^2}{3} - y^2 = 26$

$26y^2 = 26$; $\frac{16y^2-3y^2}{3} = 26$

$y^2 = 1$; $\frac{13y^2}{3} = 26$

$y^2 = 1$; $y^2 = 26 \times \frac{3}{13}$

$y^2 = 1$; $y^2 = 6$

$y = \pm 1$; $y = \pm\sqrt{6}$

Putting the values of y in equation (iii) and (iv)

$x = 3y$; $x = -\frac{4y}{3}$

$x = 3(\pm 1)$; $x = -\frac{4}{3} \times \pm\sqrt{6}$

$x = \pm 3$; $x = \mp\frac{4\sqrt{6}}{3}$

So,

$$S.S = \left\{(3,\ 1),\ (-3,-1), \left(-\frac{4\sqrt{6}}{3}, \sqrt{6}\right),\ \left(\frac{4\sqrt{6}}{3}, -\sqrt{6}\right)\right\}$$

12. $x^2 + xy = 5$; $y^2 + xy = 3$

$x^2 + xy = 5$ -------------------------- (i)

$y^2 + xy = 3$ --------------------------- (ii)

Adding equation (i) and (ii)

$x^2 + 2xy + y^2 = 8$

$(x + y)^2 = 8$

taking square root on both sides

$x + y = \pm\sqrt{8}$

$y = \pm 2\sqrt{2} - x$ ----------------------(iii)

putting the values of y in equation (i)

$x^2 + xy = 5$

$x^2 + x(\pm 2\sqrt{2} - x) = 5$

$x^2 \pm 2\sqrt{2}x - x^2 = 5$

$\pm 2\sqrt{2}x = 5$

$x = \pm \frac{5}{2\sqrt{2}}$

Putting the values of x in equation (iii)

$y = \pm 2\sqrt{2} - x$

$y = \pm 2\sqrt{2} - \left(\pm \frac{5}{2\sqrt{2}}\right)$

$y = \pm 2\sqrt{2} \mp \frac{5}{2\sqrt{2}}$

$y = +2\sqrt{2} - \frac{5}{2\sqrt{2}}$; $y = -2\sqrt{2} + \frac{5}{2\sqrt{2}}$

$y = \frac{(2\sqrt{2})^2 - 5}{2\sqrt{2}}$; $y = \frac{-(2\sqrt{2})^2 + 5}{2\sqrt{2}}$

$y = \frac{8-5}{2\sqrt{2}}$; $y = \frac{-8+5}{2\sqrt{2}}$

$y = \frac{3}{2\sqrt{2}}$; $y = \frac{-3}{2\sqrt{2}}$

So,

$$\text{S.S} = \left\{\left(\frac{5}{2\sqrt{2}}, \frac{3}{2\sqrt{2}}\right), \left(-\frac{5}{2\sqrt{2}}, -\frac{3}{2\sqrt{2}}\right)\right\}$$

13. $x^2 - 2xy = 7$; $xy + 3y^2 = 2$

$x^2 - 2xy = 7$ -------------------------- (i)

$xy + 3y^2 = 2$ ------------------------- (ii)

multiplying equ (i) by 2 and (ii) by 7 we get

$2x^2 - 4xy = 14$ -------------------------------- (iii)

$7xy + 21y^2 = 14$ ------------------------------ (iv)

subtracting (iv) from (iii)

$2x^2 - 11xy - 21y^2 = 0$

$2x^2 - 14xy + 3xy - 21y^2 = 0$

$2x(x - 7y) + 3y(x - 7y) = 0$

$(x - 7y)(2x + 3y) = 0$

$x - 7y = 0$;	$2x + 3y = 0$
$x = 7y$;	$2x = -3y$
$x = 7y$---(v) ;	$x = -\frac{3y}{2}$ -------- (vi)

put in equ (ii)

$xy + 3y^2 = 2$;	$xy + 3y^2 = 2$
$(7y)y + 3y^2 = 2$;	$\left(-\frac{3y}{2}\right)y + 3y^2 = 2$
$7y^2 + 3y^2 = 2$;	$-\frac{3y^2}{2} + 3y^2 = 2$
$10y^2 = 2$;	$\frac{-3y^2+6y^2}{2} = 2$
$y^2 = \frac{2}{10}$;	$3y^2 = 4$
$y = \pm\frac{1}{\sqrt{5}}$;	$y^2 = \frac{4}{3}$
$y = \pm\frac{1}{\sqrt{5}}$;	$y = \pm\frac{2}{\sqrt{3}}$

put the values in equation (v) and (vi) respectively.

$x = 7y$;	$x = -\frac{3y}{2}$
$x = 7\left(\pm\frac{1}{\sqrt{5}}\right)$;	$x = -\frac{3}{2} \times \pm\frac{2}{\sqrt{3}}$
$x = \pm\frac{7}{\sqrt{5}}$;	$x = \mp\frac{3}{\sqrt{3}}$
$x = \pm\frac{7}{\sqrt{5}}$;	$x = \mp\sqrt{3}$

So,

$$\text{S.S} = \left\{\left(\frac{7}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right), \left(-\frac{7}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right), \left(\sqrt{3}, -\frac{2}{\sqrt{3}}\right), \left(-\sqrt{3}, \frac{2}{\sqrt{3}}\right)\right\}$$

Exercise 2.8

1. The product of two positive consecutive numbers is 182. Find the numbers.

Let 1st No $= x$
2nd No $= x + 1$
According to the given condition

$$x(x+1) = 182$$
$$x^2 + x = 182$$
$$x^2 + x - 182 = 0$$
$$x^2 + 14x - 13x - 182 = 0$$
$$x(x+14) - 13(x+14) = 0$$
$$(x+14)(x-13) = 0$$

$x + 14 = 0$; $x - 13 = 0$
$x = -14$; $x = 13$

As, the numbers are positive so, $x = 13$

So 1st No $= x = 13$
2nd No $= x + 1 = 14$

2. The sum of the squares of three positive consecutive numbers is 77. Find them.

Let 1st No $= x$
2nd No $= x + 1$
3rd No $= x + 2$
According to the given condition

$$x^2 + (x+1)^2 + (x+2)^2 = 77$$
$$x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 = 77$$
$$3x^2 + 6x + 5 = 77$$
$$3x^2 + 6x - 72 = 0$$
$$3x^2 + 18x - 12x - 72 = 0$$
$$3x(x+6) - 12(x+6) = 0$$
$$(x+6)(3x-12) = 0$$
$$(x+6)(3x-12) = 0$$

$x + 6 = 0$; $3x - 12 = 0$
$x = -6$; $x = 4$

As, the numbers are positive so, $x = 4$

So 1st No $= x = 4$
2nd No $= x + 1 = 5$
3rd No $= x + 2 = 6$

3. The sum of five times a number and the square of the number is 204. Find the number.

Let the No $= x$
According to the given condition

$$5x + x^2 = 204$$

$$x^2 + 5x - 204 = 0$$
$$x^2 + 17x - 12x - 204 = 0$$
$$x(x + 17) - 12(x + 17) = 0$$
$$(x + 17)(x - 12) = 0$$

$x + 17 = 0$; $x - 12 = 0$

$x = -17$; $x = 12$

4. The product of five less than three times a certain number and one less than four times the number is 7. Find the number.

Let the No $= x$

According to the given condition

$$(3x - 5)(4x - 1) = 7$$
$$12x^2 - 3x - 20x + 5 = 7$$
$$12x^2 - 23x - 2 = 0$$
$$12x^2 - 24x + x - 2 = 0$$
$$12x(x - 2) + 1(x - 2) = 0$$
$$(x - 2)(12x + 1) = 0$$

$x - 2 = 0$; $12x + 1 = 0$

$x = 2$; $x = -\frac{1}{12}$

So the No $= x = 2$; $x = -\frac{1}{12}$

5. The difference of a number and its reciprocal is $\frac{15}{4}$. Find the number.

Let the No $= x$

According to the given condition

$$x - \frac{1}{x} = \frac{15}{4}$$
$$\frac{x^2 - 1}{x} = \frac{15}{4}$$
$$4x^2 - 4 = 15x$$
$$4x^2 - 15x - 4 = 0$$
$$4x^2 - 16x + x - 4 = 0$$
$$4x(x - 4) + 1(x - 4) = 0$$
$$(x - 4)(4x + 1) = 0$$

$x - 4 = 0$; $4x + 1 = 0$

$x = 4$; $x = -\frac{1}{4}$

So the No $= x = 4$; $x = -\frac{1}{4}$

6. The sum of the squares of two digits of a positive integral number is 65 and the number is 9 times the sum of its digits. Find the number

Let the digit at 10's place $= x$

And the digit at 1's place $= y$

Then, the Number is given by $= 10x + y$

According to the first condition

$$x^2 + y^2 = 65 \text{ ------------------ (i)}$$

According to the second condition

$$10x + y = 9(x + y)$$

$$10x + y = 9x + 9y$$

$$10x + y - 9x - 9y = 0$$

$$x - 8y = 0$$

$$x = 8y \text{ -------------------------- (ii)}$$

Putting this value in (i)

$$x^2 + y^2 = 65$$

$$(8y)^2 + y^2 = 65$$

$$64y^2 + y^2 = 65$$

$$65y^2 = 65$$

$$y^2 = 1$$

$$y = \pm 1$$

As the number is positive integral so $y = 1$, putting this value in (ii)

$$x = 8y$$

$$x = 8(1)$$

$$x = 8$$

So, the number $= 10x + y$

$= 10(8) + 1$

$= 81$

7. The sum of the co-ordinates of a point is 9 and sum of their squares is 45. Find the co-ordinates of the point.

Let the x co-ordinate $= x$

And the y co-ordinate $= y$

According to the first condition

$$x + y = 9$$

$$y = 9 - x \text{ ----------------- (i)}$$

According to the second condition

$$x^2 + y^2 = 45 \text{ ----------- (ii)}$$

Putting the value of y from (i) in (ii)

$$x^2 + y^2 = 45$$

$$x^2 + (9 - x)^2 = 45$$

$$x^2 + 81 - 18x + x^2 = 45$$

$$2x^2 - 18x + 36 = 0$$

$2x^2 - 12x - 6x + 36 = 0$

$2x(x - 6) - 6(x - 6) = 0$

$(x - 6)(2x - 6) = 0$

$x - 6 = 0$; $2x - 6 = 0$

$x = 6$; $x = 3$

Put in equation (i)

$y = 9 - x$; $y = 9 - x$

$y = 9 - 6$; $y = 9 - 3$

$y = 3$; $y = 6$

So, the point is $(3, 6)$ or $(6, 3)$

8. Find two integers whose sum is 9 and the difference of their squares is also 9.

Let 1st integer $= x$

And 2nd integer $= y$

According to the first condition

$x + y = 9$

$y = 9 - x$ ----------------- (i)

According to the second condition

$x^2 - y^2 = 9$ ----------- (ii)

Putting the value of y from (i) in (ii)

$x^2 - y^2 = 9$

$x^2 - (9 - x)^2 = 9$

$x^2 - (81 - 18x + x^2) = 9$

$x^2 - 81 + 18x - x^2 = 9$

$-81 + 18x = 9$

$18x = 90$

$x = 5$

Put in equation (i)

$y = 9 - x$

$y = 9 - 5$

$y = 4$

So, the integers are 5, 4

9. Find two integers whose difference is 4 and whose squares differ by 72.

Let 1st integer $= x$

And 2nd integer $= y$

According to the first condition

$x - y = 4$

$-y = 4 - x$

$y = x - 4$ ----------------- (i)

According to the second condition

$x^2 - y^2 = 72$ ----------- (ii)

Putting the value of y from (i) in (ii)

$x^2 - y^2 = 72$

$x^2 - (x-4)^2 = 72$

$x^2 - (x^2 - 8x + 16) = 72$

$x^2 - x^2 + 8x - 16 = 72$

$8x = 72 + 16$

$8x = 88$

$x = 11$

Put in equation (i)

$y = x - 4$

$y = 11 - 4$

$y = 7$

So, the integers are 11, 7

10. Find the dimensions of a rectangle, whose perimeter is 80cm and its area is 375 cm^2.

Let the 1st dimension of rectangle $= x$

And the 2nd dimension of rectangle $= y$

According to the first condition

$Perimeter = 80$

as we know the $Perimeter = 2(x+y)$ So,

$2(x+y) = 80$

$x + y = 40$

$y = 40 - x$ ---------------- (i)

According to the second condition

$Area = 375$

as we know the $Area = x \times y$ So,

$xy = 375$ --------------------- (ii)

Putting the value of y from (i) in (ii)

$x(40 - x) = 375$

$40x - x^2 = 375$

$0 = 375 - 40x + x^2$

$x^2 - 40x + 375 = 0$

$x^2 - 25x - 15x + 375 = 0$

$x(x - 25) - 15(x - 25) = 0$

$(x - 25)(x - 15) = 0$

$x - 25 = 0$; $x - 15 = 0$

$x = 25$; $x = 15$

Put in equation (i)

$y = 40 - x$; $y = 40 - x$

$y = 40 - 25$; $y = 40 - 15$

$y = 15$; $y = 25$

we have 25cm by 15cm or 15cm by 25cm as dimensions.

Exercise 3.1

1. Express the following as ratio a:b and as a fraction in its simplest (lowest) form.

(i) Rs. 750, Rs. 1250

$750 : 1250$

$\frac{750}{10} : \frac{1250}{10}$

$\frac{75}{5} : \frac{125}{5}$

$\frac{15}{5} : \frac{25}{5}$

$3 : 5 = \frac{3}{5}$

(ii) 450cm, 3m

450cm : 3x100 cm

$\frac{450}{10} : \frac{300}{10}$

$\frac{45}{5} : \frac{30}{5}$

$\frac{9}{3} : \frac{6}{3}$

$3 : 2 = \frac{3}{2}$

(iii) 4kg , 2kg 750gm

4x1000gm : 2x1000gm + 750gm

$\frac{4000}{10} : \frac{2750}{10}$

$\frac{400}{5} : \frac{275}{5}$

$\frac{80}{5} : \frac{55}{5}$

$16 : 11 = \frac{16}{11}$

(iv) 27min. 30 sec , 1 hour

27x60sec+30sec : 1x60x60 sec

$\frac{1650}{10} : \frac{3600}{10}$

$\frac{165}{5} : \frac{360}{5}$

$\frac{33}{3} : \frac{72}{3}$

$11 : 24 = \frac{11}{24}$

(v) 75° , 225°

$75^{\circ} : 225^{\circ}$

$\frac{75}{5} : \frac{225}{5}$

$\frac{15}{5} : \frac{45}{5}$

$\frac{3}{3} : \frac{9}{3}$

$1 : 3 = \frac{1}{3}$

Q. 2: In a class of 60 students, 25 students are girls and remaining students are boys. Compute the ratio of

Total students = 60

Girls = 25

Boys = 60 – 25 = 35

(i) Boys to total students

Boys	:	Total Students
35	:	60
$\frac{35}{5}$	:	$\frac{60}{5}$
7	:	12

(ii) Boys to girls

Boys	:	Girls
35	:	25
$\frac{35}{5}$	:	$\frac{25}{5}$
7	:	5

Q. 3: If $3(4x - 5y) = 2x - 7y$, find the ratio $x : y$.

$3(4x - 5y) = 2x - 7y$

$12x - 15y = 2x - 7y$

Dividing by y on both sides

$12\frac{x}{y} - 15\frac{y}{y} = 2\frac{x}{y} - 7\frac{y}{y}$

$12\frac{x}{y} - 15 = 2\frac{x}{y} - 7$

$12\frac{x}{y} - 2\frac{x}{y} = -7 + 15$

$10\frac{x}{y} = 8$

$\frac{x}{y} = \frac{8}{10}$

$\frac{x}{y} = \frac{4}{5}$

$x : y = 4 : 5$

Q. 4: Find the value of p, if the ratios $2p + 5 : 3p + 4$ and $3 : 4$ are equal.

$\frac{2p+5}{3p+4} = \frac{3}{4}$

$4(3p + 5) = 3(p + 4)$

$12p + 20 = 3p + 12$

$12p - 3p = 12 - 20$

$9p = -8$

$p = \frac{-8}{9}$

Q. 5: If the ratios $3x + 1 : 6 + 4x$ and $2 : 5$ are equal. Find the value of x.

$\frac{3x+1}{6+4x} = \frac{2}{5}$

$5(3x + 1) = 2(6 + 4x)$

$15x + 5 = 12 + 8x$

$15x - 8x = 12 - 5$

$7x = 7$

$x = 1$

Q. 6: Two numbers are in ratio $5 : 8$. If 9 is added to each number, we get a new ratio $8 : 11$. Find the numbers

Let the 1st No $= x$

and the 2nd No $= y$

According to first condition

$x : y = 5 : 8$

$\frac{x}{y} = \frac{5}{8}$ ------------------------ (i)

According to 2nd condition

$x + 9 : y + 9 = 8 : 11$

$\frac{x+9}{y+9} = \frac{8}{11}$

$11(x + 9) = 8(y + 9)$

$11x + 99 = 8y + 72$

$11x - 8y = 72 - 99$

$11x - 8y = -27$

Putting the value of x from equation (i)

$11\left(\frac{5}{8}y\right) - 8y = -27$

$\frac{55y}{8} - 8y = -27$

$\frac{55y - 64y}{8} = -27$

$\frac{-9y}{8} = -27$

$y = -27 \times \frac{8}{-9}$

$y = 24$

Putting the value of y in equation (i)

$\frac{x}{y} = \frac{5}{8}$

$\frac{x}{24} = \frac{5}{8}$

$x = \frac{5}{8} \times 24$

$x = 15$

So,

The 1st No $= x = 15$

The 2nd No $= y = 24$

Q. 7: If 10 is added in each number of the ratio $4 : 13$, we get $1 : 2$. What are the numbers?

Let the 1st No $= x$

and the 2nd No $= y$

According to first condition

$x : y = 4 : 13$

$\frac{x}{y} = \frac{4}{13}$ ----------------------- (i)

According to 2nd condition

$x + 10 : y + 10 = 1 : 2$

$\frac{x+10}{y+10} = \frac{1}{2}$

$2(x + 10) = 1(y + 10)$

$2x + 20 = y + 10$

$2x - y = 10 - 20$

$2x - y = -10$

Putting the value of x from equation (i)

$2\left(\frac{4}{13}y\right) - y = -10$

$\frac{8y}{13} - y = -10$

$\frac{8y-13y}{13} = -10$

$\frac{-5y}{13} = -10$

$y = -10 \times \frac{13}{-5}$

$y = 26$

Putting the value of y in equation (i)

$\frac{x}{y} = \frac{4}{13}$

$\frac{x}{26} = \frac{4}{13}$

$x = \frac{4}{13} \times 26$

$x = 8$

So,

The 1st No $= x = 8$

The 2nd No $= y = 26$

Q. 8: Find the cost of 8kg of mangoes, if 5kg of mangoes cost Rs. 250

let the cost of 8kg of mangoes be x-rupees

$8kg : 5kg :: Rs. x : Rs. 250$

$8kg : 5kg = Rs. x : Rs. 250$

Product of extremes = Product of means

$8 \times 250 = 5x$

$\frac{8 \times 250}{5} = x$

$x = Rs. 400$

Q. 9: If $a : b = 7 : 6$, find the value of $3a + 5b : 7b - 5a$

As given that $a : b = 7 : 6$ or

$\frac{a}{b} = \frac{7}{6}$

Now

$3a + 5b : 7b - 5a = \frac{3a+5b}{7b-5a}$

Dividing numerator and denominator by b

$$= \frac{\frac{3a+5b}{b}}{\frac{7b-5a}{b}}$$

$$= \frac{3\left(\frac{a}{b}\right)+5\left(\frac{b}{b}\right)}{7\left(\frac{b}{b}\right)-5\left(\frac{a}{b}\right)}$$

$$= \frac{3\left(\frac{a}{b}\right)+5}{7-5\left(\frac{a}{b}\right)}$$

As $\frac{a}{b} = \frac{7}{6}$ so,

$$= \frac{3\left(\frac{7}{6}\right)+5}{7-5\left(\frac{7}{6}\right)}$$

$$= \frac{\frac{7}{2}+5}{7-\frac{35}{6}}$$

$$= \frac{\frac{7+10}{2}}{\frac{42-35}{6}}$$

$$= \frac{\frac{17}{2}}{\frac{7}{6}}$$

$$= \frac{17}{2} \times \frac{6}{7}$$

$$= \frac{51}{7} = 51:7$$

Q. 10: Complete the following

(i) If $\frac{24}{7} = \frac{6}{x}, then\ 4x = 7$

(ii) If $\frac{5a}{3x} = \frac{15b}{y}, then\ ay = 9bx$

(iii) If $\frac{9pq}{2lm} = \frac{18p}{5m}, then\ 5q = 4l$

Q. 11: Find x in the following proportions.

(i) $3x - 2 : 4 \ :: \ 2x + 3 : \ 7$

Product of extremes = Product of means

$(3x-2)7 = 4(2x+3)$

$21x - 14 = 8x + 12$

$21x - 8x = 12 + 14$

$13x = 26$

$x = 2$

(ii) $\frac{3x-1}{7} : \frac{3}{5} \ :: \ \frac{2x}{3} : \frac{7}{5}$

Product of extremes = Product of means

$\left(\frac{3x-1}{7}\right)\frac{7}{5} = \frac{3}{5}\left(\frac{2x}{3}\right)$

$\frac{3x-1}{5} = \frac{2x}{5}$

$3x - 1 = 2x$

$x = 1$

(iii) $\frac{x-3}{2} : \frac{5}{x-1} \ :: \ \frac{x-1}{3} : \frac{4}{x+4}$

Product of extremes = Product of means

$\left(\frac{x-3}{2}\right)\frac{4}{x+4} = \frac{5}{x-1}\left(\frac{x-1}{3}\right)$

$\frac{2x-6}{x+4} = \frac{5}{3}$

$3(2x-6) = 5(x+4)$

$6x-18 = 5x+20$

$6x-5x = 20+18$

$x = 38$

(iv) $p^2+pq+q^2 : x :: \frac{p^3-q^3}{p+q} : (p-q)^2$

Product of extremes = Product of means

$(p^2+pq+q^2)(p-q)^2 = x \times \frac{p^3-q^3}{p+q}$

$(p^2+pq+q^2)(p-q)(p-q) = x \times \frac{p^3-q^3}{p+q}$

$(p^3-q^3)(p-q) = x \times \frac{p^3-q^3}{p+q}$

$(p^3-q^3)(p-q) \times \frac{p+q}{p^3-q^3} = x$

$x = (p-q)(p+q)$

$x = p^2-q^2$

(v) $8-x : 11-x :: 16-x : 25-x$

Product of extremes = Product of means

$(8-x)(25-x) = (11-x)(16-x)$

$200-8x-25x+x^2 = 176-11x-16x+x^2$

$200-33x+x^2 = 176-27x+x^2$

$-33x+x^2+27x-x^2 = 176-200$

$-6x = -24$

$x = 4$

Exercise 3.2

Q. 1: If y varies directly as x, and y = 8 when x=2, find

(i) y in terms of x

$$y \propto x$$
$$y = kx \text{ ---------- (i)}$$
$$8 = k \times 2$$
$$k = 4$$

So, equation (i) becomes

$$y = 4x$$

(ii) y when x = 5

$$y = 4x$$
$$y = 4 \times 5$$
$$y = 20$$

(iii) x when y = 28

$$y = 4x$$
$$28 = 4x$$
$$x = 7$$

Q. 2: If $y \propto x$, and y = 7 when x = 3 find

(i) y in terms of x

$$y \propto x$$
$$y = kx \text{ --------- (i)}$$
$$7 = k \times 3$$
$$k = \frac{7}{3}$$

So equation (i) becomes

$$y = \frac{7}{3} \times x$$

(ii) x when y = 35 and y when x = 18

when y = 35

$$y = \frac{7}{3} \times x$$
$$35 = \frac{7}{3} \times x$$
$$\frac{3}{7} \times 35 = x$$
$$15 = x$$
$$x = 15$$

put x = 18

$$y = kx$$
$$y = \frac{7}{3} \times 18$$
$$y = 42$$

Q. 3: If $R \propto T$, and R = 5 when T = 8, find the equation connection R and T. Also find R when T = 64 and T when R = 20.

$$R \propto T$$
$$R = kT$$
When R = 5 and T = 8
$$R = kT$$
$$5 = k \times 8$$
$$k = \frac{5}{8}$$
So,
$$R = \frac{5}{8}T$$
put T = 64
$$R = \frac{5}{8} \times 64$$
$$R = 40$$
when R = 20
$$20 = \frac{5}{8} \times T$$
$$\frac{8}{5} \times 20 = T$$
$$32 = T$$
$$T = 32$$

Q. 4: If $R \propto T^2$, and R = 8 when T = 3, find R when T = 6.
$$R \propto T^2$$
$$R = kT^2$$
When R = 8 and T = 3
$$R = kT^2$$
$$8 = k \times 3^2$$
$$8 = k \times 9$$
$$k = \frac{8}{9}$$
So,
$$R = \frac{8}{9}T^2$$
put T = 6
$$R = \frac{8}{9} \times 6^2$$
$$R = \frac{8}{9} \times 36$$
$$R = 32$$

Q. 5: If $V \propto R^3$, and V = 5 when R = 3, find R when V = 625.
$$V \propto R^3$$
$$V = kR^3$$
When V = 5 and R = 3
$$V = kR^3$$
$$5 = k \times 3^3$$
$$5 = k \times 27$$
$$k = \frac{5}{27}$$

So,

$$V = \frac{5}{27}R^3$$

put V = 625

$$625 = \frac{5}{27} \times R^3$$

$$R^3 = \frac{27}{5} \times 625$$

$$R^3 = 27 \times 125$$

Taking cube root on both sides

$$R = 3 \times 5$$

$$R = 15$$

Q. 6: If w varies directly as u^3 and w = 81 when u = 3. Find w when u = 5

$$w \propto u^3$$

$$w = ku^3$$

When w = 81 and u = 3

$$w = ku^3$$

$$81 = k \times 3^3$$

$$81 = k \times 27$$

$$k = \frac{81}{27}$$

$$k = 3$$

So,

$$w = 3u^3$$

put u = 5

$$w = 3 \times 5^3$$

$$w = 3 \times 125$$

$$w = 375$$

Q. 7: if y varies inversely as x and y = 7 when x = 2, find y when x = 126.

$$y \propto \frac{1}{x}$$

$$y = \frac{k}{x} \text{ ---------- (i)}$$

When y = 7 and x = 2

$$y = \frac{k}{x}$$

$$7 = \frac{k}{2}$$

$$14 = k$$

$$k = 14$$

So,

$$y = \frac{14}{x}$$

When x = 126

$$y = \frac{14}{126}$$

$$y = \frac{1}{9}$$

Q. 8: **if $y \propto \frac{1}{x}$ and y = 4 when x = 3, find x when y = 24.**

$$y \propto \frac{1}{x}$$
$$y = \frac{k}{x}$$

When y = 4 and x = 3

$$y = \frac{k}{x}$$
$$4 = \frac{k}{3}$$
$$12 = k$$
$$k = 12$$

So,

$$y = \frac{12}{x}$$

When y = 24

$$24 = \frac{12}{x}$$
$$x = \frac{12}{24}$$
$$x = \frac{1}{2}$$

Q. 9: **if $w \propto \frac{1}{z}$ and w = 5 when z = 7, find w when z = $\frac{175}{4}$.**

$$w \propto \frac{1}{z}$$
$$w = \frac{k}{z}$$

When w = 5 and z = 7

$$w = \frac{k}{z}$$
$$5 = \frac{k}{7}$$
$$35 = k$$
$$k = 35$$

So,

$$w = \frac{35}{z}$$

When z = $\frac{175}{4}$

$$w = \frac{35}{175/4}$$
$$w = 35 \times \frac{4}{175}$$
$$w = \frac{4}{5}$$

Q. 10: **if $A \propto \frac{1}{r^2}$ and A = 2 when r = 3, find r when A = 72.**

$$A \propto \frac{1}{r^2}$$
$$A = \frac{k}{r^2}$$

When A = 2 and r = 3

$$A = \frac{k}{r^2}$$

$$2 = \frac{k}{3^2}$$
$$2 = \frac{k}{9}$$
$$18 = k$$
$$k = 18$$

So,

$$A = \frac{18}{r^2}$$

When A = 72

$$A = \frac{k}{r^2}$$
$$72 = \frac{18}{r^2}$$
$$r^2 = \frac{18}{72}$$
$$r^2 = \frac{1}{4}$$
$$r = \pm\frac{1}{2}$$

Q. 11: if $a \propto \frac{1}{b^2}$ and a = 3 when b = 4, find a when b = 8.

$$a \propto \frac{1}{b^2}$$
$$a = \frac{k}{b^2}$$

When a = 3 and b = 4

$$a = \frac{k}{b^2}$$
$$3 = \frac{k}{4^2}$$
$$3 = \frac{k}{16}$$
$$48 = k$$
$$k = 48$$

So,

$$a = \frac{48}{b^2}$$

When b = 8

$$a = \frac{k}{b^2}$$
$$a = \frac{48}{8^2}$$
$$a = \frac{48}{64}$$
$$a = \frac{3}{4}$$

Q. 12: if $V \propto \frac{1}{r^3}$ and V = 5 when r = 3, find V when r = 6 and r when V = 320.

$$V \propto \frac{1}{r^3}$$
$$V = \frac{k}{r^3}$$

When V = 5 and r = 3

$$V = \frac{k}{r^3}$$

$$5 = \frac{k}{3^3}$$
$$5 = \frac{k}{27}$$
$$135 = k$$
$$k = 135$$

So,

$$V = \frac{135}{r^3}$$

When r = 6

$$V = \frac{135}{6^3}$$
$$V = \frac{135}{216}$$
$$V = \frac{5}{8}$$

When V = 320

$$320 = \frac{135}{r^3}$$
$$r^3 = \frac{135}{320}$$
$$r^3 = \frac{27}{64}$$
$$r = \frac{3}{4}$$

Q. 13: if $m \propto \frac{1}{n^3}$ and m = 2 when n = 4, find m when n = 6 and n when m = 432.

$$m \propto \frac{1}{n^3}$$
$$m = \frac{k}{n^3}$$

When m = 2 and n = 4

$$m = \frac{k}{n^3}$$
$$2 = \frac{k}{4^3}$$
$$2 = \frac{k}{64}$$
$$128 = k$$
$$k = 128$$

So,

$$m = \frac{128}{n^3}$$

When n = 6

$$m = \frac{128}{6^3}$$
$$m = \frac{128}{216}$$
$$m = \frac{16}{27}$$

When m = 432

$$m = \frac{k}{n^3}$$
$$432 = \frac{128}{n^3}$$

$n^3 = \frac{128}{432}$

$n^3 = \frac{8}{27}$

$n = \frac{2}{3}$

Exercise 3.3

Q. 1: Find a third proportional to

(i) 6, 12

Let x be the third proportional,

$$6 : 12 :: 12 : x$$

Product of Extremes = Product of Means

$$6x = 12 \times 12$$

$$x = \frac{12 \times 12}{6}$$

$$x = 24$$

(ii) $a^3, 3a^2$

Let x be the third proportional,

$$a^3 : 3a^2 :: 3a^2 : x$$

Product of Extremes = Product of Means

$$a^3 \times x = 3a^2 \times 3a^2$$

$$x = \frac{3a^2 \times 3a^2}{a^3}$$

$$x = 9a$$

(iii) $a^2 - b^2, a - b$

Let x be the third proportional,

$$a^2 - b^2 : a - b :: a - b : x$$

Product of Extremes = Product of Means

$$a^2 - b^2 \times x = a - b \times a - b$$

$$x = \frac{a-b \times a-b}{a^2-b^2}$$

$$x = \frac{a-b}{a+b}$$

(iv) $(x-y)^2, x^3 - y^3$

Let c be the third proportional,

$$(x-y)^2 : x^3 - y^3 :: x^3 - y^3 : c$$

Product of Extremes = Product of Means

$$(x-y)^2 \times c = x^3 - y^3 \times x^3 - y^3$$

$$c = \frac{x^3-y^3 \times x^3-y^3}{(x-y)^2}$$

$$c = (x^2 + xy + y^2)(x^2 + xy + y^2)$$

(v) $(x+y)^2, x^2 - xy - 2y^2$

Let c be the third proportional,

$$(x+y)^2 : x^2 - xy - 2y^2 :: x^2 - xy - 2y^2 : c$$

Product of Extremes = Product of Means

$$(x+y)^2 \times c = x^2 - xy - 2y^2 \times x^2 - xy - 2y^2$$

$$c = \frac{(x^2-xy-2y^2)(x^2-xy-2y^2)}{(x+y)^2}$$

$$c = \frac{(x^2-2xy+xy-2y^2)(x^2-2xy+xy-2y^2)}{(x+y)^2}$$

$$= \frac{\big(x(x-2y)+y(x-2y)\big)\big(x(x-2y)+y(x-2y)\big)}{(x+y)^2}$$

$$= \frac{\big((x-2y)(x+y)\big)\big((x-2y)(x+y)\big)}{(x+y)^2}$$

$$= \frac{(x-2y)(x+y)(x-2y)(x+y)}{(x+y)^2}$$

$$= (x-2y)(x-2y)$$

(vi) $\frac{p^2-q^2}{p^3+q^3}, \frac{p-q}{p^2-pq+q^2}$

Let c be the third proportional,

$$\frac{p^2-q^2}{p^3+q^3} : \frac{p-q}{p^2-pq+q^2} :: \frac{p-q}{p^2-pq+q^2} : c$$

Product of Extremes = Product of Means

$$\frac{p^2-q^2}{p^3+q^3} \times c = \frac{p-q}{p^2-pq+q^2} \times \frac{p-q}{p^2-pq+q^2}$$

$$c = \frac{p-q}{p^2-pq+q^2} \times \frac{p-q}{p^2-pq+q^2} \times \frac{p^3+q^3}{p^2-q^2}$$

$$= \frac{p-q}{p^2-pq+q^2} \times \frac{p-q}{p^2-pq+q^2} \times \frac{(p+q)(p^2-pq+q^2)}{(p-q)(p+q)}$$

$$= \frac{p-q}{p^2-pq+q^2}$$

Q. 2: Find a fourth proportional to

(i) 5, 8, 15

Let x be the fourth proportional,

$$5 : 8 :: 15 : x$$

Product of Extremes = Product of Means

$$5x = 8 \times 15$$

$$x = \frac{8 \times 15}{5}$$

$$x = 24$$

(ii) $4x^4, 2x^3, 18x^5$

Let c be the fourth proportional,

$$4x^4 : 2x^3 :: 18x^5 : c$$

Product of Extremes = Product of Means

$$4x^4 \times c = 2x^3 \times 18x^5$$

$$c = \frac{2x^3 \times 18x^5}{4x^4}$$

$$c = \frac{9x^8}{x^4}$$

$$c = 9x^4$$

(iii) $15a^5b^6, 10a^2b^5, 21a^3b^3$

Let c be the fourth proportional,

$$15a^5b^6 : 10a^2b^5 :: 21a^3b^3 : c$$

Product of Extremes = Product of Means

$$15a^5b^6 \times c = 10a^2b^5 \times 21a^3b^3$$

$c = \frac{10a^2b^5 \times 21a^3b^3}{15a^5b^6}$

$c = \frac{2a^2b^5 \times 7a^3b^3}{a^5b^6}$

$c = 14a^{2+3-5}b^{5+3-6}$

$c = 14b^2$

(iv) $x^2 - 11x + 24, x - 3, 5x^4 - 40x^3$

Let c be the fourth proportional,

$x^2 - 11x + 24 : x - 3 :: 5x^4 - 40x^3 : c$

Product of Extremes = Product of Means

$x^2 - 11x + 24 \times c = x - 3 \times 5x^4 - 40x^3$

$c = \frac{x-3 \times 5x^3(x-8)}{x^2-8x-3x+24}$

$c = \frac{x-3 \times 5x^3(x-8)}{x(x-8)-3(x-8)}$

$c = \frac{x-3 \times 5x^3(x-8)}{(x-8)(x-3)}$

$c = 5x^3$

(v) $p^3 + q^3, p^2 - q^2, p^2 - pq + q^2$

Let c be the fourth proportional,

$p^3 + q^3 : p^2 - q^2 :: p^2 - pq + q^2 : c$

Product of Extremes = Product of Means

$p^3 + q^3 \times c = p^2 - q^2 \times p^2 - pq + q^2$

$c = \frac{(p^2-q^2)(p^2-pq+q^2)}{p^3+q^3}$

$c = \frac{(p-q)(p+q)(p^2-pq+q^2)}{p^3+q^3}$

$c = \frac{(p-q)(p^3+q^3)}{p^3+q^3}$

$c = p - q$

(vi) $(p^2 - q^2)(p^2 + pq + q^2), p^3 + q^3, p^3 - q^3$

Let c be the fourth proportional,

$(p^2 - q^2)(p^2 + pq + q^2) : p^3 + q^3 :: p^3 - q^3 : c$

Product of Extremes = Product of Means

$(p^2 - q^2)(p^2 + pq + q^2) \times c = p^3 + q^3 \times p^3 - q^3$

$c = \frac{(p^3+q^3)(p^3-q^3)}{(p^2-q^2)(p^2+pq+q^2)}$

$c = \frac{(p+q)(p^2-pq+q^2)(p-q)(p^2+pq+q^2)}{(p+q)(p-q)(p^2+pq+q^2)}$

$c = p^2 - pq + q^2$

Q. 3: Find a mean proportional between

(i) 20, 45

Let x be the mean proportional,

$20 : x :: x : 45$

Product of means = Product of extremes

$x^2 = 20 \times 45$

$x^2 = 900$

$x = \pm 30$

(ii) $20x^3y^5, 5x^7y$

Let c be the mean proportional,

$20x^3y^5 : c :: c : 5x^7y$

Product of means = Product of extremes

$c^2 = 20x^3y^5 \times 5x^7y$

$c^2 = 100x^{10}y^6$

$c = \pm 10x^5y^3$

(iii) $15p^4qr^3, 135q^5r^7$

Let c be the mean proportional,

$15p^4qr^3 : c :: c : 135q^5r^7$

Product of means = Product of extremes

$c^2 = 15p^4qr^3 \times 135q^5r^7$

$c^2 = 2025p^4q^6r^{10}$

$c = \pm 45p^2q^3r^5$

(iv) $x^2 - y^2, \frac{x-y}{x+y}$

Let c be the mean proportional,

$x^2 - y^2 : c :: c : \frac{x-y}{x+y}$

Product of Extremes = Product of Means

$c^2 = x^2 - y^2 \times \frac{x-y}{x+y}$

$c^2 = \frac{(x-y)(x+y)(x-y)}{(x+y)}$

$c^2 = (x-y)^2$

$c = \pm(x-y)$

Q. 4: Find the values of the letter involved in the following continued proportions.

(i) 5, p, 45

$5 : p :: p : 45$

Product of means = Product of extremes

$p^2 = 5 \times 45$

$p^2 = 225$

$p = \pm 15$

(ii) 8, x, 18

$8 : x :: x : 18$

Product of means = Product of extremes

$x^2 = 8 \times 18$

$x^2 = 144$

$x = \pm 12$

(iii) $12,\ 3p-6,\ 27$

Let c be the mean proportional,

$12 : 3p-6 :: 3p-6 : 27$

Product of means = Product of extremes

$(3p-6)^2 = 12 \times 27$

$(3p-6)^2 = 324$

$3p-6 = \pm 18$

$3p-6 = 18$; $3p-6 = -18$

$3p = 24$; $3p = -12$

$p = 8$; $p = -4$

(iv) $7,\ m-3,\ 28$

Let c be the mean proportional,

$7 : m-3 :: m-3 : 28$

Product of means = Product of extremes

$(m-3)^2 = 7 \times 28$

$(m-3)^2 = 196$

$m-3 = \pm 14$

$m-3 = 14$; $m-3 = -14$

$m = 17$; $m = -11$

Exercise 3.4

Q. 1: Prove that a : b = c : d, if

(i) $\frac{4a+5b}{4a-5b} = \frac{4c+5d}{4c-5d}$

By componendo-dividendo

$$\frac{(4a+5b)+(4a-5b)}{(4a+5b)-(4a-5b)} = \frac{(4c+5d)+(4c-5d)}{(4c+5d)-(4c-5d)}$$

$$\frac{4a+5b+4a-5b}{4a+5b-4a+5b} = \frac{4c+5d+4c-5d}{4c+5d-4c+5d}$$

$$\frac{8a}{10b} = \frac{8c}{10d}$$

Multiplying by $\frac{10}{8}$

$$\frac{a}{b} = \frac{c}{d}$$

$$a : b = c : d$$

(ii) $\frac{2a+9b}{2a-9b} = \frac{2c+9d}{2c-9d}$

By componendo-dividendo

$$\frac{(2a+9b)+(2a-9b)}{(2a+9b)-(2a-9b)} = \frac{(2c+9d)+(2c-9d)}{(2c+9d)-(2c-9d)}$$

$$\frac{2a+9b+2a-9b}{2a+9b-2a+9b} = \frac{2c+9d+2c-9d}{2c+9d-2c+9d}$$

$$\frac{4a}{18b} = \frac{4c}{18d}$$

Multiplying by $\frac{18}{4}$

$$\frac{a}{b} = \frac{c}{d}$$

$$a : b = c : d$$

(iii) $\frac{ac^2+bd^2}{ac^2-bd^2} = \frac{c^3+d^3}{c^3-d^3}$

By componendo-dividendo

$$\frac{(ac^2+bd^2)+(ac^2-bd^2)}{(ac^2+bd^2)-(ac^2-bd^2)} = \frac{(c^3+d^3)+(c^3-d^3)}{(c^3+d^3)-(c^3-d^3)}$$

$$\frac{ac^2+bd^2+ac^2-bd^2}{ac^2+bd^2-ac^2+bd^2} = \frac{c^3+d^3+c^3-d^3}{c^3+d^3-c^3+d^3}$$

$$\frac{2ac^2}{2bd^2} = \frac{2c^3}{2d^3}$$

$$\frac{ac^2}{bd^2} = \frac{c^3}{d^3}$$

Multiplying by $\frac{d^2}{c^2}$

$$\frac{a}{b} = \frac{c}{d}$$

$$a : b = c : d$$

(iv) $\frac{a^2c+b^2d}{a^2c-b^2d} = \frac{ac^2+bd^2}{ac^2-bd^2}$

By componendo-dividendo

$$\frac{(a^2c+b^2d)+(a^2c-b^2d)}{(a^2c+b^2d)-(a^2c-b^2d)} = \frac{(ac^2+bd^2)+(ac^2-bd^2)}{(ac^2+bd^2)-(ac^2-bd^2)}$$

$$\frac{a^2c+b^2d+a^2c-b^2d}{a^2c+b^2d-a^2c+b^2d} = \frac{ac^2+bd^2+ac^2-bd^2}{ac^2+bd^2-ac^2+bd^2}$$

$$\frac{2a^2c}{2b^2d} = \frac{2ac^2}{2bd^2}$$

$$\frac{a^2c}{b^2d} = \frac{ac^2}{bd^2}$$

Multiplying by $\frac{bd}{ac}$

$$\frac{a}{b} = \frac{c}{d}$$

$$a : b = c : d$$

(v) $\frac{pa+qb}{pa-qb} = \frac{pc+qd}{pc-qd}$

By componendo-dividendo

$$\frac{(pa+qb)+(pa-qb)}{(pa+qb)-(pa-qb)} = \frac{(pc+qd)+(pc-qd)}{(pc+qd)-(pc-qd)}$$

$$\frac{pa+qb+pa-qb}{pa+qb-pa+qb} = \frac{pc+qd+pc-qd}{pc+qd-pc+qd}$$

$$\frac{2pa}{2qb} = \frac{2pc}{2qd}$$

$$\frac{pa}{qb} = \frac{pc}{qd}$$

Multiplying by $\frac{q}{p}$

$$\frac{a}{b} = \frac{c}{d}$$

$$a : b = c : d$$

(vi) $\frac{a+b+c+d}{a+b-c-d} = \frac{a-b+c-d}{a-b-c+d}$

By componendo-dividendo

$$\frac{(a+b+c+d)+(a+b-c-d)}{(a+b+c+d)-(a+b-c-d)} = \frac{(a-b+c-d)+(a-b-c+d)}{(a-b+c-d)-(a-b-c+d)}$$

$$\frac{a+b+c+d+a+b-c-d}{a+b+c+d-a-b+c+d} = \frac{a-b+c-d+a-b-c+d}{a-b+c-d-a+b+c-d}$$

$$\frac{2a+2b}{2c+2d} = \frac{2a-2b}{2c-2d}$$

$$\frac{a+b}{c+d} = \frac{a-b}{c-d}$$

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}$$

By componendo-dividendo

$$\frac{(a+b)+(a-b)}{(a+b)-(a-b)} = \frac{(c+d)+(c-d)}{(c+d)-(c-d)}$$

$$\frac{a+b+a-b}{a+b-a+b} = \frac{c+d+c-d}{c+d-c+d}$$

$$\frac{2a}{2b} = \frac{2c}{2d}$$

$$\frac{a}{b} = \frac{c}{d}$$

$$a : b = c : d$$

(vii) $\frac{2a+3b+2c+3d}{2a+3b-2c-3d} = \frac{2a-3b+2c-3d}{2a-3b-2c+3d}$

By componendo-dividendo

$$\frac{(2a+3b+2c+3d)+(2a+3b-2c-3d)}{(2a+3b+2c+3d)-(2a+3b-2c-3d)} = \frac{(2a-3b+2c-3d)+(2a-3b-2c+3d)}{(2a-3b+2c-3d)-(2a-3b-2c+3d)}$$

$$\frac{2a+3b+2c+3d+2a+3b-2c-3d}{2a+3b+2c+3d-2a-3b+2c+3d} = \frac{2a-3b+2c-3d+2a-3b-2c+3d}{2a-3b+2c-3d-2a+3b+2c-3d}$$

$$\frac{4a+6b}{4c+6d} = \frac{4a-6b}{4c-6d}$$

$$\frac{4a+6b}{4a-6b} = \frac{4c+6d}{4c-6d}$$

By componendo-dividendo

$$\frac{(4a+6b)+(4a-6b)}{(4a+6b)-(4a-6b)} = \frac{(4c+6d)+(4c-6d)}{(4c+6d)-(4c-6d)}$$

$$\frac{4a+6b+4a-6b}{4a+6b-4a+6b} = \frac{4c+6d+4c-6d}{4c+6d-4c+6d}$$

$$\frac{8a}{12b} = \frac{8c}{12d}$$

$$\frac{2a}{3b} = \frac{2c}{3d}$$

Multiplying by $\frac{3}{2}$

$$\frac{a}{b} = \frac{c}{d}$$

$$a : b = c : d$$

(viii) $\frac{a^2+b^2}{a^2-b^2} = \frac{ac+bd}{ac-bd}$

By componendo-dividendo

$$\frac{(a^2+b^2)+(a^2-b^2)}{(a^2+b^2)-(a^2-b^2)} = \frac{(ac+bd)+(ac-bd)}{(ac+bd)-(ac-bd)}$$

$$\frac{a^2+b^2+a^2-b^2}{a^2+b^2-a^2+b^2} = \frac{ac+bd+ac-bd}{ac+bd-ac+bd}$$

$$\frac{2a^2}{2b^2} = \frac{2ac}{2bd}$$

$$\frac{a^2}{b^2} = \frac{ac}{bd}$$

Multiplying by $\frac{b}{a}$

$$\frac{a}{b} = \frac{c}{d}$$

$$a : b = c : d$$

Q. 2: Using theorem of componendo-dividendo

(i) Find the value of $\frac{x+2y}{x-2y} + \frac{x+2z}{x-2z}$, if $x = \frac{4yz}{y+z}$

$$x = \frac{4yz}{y+z} \text{ ------------ (i)}$$

From equation (i)

$$x = \frac{2y \times 2z}{y+z}$$

$$\frac{x}{2y} = \frac{2z}{y+z}$$

By applying componendo-dividendo theorem

$$\frac{x+2y}{x-2y} = \frac{2z+y+z}{2z-y-z}$$

$$\frac{x+2y}{x-2y} = \frac{y+3z}{z-y} \text{ ----------- (ii)}$$

From equation (i)

$$x = \frac{2y \times 2z}{y+z}$$

$$\frac{x}{2z} = \frac{2y}{y+z}$$

By applying componendo-dividendo theorem

$\frac{x+2z}{x-2z} = \frac{2y+y+z}{2y-y-z}$

$\frac{x+2z}{x-2z} = \frac{z+3y}{y-z}$ ----------- (iii)

Adding equation (ii) and (iii)

$\frac{x+2y}{x-2y} + \frac{x+2z}{x-2z} = \frac{y+3z}{z-y} + \frac{z+3y}{y-z}$

$= -\frac{y+3z}{y-z} + \frac{z+3y}{y-z}$

$= \frac{z+3y}{y-z} - \frac{y+3z}{y-z}$

$= \frac{z+3y-y-3z}{y-z}$

$= \frac{2y-2z}{y-z}$

$= \frac{2(y-z)}{y-z}$

$= 2$

(ii) Find the value of $\frac{m+5n}{m-5n} + \frac{m+5p}{m-5p}$, if $m = \frac{10np}{n+p}$

$m = \frac{10np}{n+p}$ ------------ (i)

From equation (i)

$m = \frac{5n \times 2p}{n+p}$

$\frac{m}{5n} = \frac{2p}{n+p}$

By applying componendo-dividendo theorem

$\frac{m+5n}{m-5n} = \frac{2p+n+p}{2p-n-p}$

$\frac{m+5n}{m-5n} = \frac{3p+n}{p-n}$ ----------- (ii)

From equation (i)

$m = \frac{2n \times 5p}{n+p}$

$\frac{m}{5p} = \frac{2n}{n+p}$

By applying componendo-dividendo theorem

$\frac{m+5p}{m-5p} = \frac{2n+n+p}{2n-n-p}$

$\frac{m+5p}{m-5p} = \frac{3n+p}{n-p}$ ----------- (iii)

Adding equation (ii) and (iii)

$\frac{m+5n}{m-5n} + \frac{m+5p}{m-5p} = \frac{3p+n}{p-n} + \frac{3n+p}{n-p}$

$= -\frac{3p+n}{n-p} + \frac{3n+p}{n-p}$

$= \frac{3n+p}{n-p} - \frac{3p+n}{n-p}$

$= \frac{3n+p-3p-n}{n-p}$

$= \frac{2n-2p}{n-p}$

$= \frac{2(n-p)}{n-p}$

$= 2$

(iii) Find the value of $\frac{x-6a}{x+6a} - \frac{x+6b}{x-6b}$, if $x = \frac{12ab}{a-b}$

$x = \frac{12ab}{a-b}$ ------------ (i)

From equation (i)

$x = \frac{6a \times 2b}{a-b}$

$\frac{x}{6a} = \frac{2b}{a-b}$

By applying componendo-dividendo theorem

$\frac{x+6a}{x-6a} = \frac{2b+a-b}{2b-a+b}$

$\frac{x+6a}{x-6a} = \frac{a+b}{3b-a}$

$\frac{x-6a}{x+6a} = \frac{3b-a}{a+b}$ ------------ (ii)

From equation (i)

$x = \frac{6b \times 2a}{a-b}$

$\frac{x}{6b} = \frac{2a}{a-b}$

By applying componendo-dividendo theorem

$\frac{x+6b}{x-6b} = \frac{2a+a-b}{2a-a+b}$

$\frac{x+6b}{x-6b} = \frac{3a-b}{a+b}$

$\frac{x+6b}{x-6b} = \frac{3a-b}{a+b}$ ----------- (iii)

Subtracting equation (iii) from (ii)

$\frac{x-6a}{x+6a} - \frac{x+6b}{x-6b} = \frac{3b-a}{a+b} - \frac{3a-b}{a+b}$

$= \frac{3b-a}{a+b} - \frac{3a-b}{a+b}$

$= \frac{-4a+4b}{a+b}$

$= \frac{4(b-a)}{a+b}$

(iv) Find the value of $\frac{x-3y}{x+3y} - \frac{x+3z}{x-3z}$, if $x = \frac{3yz}{y-z}$

$x = \frac{3yz}{x-y}$ ------------ (i)

From equation (i)

$x = \frac{3y \times z}{y-z}$

$\frac{x}{3y} = \frac{z}{y-z}$

By applying componendo-dividendo theorem

$\frac{x+3y}{x-3y} = \frac{z+y-z}{z-y+z}$

$\frac{x+3y}{x-3y} = \frac{y}{2z-y}$

$\frac{x-3y}{x+3y} = \frac{2z-y}{y}$ ----------- (ii)

From equation (i)

$$x = \frac{3z \times y}{y-z}$$

$$\frac{x}{3z} = \frac{y}{y-z}$$

By applying componendo-dividendo theorem

$$\frac{x+3z}{x-3z} = \frac{y+y-z}{y-y+z}$$

$$\frac{x+3z}{x-3z} = \frac{2y-z}{z}$$

$$\frac{x+3z}{x-3z} = \frac{2y-z}{z} \text{ ----------- (iii)}$$

Subtracting equation (iii) from (ii)

$$\frac{x-3y}{x+3y} - \frac{x+3z}{x-3z} = \frac{2z-y}{y} - \frac{2y-z}{z}$$

$$= \frac{z(2z-y)-y(2y-z)}{yz}$$

$$= \frac{2z^2-yz-2y^2+yz}{yz}$$

$$= \frac{2(z^2-y^2)}{yz}$$

(v) Find the value of $\frac{s-3p}{s+3p} + \frac{s+3q}{s-3q}$, if $s = \frac{6pq}{p-q}$

$$s = \frac{6pq}{p-q} \text{ ------------ (i)}$$

From equation (i)

$$s = \frac{3p \times 2q}{p-q}$$

$$\frac{s}{3p} = \frac{2q}{p-q}$$

By applying componendo-dividendo theorem

$$\frac{s+3p}{s-3p} = \frac{2q+p-q}{2q-p+q}$$

$$\frac{s+3p}{s-3p} = \frac{p+q}{3q-p}$$

$$\frac{s-3p}{s+3p} = \frac{3q-p}{p+q} \text{ ----------- (ii)}$$

From equation (i)

$$s = \frac{2p \times 3q}{p-q}$$

$$\frac{s}{3q} = \frac{2p}{p-q}$$

By applying componendo-dividendo theorem

$$\frac{s+3q}{s-3q} = \frac{2p+p-q}{2p-p+q}$$

$$\frac{s+3q}{s-3q} = \frac{3p-q}{p+q}$$

$$\frac{s+3q}{s-3q} = \frac{3p-q}{p+q} \text{ ----------- (iii)}$$

Adding equation (ii) and (iii)

$$\frac{s-3p}{s+3p}+\frac{s+3q}{s-3q} = \frac{3q-p}{p+q}+\frac{3p-q}{p+q}$$
$$= \frac{3q-p+3p-q}{p+q}$$
$$= \frac{2p+2q}{p+q}$$
$$= \frac{2(p+q)}{p+q}$$
$$= 2$$

(vi) Solve $\frac{(x-2)^2-(x-4)^2}{(x-2)^2+(x-4)^2} = \frac{12}{13}$

$$\frac{(x-2)^2-(x-4)^2}{(x-2)^2+(x-4)^2} = \frac{12}{13}$$
$$\frac{(x-2)^2-(x-4)^2}{(x-2)^2+(x-4)^2} = \frac{12}{13}$$

By componendo-dividendo theorem

$$\frac{(x-2)^2-(x-4)^2+(x-2)^2+(x-4)^2}{(x-2)^2-(x-4)^2-(x-2)^2-(x-4)^2} = \frac{12+13}{12-13}$$
$$\frac{2(x-2)^2}{-2(x-4)^2} = \frac{25}{-1}$$
$$\frac{(x-2)^2}{(x-4)^2} = 25$$

Taking root on b.s

$$\frac{x-2}{x-4} = \pm 5$$

$\frac{x-2}{x-4} = 5$; $\frac{x-2}{x-4} = -5$

$x-2 = 5(x-4)$; $x-2 = -5(x-4)$

$x-2 = 5x-20$; $x-2 = -5x+20$

$x-5x = 2-20$; $x+5x = 2+20$

$-4x = -18$; $6x = 22$

$x = \frac{-18}{-4}$; $x = \frac{22}{6}$

$x = \frac{9}{2}$; $x = \frac{11}{3}$

S.S = $\left\{\frac{9}{2}, \frac{11}{3}\right\}$

(vii) Solve $\frac{\sqrt{x^2+2}+\sqrt{x^2-2}}{\sqrt{x^2+2}-\sqrt{x^2-2}} = 2$

$$\frac{\sqrt{x^2+2}+\sqrt{x^2-2}}{\sqrt{x^2+2}-\sqrt{x^2-2}} = 2$$

By componendo-dividendo theorem

$$\frac{\sqrt{x^2+2}+\sqrt{x^2-2}+\sqrt{x^2+2}-\sqrt{x^2-2}}{\sqrt{x^2+2}+\sqrt{x^2-2}-\sqrt{x^2+2}+\sqrt{x^2-2}} = \frac{2+1}{2-1}$$
$$\frac{2\sqrt{x^2+2}}{2\sqrt{x^2-2}} = \frac{3}{1}$$
$$\frac{\sqrt{x^2+2}}{\sqrt{x^2-2}} = 3$$

Taking square on b.s

$$\frac{x^2+2}{x^2-2} = 9$$
$$x^2+2 = 9(x^2-2)$$

$$x^2 + 2 = 9x^2 - 18$$
$$x^2 - 9x^2 = -18 - 2$$
$$-8x^2 = -20$$
$$x^2 = \frac{-20}{-8}$$
$$x^2 = \frac{5}{2}$$
$$x = \pm\sqrt{\frac{5}{2}}$$

If we check the given equation for this value the value doesn't satisfy the equation so the given solution is extraneous. So, s.s = { }

(viii) Solve $\frac{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}}{\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}} = \frac{1}{3}$

$$\frac{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}}{\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}} = \frac{1}{3}$$

By componendo-dividendo theorem

$$\frac{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}+\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}}{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}-\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}} = \frac{1+3}{1-3}$$
$$\frac{2\sqrt{x^2+8p^2}}{-2\sqrt{x^2-p^2}} = \frac{4}{-2}$$
$$\frac{\sqrt{x^2+8p^2}}{\sqrt{x^2-p^2}} = 2$$

Taking square on b.s

$$\frac{x^2+8p^2}{x^2-p^2} = 4$$
$$x^2 + 8p^2 = 4(x^2 - p^2)$$
$$x^2 + 8p^2 = 4x^2 - 4p^2$$
$$x^2 - 4x^2 = -4p^2 - 8p^2$$
$$-3x^2 = -12p^2$$
$$x^2 = 4p^2$$
$$x = \pm 2p$$

S.S = $\{2p, -2p\}$

(ix) Solve $\frac{(x+5)^3-(x-3)^3}{(x+5)^3+(x-3)^3} = \frac{13}{14}$

$$\frac{(x+5)^3-(x-3)^3}{(x+5)^3+(x-3)^3} = \frac{13}{14}$$

By componendo-dividendo theorem

$$\frac{(x+5)^3-(x-3)^3+(x+5)^3+(x-3)^3}{(x+5)^3-(x-3)^3-(x+5)^3-(x-3)^3} = \frac{13+14}{13-14}$$
$$\frac{2(x+5)^3}{-2(x-3)^3} = \frac{27}{-1}$$
$$\frac{(x+5)^3}{(x-3)^3} = 27$$

Taking cube root on b.s

$$\frac{x+5}{x-3} = 3$$
$$x + 5 = 3(x - 3)$$

$$\begin{aligned} x+5 &= 3x-9 \\ x-3x &= -9-5 \\ -2x &= -14 \\ x &= 7 \end{aligned}$$

S.S = {7}

Exercise 3.5

Q. 1: If s varies directly as u^2 and inversely as v and s = 7 when u = 3, v = 2. Find the value of s when u = 6 and v = 10.

$$s \propto u^2$$
$$s \propto \frac{1}{v}$$
$$s = k\frac{u^2}{v} \text{ ---------- (i)}$$

Put s = 7, u = 3, v = 2

$$7 = k\frac{3^2}{2}$$
$$7 = k\frac{9}{2}$$
$$k = \frac{14}{9}$$

So, equation (i) becomes

$$s = \frac{14u^2}{9v} \text{ -------- (ii)}$$

Put u = 6 and v = 10 in equation (ii)

$$s = \frac{14(6)^2}{9(10)}$$
$$s = \frac{14(36)}{9(10)}$$
$$s = \frac{28}{5}$$

Q. 2: If w varies jointly as x, y^2 and z and w = 5 when x =2, y = 3, z = 10. Find w when x = 4, y = 7 and z = 3.

$$w \propto x$$
$$w \propto y^2$$
$$w \propto z$$
$$w = kxy^2z \text{ ---------- (i)}$$

Put w = 5, x = 2, y = 3, z = 10

$$5 = k(2)(3)^2(10)$$
$$5 = k(180)$$
$$k = \frac{5}{180}$$
$$k = \frac{1}{36}$$

So, equation (i) becomes

$$w = \frac{xy^2z}{36} \text{ -------- (ii)}$$

Put x = 4, y = 7 and z = 3 in equation (ii)

$$s = \frac{4(7)^2(3)}{36}$$
$$s = \frac{49}{3}$$

Q. 3: If y varies directly as x^3 and inversely as z^2 and t, and y = 16 when x = 4, z = 2, t = 3. Find the value of y when x = 2, z = 3 and t = 4.

$$y \propto x^3$$

$$y \propto \frac{1}{z^2}$$

$$y \propto \frac{1}{t}$$

$$y = k\frac{x^3}{z^2 t} \text{ ---------- (i)}$$

Put y=16, x = 4, z = 2, t = 3

$$16 = k\frac{4^3}{2^2.3}$$

$$16 = k\frac{64}{12}$$

$$k = \frac{12}{64} \times 16$$

$$k = 3$$

So, equation (i) becomes

$$y = \frac{3x^3}{z^2 t} \text{ -------- (ii)}$$

Put x = 2, z = 3 and t = 4 in equation (ii)

$$y = \frac{3(2)^3}{(3)^2(4)}$$

$$y = \frac{3(8)}{9(4)}$$

$$y = \frac{2}{3}$$

Q. 4: If u varies directly as x^2 and inversely as the product yz^3, and u = 2 when x = 8, y = 7, z = 2. Find the value of u when x = 6, y = 3, z = 2.

$$u \propto x^2$$

$$u \propto \frac{1}{yz^3}$$

$$u = k\frac{x^2}{yz^3} \text{ ---------- (i)}$$

Put u = 2, x = 8, y = 7, z = 2

$$2 = k\frac{8^2}{7.\ 2^3}$$

$$2 = k\frac{64}{56}$$

$$k = \frac{7}{8} \times 2$$

$$k = \frac{7}{4}$$

So, equation (i) becomes

$$u = \frac{7x^2}{4yz^3} \text{ -------- (ii)}$$

Put x = 6, y = 3 and z = 2 in equation (ii)

$$u = \frac{7(6)^2}{4(3)(2)^3}$$

$$y = \frac{7(36)}{12(8)}$$

$$y = \frac{21}{8}$$

Q. 5: If v varies directly as the product xy^3 and inversely as z^2 and v = 27 when x = 7, y = 6, z = 7. Find the value of v when x = 6, y = 2, z = 3.

$$v \propto xy^3$$

$$v \propto \frac{1}{z^2}$$

$$u = k\frac{xy^3}{z^2} \text{ ---------- (i)}$$

Put v = 27, x = 7, y = 6, z = 7

$$27 = k\frac{(7)(6)^3}{7^2}$$

$$27 = k\frac{216}{7}$$

$$k = \frac{7}{216} \times 27$$

$$k = \frac{7}{8}$$

So, equation (i) becomes

$$u = \frac{7xy^3}{8z^2} \text{ -------- (ii)}$$

Put x = 6, y = 2 and z = 3 in equation (ii)

$$u = \frac{7(6)(2)^3}{8(3)^2}$$

$$y = \frac{7(6)(8)}{8(9)}$$

$$y = \frac{14}{3}$$

Q. 6: If w varies inversely as the cube of u, and w = 5 when u = 3. Find w when u = 6.

$$w \propto \frac{1}{u^3}$$

$$w = \frac{k}{u^3} \text{ ---------- (i)}$$

Put w = 5 and u = 3

$$5 = \frac{k}{(3)^3}$$

$$5 = \frac{k}{27}$$

$$k = 135$$

So, equation (i) becomes

$$w = \frac{135}{u^3} \text{ -------- (ii)}$$

Put u = 6 in equation (ii)

$$w = \frac{135}{(6)^3}$$

$$w = \frac{135}{216}$$

$$w = \frac{5}{8}$$

Exercise 3.6

Q. 1: If a : b = c : d, (a, b, c, d ≠ 0), then show that

(i) $\frac{4a-9b}{4a+9b} = \frac{4c-9d}{4c+9d}$

as $a : b = c : d$

Let $\frac{a}{b} = \frac{c}{d} = k$

Then $a = bk$ and $c = dk$

$\frac{4a-9b}{4a+9b} = \frac{4c-9d}{4c+9d}$

Putting the values

$\frac{4(bk)-9b}{4(bk)+9b} = \frac{4(dk)-9d}{4(dk)+9d}$

$\frac{4bk-9b}{4bk+9b} = \frac{4dk-9d}{4dk+9d}$

$\frac{b(4k-9)}{b(4k+9)} = \frac{d(4k-9)}{d(4k+9)}$

$\frac{4k-9}{4k+9} = \frac{4k-9}{4k+9}$

L.H.S = R.H.S

(ii) $\frac{6a-5b}{6a+5b} = \frac{6c-5d}{6c+5d}$

as $a : b = c : d$

Let $\frac{a}{b} = \frac{c}{d} = k$

Then $a = bk$ and $c = dk$

$\frac{6a-5b}{6a+5b} = \frac{6c-5d}{6c+5d}$

Putting the values

$\frac{6(bk)-5b}{6(bk)+5b} = \frac{6(dk)-5d}{6(dk)+5d}$

$\frac{6bk-5b}{6bk+5b} = \frac{6dk-5d}{6dk+5d}$

$\frac{b(6k-5)}{b(6k+5)} = \frac{d(6k-5)}{d(6k+5)}$

$\frac{6k-5}{6k+5} = \frac{6k-5}{6k+5}$

L.H.S = R.H.S

(iii) $\frac{a}{b} = \sqrt{\frac{a^2+c^2}{b^2+d^2}}$

as $a : b = c : d$

Let $\frac{a}{b} = \frac{c}{d} = k$

Then $a = bk$ and $c = dk$

$\frac{a}{b} = \sqrt{\frac{a^2+c^2}{b^2+d^2}}$

Putting the values

$\frac{bk}{b} = \sqrt{\frac{(bk)^2+(dk)^2}{b^2+d^2}}$

$k = \sqrt{\frac{b^2k^2+d^2k^2}{b^2+d^2}}$

$k = \sqrt{\frac{k^2(b^2+d^2)}{b^2+d^2}}$

$k = \sqrt{k^2}$

$k = k$

L.H.S = R.H.S

(iv) $a^6 + c^6 : b^6 + d^6 = a^3c^3 : b^3d^3$

$\frac{a^6+c^6}{b^6+d^6} = \frac{a^3c^3}{b^3d^3}$

as $a : b = c : d$

Let $\frac{a}{b} = \frac{c}{d} = k$

Then $a = bk$ and $c = dk$

Putting the values

$\frac{(bk)^6+(dk)^6}{b^6+d^6} = \frac{(bk)^3(dk)^3}{b^3d^3}$

$\frac{b^6k^6+d^6k^6}{b^6+d^6} = \frac{b^3k^3d^3k^3}{b^3d^3}$

$\frac{k^6(b^6+d^6)}{b^6+d^6} = \frac{b^3d^3k^6}{b^3d^3}$

$k^6 = k^6$

L.H.S = R.H.S

(v) $p(a+b) + qb : p(c+d) + qd = a : c$

$\frac{p(a+b)+qb}{p(c+d)+qd} = \frac{a}{c}$

as $a : b = c : d$

Let $\frac{a}{b} = \frac{c}{d} = k$

Then $a = bk$ and $c = dk$

Putting the values

$\frac{p(bk+b)+qb}{p(dk+d)+qd} = \frac{bk}{dk}$

$\frac{b[p(k+1)+q]}{d[p(k+1)+q]} = \frac{bk}{dk}$

$\frac{b}{d} = \frac{b}{d}$

L.H.S = R.H.S

(vi) $a^2 + b^2 : \frac{a^3}{a+b} = c^2 + d^2 : \frac{c^3}{c+d}$

$\frac{a^2+b^2}{\frac{a^3}{a+b}} = \frac{c^2+d^2}{\frac{c^3}{c+d}}$

as $a : b = c : d$

Let $\frac{a}{b} = \frac{c}{d} = k$

Then $a = bk$ and $c = dk$

Putting the values

$$\frac{\frac{(bk)^2+b^2}{(bk)^3}}{bk+b} = \frac{\frac{(dk)^2+d^2}{(dk)^3}}{dk+d}$$

$$\frac{\frac{b^2k^2+b^2}{b^3k^3}}{bk+b} = \frac{\frac{d^2k^2+d^2}{d^3k^3}}{dk+d}$$

$$\frac{\frac{b^2(k^2+1)}{b^3k^3}}{b(k+1)} = \frac{\frac{d^2(k^2+1)}{d^3k^3}}{d(k+1)}$$

$$\frac{\frac{b^2(k^2+1)}{b^2k^3}}{(k+1)} = \frac{\frac{d^2(k^2+1)}{d^2k^3}}{(k+1)}$$

$$\frac{\frac{(k^2+1)}{k^3}}{(k+1)} = \frac{\frac{(k^2+1)}{k^3}}{(k+1)}$$

$$\frac{(k^2+1)(k+1)}{k^3} = \frac{(k^2+1)(k+1)}{k^3}$$

L.H.S = R.H.S

(vii) $\frac{a}{a-b} : \frac{a+b}{b} = \frac{c}{c-d} : \frac{c+d}{d}$

$$\frac{\frac{a}{a-b}}{\frac{a+b}{b}} = \frac{\frac{c}{c-d}}{\frac{c+d}{d}}$$

as $a : b = c : d$

Let $\frac{a}{b} = \frac{c}{d} = k$

Then $a = bk$ and $c = dk$

Putting the values

$$\frac{\frac{bk}{bk-b}}{\frac{bk+b}{b}} = \frac{\frac{dk}{dk-d}}{\frac{dk+d}{d}}$$

$$\frac{\frac{bk}{b(k-1)}}{\frac{b(k+1)}{b}} = \frac{\frac{dk}{d(k-1)}}{\frac{d(k+1)}{d}}$$

$$\frac{\frac{k}{(k-1)}}{(k+1)} = \frac{\frac{k}{(k-1)}}{(k+1)}$$

$$\frac{k}{(k-1)(k+1)} = \frac{k}{(k-1)(k+1)}$$

L.H.S = R.H.S

Q. 2: If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ (a, b, c, d, e, f $\neq$ 0), then show that

(i) $\frac{a}{b} = \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}$

as $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

Then $a = bk, c = dk$ and $e = fk$

Putting the values

$$\frac{bk}{b} = \sqrt{\frac{(bk)^2+(dk)^2+(fk)^2}{b^2+d^2+f^2}}$$

$$k = \sqrt{\frac{b^2k^2+d^2k^2+f^2k^2}{b^2+d^2+f^2}}$$

$k = \sqrt{\frac{k^2(b^2+d^2+f^2)}{b^2+d^2+f^2}}$

$k = \sqrt{k^2}$

$k = k$

L.H.S = R.H.S

(ii) $\frac{ac+ce+ea}{bd+df+fb} = \left[\frac{ace}{bdf}\right]^{2/3}$

as $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

Then $a = bk$, $c = dk$ and $e = fk$

Putting the values

$\frac{bkdk+dkfk+fkbk}{bd+df+fb} = \left[\frac{bkdkfk}{bdf}\right]^{2/3}$

$\frac{k^2(bd+df+fb)}{bd+df+fb} = \left[\frac{k^3bdf}{bdf}\right]^{2/3}$

$k^2 = [k^3]^{2/3}$

$k^2 = k^2$

L.H.S = R.H.S

(iii) $\frac{ac}{bd} + \frac{ce}{df} + \frac{ea}{fb} = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$

as $\frac{a}{b} = \frac{c}{d} = \frac{e}{g}$

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{g} = k$

Then $a = bk$, $c = dk$ and $e = fk$

Putting the values

$\frac{bkdk}{bd} + \frac{dkfk}{df} + \frac{fkbk}{fb} = \frac{b^2k^2}{b^2} + \frac{d^2k^2}{d^2} + \frac{f^2k^2}{f^2}$

$\frac{bdk^2}{bd} + \frac{dfk^2}{df} + \frac{fbk^2}{fb} = \frac{b^2k^2}{b^2} + \frac{d^2k^2}{d^2} + \frac{f^2k^2}{f^2}$

$k^2 + k^2 + k^2 = k^2 + k^2 + k^2$

$3k^2 = 3k^2$

$k^2 = k^2$

L.H.S = R.H.S

Exercise 3.7

Q. 1: The surface area A of a cube varies directly as the square of the length l of an edge and A = 27 square units when l = 3 units.

Find (i) A when l = 4 units (ii) l when A = 12 sq. units.

$A \propto l^2$

$A = kl^2$ ----------- (i)

When A = 27 square units, l = 3 units

$(27) = k(3)^2$

$(27) = k(9)$

$k = 3$

So equation (i) becomes

$A = 3l^2$

(i) When l = 4 units

$A = 3(4)^2$

$A = 3(16)$

$A = 48$ sq. units

(ii) When A = 12 sq. units

$12 = 3l^2$

$4 = l^2$

$l = 2$

Q. 2: The surface area S of a sphere varies directly as the square of radius r, and S = 16π when r = 2. Find r when $S = 36\pi$.

$S \propto r^2$

$S = kr^2$ ----------- (i)

S = 16π when r = 2

$(16\pi) = k(2)^2$

$(16\pi) = k(4)$

$k = 4\pi$

So equation (i) becomes

$S = 4\pi r^2$

When S = 36π units

$36\pi = 4\pi r^2$

$r^2 = 9$

$r = 3$

Q. 3: In Hook's law the force F applied to stretch a spring varies directly as the amount of elongation S and $F = 32lb$ when $S = 1.6\ in$. Find (i) S when $F = 50\ lb$ (ii) F when $S = 0.8in$.

$F \propto S$

$F = kS$ ----------- (i)

F = 32lb when S = 1.6

$(32) = k(1.6)$

$k = \frac{32}{1.6}$

$k = 20$

So equation (i) becomes

$F = 20S$

(i) When F = 50lb

$50lb = 20S$

$S = 2.5$ in

(ii) When S = 0.8in

$F = 20(0.8)$

$F = 16\ lb$

Q. 4: The intensity I of light from a given source varies inversely as the square of the distance d from it. If the intensity is 20 candlepower at a distance of 12ft. from the source, find the intensity at a point 8ft. from the source.

$I \propto \frac{1}{d^2}$

$I = \frac{k}{d^2}$ ----------- (i)

I = 20 when d = 12ft

$(20) = \frac{k}{(12)^2}$

$20 = \frac{k}{144}$

$k = 2880$

So equation (i) becomes

$I = \frac{2880}{d^2}$

When d = 8ft

$I = \frac{2880}{8^2}$

$I = \frac{2880}{64}$

$I = 45$cp

Q. 5: The pressure P in a body of fluid varies directly as the depth d. If the pressure exerted on the bottom of a tank by a column of fluid 5ft. high is 2.25 lb/sq. in, how deep must the fluid be to exert a pressure of 9 lb/sq. in?

$P \propto d$

$P = kd$ ----------- (i)

d = 5ft when P = 2.25 lb/sq. in

$2.25 = k(5)$

$k = \frac{2.25}{5}$

$k = 0.45$

So equation (i) becomes

$P = 0.45d$

When P = 9 lb/sq. in

$9 = 0.45d$

$$d = \frac{9}{0.45}$$

$$d = 20$$

Q. 6: Labour costs c varies jointly as the number of workers n and the average number of days d, if the cost of 800 workers for 13 days is Rs. 286000, then find the labour cost of 600 workers for 18 days.

$$c \propto nd$$

$$c = knd \text{ ----------- (i)}$$

c = Rs. 286000, n = 800, d = 13

$$286000 = k(800)(13)$$

$$k = \frac{286000}{10400}$$

$$k = 27.5$$

So equation (i) becomes

$$c = 27.5nd$$

When n = 600, d = 18

$$c = 27.5(600)(18)$$

$$c = 297000$$

Q. 7: The supporting load c of a pillar varies as the fourth power of its diameter d and inversely as the square of its length l. A pillar of diameter 6 inch and of height 30 feet will support a load of 63 tons. How high a 4 inch pillar must be to support a load of 28 tons?

$$c \propto \frac{d^4}{l^2}$$

$$c = \frac{kd^4}{l^2} \text{ ----------- (i)}$$

c = 63 tons, d = 6inch, l = 30feet

$$63 = \frac{k(6)^4}{(30)^2}$$

$$k = \frac{63 \times 900}{1296}$$

$$k = 43.75$$

So equation (i) becomes

$$c = \frac{43.75d^4}{l^2}$$

When d = 4inch, c = 28 tons

$$28 = \frac{43.75(4)^4}{l^2}$$

$$l^2 = \frac{43.75(4)^4}{28}$$

$$l^2 = 400$$

$$l = 20 \text{ feet}$$

Q. 8: The time T required for an elevator to lift a weight varies jointly as the weight w and the lifting depth d varies inversely as the power p of the motor. If 25 sec. are required for a 4-hp motor to lift 500 lb through 40 ft, what power is required to lift 800 lb, through 120 ft in 40 sec.?

$$T \propto \frac{wd}{p}$$

$$T = \frac{kwd}{p} \text{ ----------- (i)}$$

T=25 sec., p = 4-hp, w = 500lb, d = 40ft

$$25 = \frac{k(500)(40)}{(4)}$$

$$k = \frac{25 \times 4}{500 \times 40}$$

$$k = 0.005$$

So equation (i) becomes

$$T = \frac{0.005wd}{p}$$

When w = 800lb, d = 120ft, T = 40 sec.

$$40 = \frac{0.005 \times 800 \times 120}{p}$$

$$p = \frac{0.005 \times 800 \times 120}{40}$$

$$p = 12 \text{ hp}$$

Q. 9: The kinetic energy (K.E.) of a body varies jointly as the mass "m" of the body and the square of its velocity "v". If the kinetic energy is 4320 ft/lb when the mass is 45 lb and the velocity is 24 ft/sec. Determine the kinetic energy of a 3000 lb automobile travelling 44 ft/sec.

$$K.E \propto mv^2$$

$$K.E = kmv^2 \text{ ----------- (i)}$$

K.E=4320 ft/lb, m = 45lb, v = 24 ft/sec.

$$4320 = k(45)(24)^2$$

$$k = \frac{4320}{45 \times 576}$$

$$k = \frac{4320}{25920}$$

$$k = \frac{1}{6}$$

So equation (i) becomes

$$K.E = \frac{mv^2}{6}$$

When m = 3000lb, v = 44 ft/sec.

$$K.E = \frac{(3000)(44)^2}{6}$$

$$K.E = 500 \times 1936$$

$$K.E = 968000 \text{ ft/lb}$$

Exercise 4.1

Resolve into partial fractions.

1. $\frac{7x-9}{(x+1)(x-3)} = \frac{A}{(x+1)} + \frac{B}{(x-3)}$ .... (i)

multiplying by $(x+1)(x-3)$ we get

$7x-9 = A(x-3) + B(x+1)$ .... (ii)

put $x = -1$ in (ii)

$7(-1) - 9 = A(-1-3) + B(-1+1)$

$-7 - 9 = A(-4)$

$-16 = -4A$

$A = \frac{-16}{-4}$

$A = 4$

put $x = 3$ in (ii)

$7(3) - 9 = A(3-3) + B(3+1)$

$21 - 9 = B(4)$

$12 = 4B$

$B = \frac{12}{4}$

$B = 3$

put the values in (i) we get

$\frac{7x-9}{(x+1)(x-3)} = \frac{4}{(x+1)} + \frac{3}{(x-3)}$

2. $\frac{x-11}{(x-4)(x+3)} = \frac{A}{(x-4)} + \frac{B}{(x+3)}$ .... (i)

multiplying by $(x-4)(x+3)$ we get

$x - 11 = A(x+3) + B(x-4)$ .... (ii)

put $x = 4$ in (ii)

$4 - 11 = A(4+3) + B(4-4)$

$-7 = A(7)$

$-7 = 7A$

$A = \frac{-7}{7}$

$A = -1$

put $x = -3$ in (ii)

$-3 - 11 = A(-3+3) + B(-3-4)$

$-14 = B(-7)$

$-14 = -7B$

$B = \frac{-14}{-7}$

$B = 2$

put the values in (i) we get

$$\frac{x-11}{(x-4)(x+3)} = \frac{-1}{(x-4)} + \frac{2}{(x+3)}$$

3. $$\frac{3x-1}{x^2-1} = \frac{3x-1}{(x-1)(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x+1)} \text{ .... (i)}$$

multiplying by $(x-1)(x+1)$ we get

$$3x-1 = A(x+1) + B(x-1) \text{ .... (ii)}$$

put $x = 1$ in (ii)

$$3(1)-1 = A(1+1) + B(1-1)$$

$$3-1 = A(2)$$

$$2 = 2A$$

$$A = \frac{2}{2}$$

$$A = 1$$

put $x = -1$ in (ii)

$$3(-1)-1 = A(-1+1) + B(-1-1)$$

$$-3-1 = B(-2)$$

$$-4 = -2B$$

$$B = \frac{-4}{-2}$$

$$B = 2$$

put the values in (i) we get

$$\frac{3x-1}{x^2-1} = \frac{1}{(x-1)} + \frac{2}{(x+1)}$$

4. $$\frac{x-5}{x^2+2x-3} = \frac{x-5}{x^2+3x-x-3} = \frac{x-5}{x(x+3)-1(x+3)}$$

$$\frac{x-5}{(x+3)(x-1)} = \frac{A}{(x+3)} + \frac{B}{(x-1)} \text{ .... (i)}$$

multiplying by $(x+3)(x-1)$ we get

$$x-5 = A(x-1) + B(x+3) \text{ .... (ii)}$$

put $x = -3$ in (ii)

$$-3-5 = A(-3-1) + B(-3+3)$$

$$-8 = A(-3-1)$$

$$-8 = -4A$$

$$A = \frac{-8}{-4}$$

$$A = 2$$

put $x = 1$ in (ii)

$$1-5 = A(1-1) + B(1+3)$$

$$-4 = B(4)$$

$$-4 = 4B$$

$$B = \frac{-4}{4}$$

$$B = -1$$

put the values in (i) we get

$$\frac{x-5}{(x+3)(x-1)} = \frac{2}{(x+3)} + \frac{-1}{(x-1)}$$

5. $\frac{3x+3}{(x-1)(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x+2)}$ .... (i)

multiplying by $(x-1)(x+2)$ we get

$3x+3 = A(x+2) + B(x-1)$ .... (ii)

put $x = 1$ in (ii)

$3(1)+3 = A(1+2) + B(1-1)$

$3+3 = A(3)$

$6 = 3A$

$A = \frac{6}{3}$

$A = 2$

put $x = -2$ in (ii)

$3(-2)+3 = A(-2+2) + B(-2-1)$

$-6+3 = B(-3)$

$-3 = -3B$

$B = \frac{-3}{-3}$

$B = 1$

put the values in (i) we get

$\frac{3x+3}{(x-1)(x+2)} = \frac{2}{(x-1)} + \frac{1}{(x+2)}$

6. $\frac{7x-25}{(x-4)(x-3)} = \frac{A}{(x-4)} + \frac{B}{(x-3)}$ .... (i)

multiplying by $(x-4)(x-3)$ we get

$7x-25 = A(x-3) + B(x-4)$ .... (ii)

put $x = 4$ in (ii)

$7(4)-25 = A(4-3) + B(4-4)$

$28-25 = A(1)$

$3 = A$

$A = 3$

put $x = 3$ in (ii)

$7(3)-25 = A(3-3) + B(3-4)$

$21-25 = B(-1)$

$-4 = -1B$

$B = \frac{-4}{-1}$

$B = 4$

put the values in (i) we get

$\frac{7x-25}{(x-4)(x-3)} = \frac{3}{(x-4)} + \frac{4}{(x-3)}$

7. $\frac{x^2+2x+1}{(x-2)(x+3)} = \frac{x^2+2x+1}{x^2+3x-2x-6} = \frac{x^2+2x+1}{x^2+x-6} = 1 + \frac{x+7}{x^2+x-6}$ .... (i)

$\frac{x+7}{(x-2)(x+3)} = \frac{A}{(x-2)} + \frac{B}{(x+3)}$ .... (ii)

multiplying by $(x-2)(x+3)$ we get

$x+7 = A(x+3) + B(x-2)$ .... (iii) put $x = 2$ in (iii)

$$2 + 7 = A(2 + 3) + B(2 - 2)$$
$$9 = A(5)$$
$$9 = 5A$$
$$A = \frac{9}{5}$$

put $x = -3$ in (iii)

$$-3 + 7 = A(-3 + 3) + B(-3 - 2)$$
$$4 = B(-5)$$
$$4 = -5B$$
$$B = \frac{4}{-5}$$

put the values in (ii) we get

$$\frac{x+7}{(x-2)(x+3)} = \frac{9}{5(x-2)} - \frac{9}{5(x+3)}$$

put the value in (i) we get

$$\frac{x^2+2x+1}{(x-2)(x+3)} = 1 + \frac{9}{5(x-2)} - \frac{9}{5(x+3)}$$

8. $\frac{6x^3+5x^2-7}{3x^2-2x-1} = 2x + 3 + \frac{8x-4}{3x^2-2x-1}$ .... (i)

$$\frac{8x-4}{3x^2-2x-1} = \frac{8x-4}{(x-1)(3x+1)} = \frac{A}{(x-1)} + \frac{B}{(3x+1)}$$ .... (ii)

multiplying by $(x - 1)(3x + 1)$ we get

$8x - 4 = A(3x + 1) + B(x - 1)$ .... (iii)

put $x = 1$ in (iii)

$$8(1) - 4 = A(3(1) + 1) + B(1 - 1)$$
$$4 = A(3 + 1)$$
$$4 = 4A$$
$$A = 1$$

put $x = -\frac{1}{3}$ in (iii)

$$8\left(-\frac{1}{3}\right) - 4 = A\left(3\left(-\frac{1}{3}\right) + 1\right) + B\left(-\frac{1}{3} - 1\right)$$
$$-\frac{8}{3} - 4 = B\left(-\frac{1}{3} - 1\right)$$
$$\frac{-8-12}{3} = B\left(\frac{-1-3}{3}\right)$$
$$\frac{-20}{3} = \frac{-4}{3}B$$
$$B = \frac{-20}{3} \times \frac{3}{-4}$$
$$B = 5$$

put the values in (ii) we get

$$\frac{8x-4}{3x^2-2x-1} = \frac{1}{(x-1)} + \frac{5}{(3x+1)}$$

put the value in (i) we get

$$\frac{6x^3+5x^2-7}{3x^2-2x-1} = 2x + 3 + \frac{1}{(x-1)} + \frac{5}{(3x+1)}$$

Exercise 4.2

Resolve into partial fractions.

1. $\frac{x^2-3x+1}{(x-2)(x-1)^2} = \frac{A}{(x-2)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$ .... (i)

multiplying by $(x-2)(x-1)^2$ we get

$x^2-3x+1 = A(x-1)^2 + B(x-1)(x-2) + C(x-2)$ .... (ii)

$x^2-3x+1 = A(x^2-2x+1) + B(x^2-2x-x+2) + C(x-2)$

$x^2-3x+1 = A(x^2-2x+1) + B(x^2-3x+2) + C(x-2)$ .... (iii)

put $x = 2$ in (ii)

$(2)^2-3(2)+1 = A(2-1)^2 + B(2-1)(2-2) + C(2-2)$

$4-6+1 = A(1)^2$

$-1 = A$

$A = -1$

put $x = 1$ in (ii)

$(1)^2-3(1)+1 = A(1-1)^2 + B(1-1)(1-2) + C(1-2)$

$1-3+1 = C(-1)$

$-1 = C(-1)$

$-1 = -C$

$C = 1$

Now, comparing coefficients of equation (iii)

x^2; $A+B = 1$

as A = -1

$-1+B = 1$

$B = 2$

put the values in (i) we get

$\frac{x^2-3x+1}{(x-2)(x-1)^2} = \frac{-1}{(x-2)} + \frac{2}{(x-1)} + \frac{1}{(x-1)^2}$

2. $\frac{x^2+7x+11}{(x+3)(x+2)^2} = \frac{A}{(x+3)} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2}$ .... (i)

multiplying by $(x+3)(x+2)^2$ we get

$x^2+7x+11 = A(x+2)^2 + B(x+2)(x+3) + C(x+3)$ .... (ii)

$x^2+7x+11 = A(x^2+4x+4) + B(x^2+3x+2x+6) + C(x+3)$

$x^2+7x+11 = A(x^2+4x+4) + B(x^2+5x+6) + C(x+3)$ .... (iii)

put $x = -3$ in (ii)

$(-3)^2+7(-3)+11 = A(-3+2)^2 + B(-3+2)(-3+3) + C(-3+3)$

$9-21+11 = A(-1)^2$

$-1 = A$

$A = -1$

put $x = -2$ in (ii)

$(-2)^2+7(-2)+11 = A(-2+2)^2 + B(-2+2)(-2+3) + C(-2+3)$

$4 - 14 + 11 = C(1)$

$1 = C(1)$

$1 = C$

$C = 1$

Now, comparing coefficients of equation (iii)

$x^2;\quad A + B = 1$

as A = -1

$-1 + B = 1$

$B = 2$

put the values in (i) we get

$$\frac{x^2+7x+11}{(x+3)(x+2)^2} = \frac{-1}{(x+3)} + \frac{2}{(x+2)} + \frac{1}{(x+2)^2}$$

3. $\frac{9}{(x-1)(x+2)^2} = \frac{A}{(x-1)} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2}$ .... (i)

multiplying by $(x-1)(x+2)^2$ we get

$9 = A(x+2)^2 + B(x+2)(x-1) + C(x-1)$ .... (ii)

$9 = A(x^2+4x+4) + B(x^2 - x + 2x - 2) + C(x-1)$

$9 = A(x^2+4x+4) + B(x^2+x+6) + C(x-1)$ .... (iii)

put $x = 1$ in (ii)

$9 = A(1+2)^2 + B(1+2)(1-1) + C(1-1)$

$9 = A(3)^2$

$9 = 9A$

$A = 1$

put $x = -2$ in (ii)

$9 = A(-2+2)^2 + B(-2+2)(2-1) + C(-2-1)$

$9 = C(-2-1)$

$9 = C(-3)$

$9 = -3C$

$C = -3$

Now, comparing coefficients of equation (iii)

$x^2;\quad A + B = 0$

as A = 1

$1 + B = 0$

$B = -1$

put the values in (i) we get

$$\frac{9}{(x-1)(x+2)^2} = \frac{1}{(x-1)} + \frac{-1}{(x+2)} + \frac{-3}{(x+2)^2}$$

4. $\frac{x^4+1}{x^2(x-1)} = \frac{x^4+1}{x^3-x^2} = x + 1 + \frac{x^2+1}{x^3-x^2}$ .... (i)

$\frac{x^2+1}{x^3-x^2} = \frac{A}{(x-1)} + \frac{B}{x} + \frac{C}{x^2}$ .... (ii)

multiplying by $(x-1)x^2$ we get

$x^2 + 1 = Ax^2 + Bx(x-1) + C(x-1)$ .... (iii)

$$x^2+1 = Ax^2 + B(x^2-x) + C(x-1) \text{ .... (iv)}$$

put $x = 1$ in (iii)

$$(1)^2 + 1 = A(1)^2 + B(1)(1-1) + C(1-1)$$
$$1 + 1 = A(1)^2$$
$$2 = A$$
$$A = 2$$

put $x = 0$ in (iii)

$$(0)^2 + 1 = A(0)^2 + B(0)(0-1) + C(0-1)$$
$$1 = C(-1)$$
$$C = -1$$

Now, comparing coefficients of equation (iv)

x^2; $\quad A + B = 1$

as A = 2

$$2 + B = 1$$
$$B = -1$$

put the values in (ii) we get

$$\frac{x^2+1}{x^3-x^2} = \frac{2}{(x-1)} + \frac{-1}{x} + \frac{-1}{x^2}$$

put this in (i) we get

$$\frac{x^4+1}{x^2(x-1)} = x + 1 + \frac{2}{(x-1)} - \frac{1}{x} - \frac{1}{x^2}$$

5. $$\frac{7x+4}{(3x+2)(x+1)^2} = \frac{A}{(3x+2)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2} \text{ .... (i)}$$

multiplying by $(3x+2)(x+1)^2$ we get

$$7x + 4 = A(x+1)^2 + B(3x+2)(x+1) + C(3x+2) \text{ .... (ii)}$$
$$7x + 4 = A(x^2+2x+1) + B(3x^2+3x+2x+2) + C(x-1)$$
$$7x + 4 = A(x^2+2x+1) + B(3x^2+5x+2) + C(x-1) \text{ .... (iii)}$$

put $x = -\frac{2}{3}$ in (ii)

$$7\left(-\frac{2}{3}\right) + 4 = A\left(-\frac{2}{3}+1\right)^2 + B\left(3\left(-\frac{2}{3}\right)+2\right)\left(-\frac{2}{3}-1\right) + C\left(3\left(-\frac{2}{3}\right)+2\right)$$
$$-\frac{14}{3} + 4 = A\left(-\frac{2}{3}+1\right)^2$$
$$\frac{-14+12}{3} = A\left(\frac{-2+3}{3}\right)^2$$
$$\frac{-2}{3} = A\left(\frac{1}{3}\right)^2$$
$$\frac{-2}{3} = \frac{1}{9}A$$
$$A = -6$$

put $x = -1$ in (ii)

$$7(-1) + 4 = A(-1+1)^2 + B(3(-1)+2)(-1-1) + C(3(-1)+2)$$
$$-7 + 4 = C(-3+2)$$
$$-3 = C(-1)$$
$$-3 = -C$$

$C \quad = 3$

Now, comparing coefficients of equation (iii)

$x^2;\quad A + 3B = 0$

as A = -6

$-6 + 3B = 0$

$3B = 6$

$B = 2$

put the values in (i) we get

$$\frac{7x+4}{(3x+2)(x+1)^2} = \frac{-6}{(3x+2)} + \frac{2}{(x+1)} + \frac{3}{(x+1)^2}$$

6. $\frac{1}{(x+1)(x-1)^2} = \frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$ .... (i)

multiplying by $(x+1)(x-1)^2$ we get

$1 = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$ .... (ii)

$1 = A(x^2 - 2x + 1) + B(x^2 - 1) + C(x+1)$ .... (iii)

put $x = -1$ in (ii)

$1 = A(-1-1)^2 + B(-1+1)(-1-1) + C(-1+1)$

$1 = A(-2)^2$

$1 = 4A$

$A = \frac{1}{4}$

put $x = 1$ in (ii)

$1 = A(1-1)^2 + B(1+1)(1-1) + C(1+1)$

$1 = C(1+1)$

$1 = C(2)$

$1 = 2C$

$C = \frac{1}{2}$

Now, comparing coefficients of equation (iii)

$x^2;\quad A + B = 0$

as A = $\frac{1}{4}$

$\frac{1}{4} + B = 0$

$B = -\frac{1}{4}$

put the values in (i) we get

$$\frac{1}{(x+1)(x-1)^2} = \frac{1}{4(x+1)} + \frac{-1}{4(x-1)} + \frac{1}{2(x-1)^2}$$

7. $\frac{3x^2+15x+16}{(x+2)^2} = \frac{3x^2+15x+16}{x^2+4x+4} = 3 + \frac{3x+4}{x^2+4x+4}$ .... (i)

$\frac{3x+4}{x^2+4x+4} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2}$ .... (ii)

multiplying by $(x+2)^2$ we get

$3x + 4 = A(x+2) + B$ .... (iii)

$3x + 4 = A(x+2) + B$ .... (iv)

put $x = -2$ in (iii)

$$3(-2)+4 = A(-2+2)+B$$
$$-6+4 = B$$
$$-2 = B$$
$$B = -2$$

Now, comparing coefficients of equation (iv)

$x;\quad A = 3$

put the values in (ii) we get

$$\frac{3x+4}{x^2+4x+4} = \frac{3}{(x+2)} + \frac{-2}{(x+2)^2}$$

put this in (i) we get

$$\frac{3x^2+15x+16}{(x+2)^2} = 3 + \frac{3}{(x+2)} - \frac{2}{(x+2)^2}$$

8. $\frac{1}{(x^2-1)(x+1)} = \frac{1}{(x-1)(x+1)^2} = \frac{A}{(x-1)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$ .... (i)

multiplying by $(x-1)(x+1)^2$ we get

$$1 = A(x+1)^2 + B(x-1)(x+1) + C(x-1) \text{ .... (ii)}$$
$$1 = A(x^2+2x+1) + B(x^2-1) + C(x-1) \text{ .... (iii)}$$

put $x = 1$ in (ii)

$$1 = A(1+1)^2 + B(1-1)(1+1) + C(1-1)$$
$$1 = A(2)^2$$
$$1 = 4A$$
$$A = \frac{1}{4}$$

put $x = -1$ in (ii)

$$1 = A(-1+1)^2 + B(-1-1)(-1+1) + C(-1-1)$$
$$1 = C(-1-1)$$
$$1 = C(-2)$$
$$1 = -2C$$
$$C = \frac{-1}{2}$$

Now, comparing coefficients of equation (iii)

$x^2;\quad A + B = 0$

as A = $\frac{1}{4}$

$$\frac{1}{4} + B = 0$$
$$B = -\frac{1}{4}$$

put the values in (i) we get

$$\frac{1}{(x^2-1)(x+1)} = \frac{1}{4(x-1)} + \frac{-1}{4(x+1)} + \frac{-1}{2(x+1)^2}$$

Exercise 4.3

Resolve into partial fractions.

1. $\frac{3x-11}{(x+3)(x^2+1)} = \frac{A}{(x+3)} + \frac{Bx+c}{(x^2+1)}$ .... (i)

multiplying by $(x+3)(x^2+1)$ we get

$3x-11 = A(x^2+1) + (Bx+c)(x+3)$ .... (ii)

$3x-11 = A(x^2+1) + B(x^2+3x) + C(x+3)$

$3x-11 = A(x^2+1) + B(x^2+3x) + C(x+3)$ .... (iii)

put $x = -3$ in (ii)

$3(-3)-11 = A((-3)^2+1) + (B(-3)+c)(-3+3)$

$-9-11 = A(9+1)$

$-20 = 10A$

$A = -2$

Now, comparing coefficients of equation (iii)

x^2; $A + B = 0$

as A = -2

$-2 + B = 0$

$B = 2$

x; $3B + C = 3$

as B = 2

$6 + C = 3$

$C = -3$

put the values in (i) we get

$\frac{3x-11}{(x+3)(x^2+1)} = \frac{-2}{(x+3)} + \frac{2x-3}{(x^2+1)}$

2. $\frac{3x+7}{(x+3)(x^2+1)} = \frac{A}{(x+3)} + \frac{Bx+c}{(x^2+1)}$ .... (i)

multiplying by $(x+3)(x^2+1)$ we get

$3x+7 = A(x^2+1) + (Bx+c)(x+3)$ .... (ii)

$3x+7 = A(x^2+1) + B(x^2+3x) + C(x+3)$

$3x+7 = A(x^2+1) + B(x^2+3x) + C(x+3)$ .... (iii)

put $x = -3$ in (ii)

$3(-3)+7 = A((-3)^2+1) + (B(-3)+c)(-3+3)$

$-9+7 = A(9+1)$

$-2 = 10A$

$A = \frac{-1}{5}$

Now, comparing coefficients of equation (iii)

x^2; $A + B = 0$

as A = $\frac{-1}{5}$

$$\frac{-1}{5} + B = 0$$
$$B = \frac{1}{5}$$
$x;$ $3B + C = 3$
as B = $\frac{1}{5}$
$$\frac{3}{5} + C = 3$$
$$C = 3 - \frac{3}{5}$$
$$C = \frac{15-3}{5}$$
$$C = \frac{12}{5}$$
put the values in (i) we get
$$\frac{3x+7}{(x+3)(x^2+1)} = \frac{\frac{-1}{5}}{(x+3)} + \frac{\frac{1}{5}x+\frac{12}{5}}{(x^2+1)}$$
$$= \frac{\frac{-1}{5}}{(x+3)} + \frac{\frac{x+12}{5}}{(x^2+1)}$$
$$= \frac{-1}{5(x+3)} + \frac{x+12}{5(x^2+1)}$$

3. $\frac{1}{(x+1)(x^2+1)} = \frac{A}{(x+1)} + \frac{Bx+c}{(x^2+1)}$ .... (i)

multiplying by $(x+1)(x^2+1)$ we get
$1 = A(x^2+1) + (Bx+c)(x+1)$ .... (ii)
$1 = A(x^2+1) + B(x^2+x) + C(x+1)$
$1 = A(x^2+1) + B(x^2+x) + C(x+1)$ .... (iii)

put $x = -1$ in (ii)
$$1 = A((-1)^2+1) + (B(-1)+c)(-1+1)$$
$$-9 + 7 = A(1+1)$$
$$-2 = 2A$$
$$A = -1$$
Now, comparing coefficients of equation (iii)
$x^2;$ $A + B = 0$
as A = -1
$$-1 + B = 0$$
$$B = 1$$
$x;$ $B + C = 0$
as B = 1
$$1 + C = 0$$
$$C = -1$$
put the values in (i) we get
$$\frac{1}{(x+1)(x^2+1)} = \frac{A}{(x+1)} + \frac{Bx+c}{(x^2+1)}$$
$$= \frac{-1}{(x+1)} + \frac{x-1}{(x^2+1)}$$

4. $\frac{9x-7}{(x+3)(x^2+1)} = \frac{A}{(x+3)} + \frac{Bx+c}{(x^2+1)}$ .... (i)

multiplying by $(x+3)(x^2+1)$ we get

$$9x-7 = A(x^2+1)+(Bx+c)(x+3) \text{ .... (ii)}$$
$$9x-7 = A(x^2+1)+B(x^2+3x)+C(x+3)$$
$$9x-7 = A(x^2+1)+B(x^2+3x)+C(x+3) \text{ .... (iii)}$$

put $x=-3$ in (ii)

$$9(-3)-7 = A((-3)^2+1)+(B(-3)+c)(-3+3)$$
$$-27-7 = A(9+1)$$
$$-34 = 10A$$
$$A = \frac{-17}{5}$$

Now, comparing coefficients of equation (iii)

x^2; $\quad A+B = 0$

as A = $\frac{-17}{5}$

$$\frac{-17}{5}+B = 0$$
$$B = \frac{17}{5}$$

x; $\quad 3B+C = 9$

as B = $\frac{17}{5}$

$$\frac{51}{5}+C = 9$$
$$C = 9-\frac{51}{5}$$
$$C = \frac{45-51}{5}$$
$$C = \frac{-6}{5}$$

put the values in (i) we get

$$\frac{9x-7}{(x+3)(x^2+1)} = \frac{\frac{-17}{5}}{(x+3)}+\frac{\frac{17}{5}x+\frac{-6}{5}}{(x^2+1)}$$
$$= \frac{\frac{-17}{5}}{(x+3)}+\frac{\frac{17x-6}{5}}{(x^2+1)}$$
$$= \frac{-17}{5(x+3)}+\frac{17x-6}{5(x^2+1)}$$

5. $\quad \frac{3x+7}{(x+3)(x^2+4)} = \frac{A}{(x+3)}+\frac{Bx+c}{(x^2+4)}$ .... (i)

multiplying by $(x+3)(x^2+4)$ we get

$$3x+7 = A(x^2+4)+(Bx+c)(x+3) \text{ .... (ii)}$$
$$3x+7 = A(x^2+4)+B(x^2+3x)+C(x+3)$$
$$3x+7 = A(x^2+4)+B(x^2+3x)+C(x+3) \text{ .... (iii)}$$

put $x=-3$ in (ii)

$$3(-3)+7 = A((-3)^2+4)+(B(-3)+c)(-3+3)$$
$$-9+7 = A(9+4)$$
$$-2 = 13A$$
$$A = \frac{-2}{13}$$

Now, comparing coefficients of equation (iii)

$x^2;$ $A + B = 0$

as A = $\frac{-2}{13}$

$\frac{-2}{13} + B = 0$

$B = \frac{2}{13}$

$x;$ $3B + C = 3$

as B = $\frac{2}{13}$

$\frac{6}{13} + C = 3$

$C = 3 - \frac{6}{13}$

$C = \frac{39-6}{13}$

$C = \frac{33}{13}$

put the values in (i) we get

$$\frac{3x+7}{(x+3)(x^2+4)} = \frac{\frac{-2}{13}}{(x+3)} + \frac{\frac{2}{13}x + \frac{33}{13}}{(x^2+4)}$$

$$= \frac{\frac{-2}{13}}{(x+3)} + \frac{\frac{2x+33}{13}}{(x^2+1)}$$

$$= \frac{-2}{13(x+3)} + \frac{2x+33}{13(x^2+1)}$$

6. $\frac{x^2}{(x+2)(x^2+4)} = \frac{A}{(x+2)} + \frac{Bx+c}{(x^2+4)}$ .... (i)

multiplying by $(x+2)(x^2+4)$ we get

$x^2 = A(x^2+4) + (Bx+c)(x+2)$ .... (ii)

$x^2 = A(x^2+4) + B(x^2+2x) + C(x+2)$

$x^2 = A(x^2+4) + B(x^2+2x) + C(x+2)$ .... (iii)

put $x = -2$ in (ii)

$(-2)^2 = A((-2)^2+4) + (B(-2)+c)(-2+3)$

$4 = A(4+4)$

$4 = 8A$

$A = \frac{1}{2}$

Now, comparing coefficients of equation (iii)

$x^2;$ $A + B = 1$

as A = $\frac{1}{2}$

$\frac{1}{2} + B = 1$

$B = 1 - \frac{1}{2}$

$B = \frac{1}{2}$

$x;$ $2B + C = 0$

as B = $\frac{1}{2}$

$1 + C = 0$

$C = -1$

put the values in (i) we get

$$\frac{x^2}{(x+2)(x^2+4)} = \frac{\frac{1}{2}}{(x+2)} + \frac{\frac{1}{2}x-1}{(x^2+4)}$$

$$= \frac{\frac{1}{2}}{(x+3)} + \frac{\frac{x-2}{2}}{(x^2+1)}$$

$$= \frac{1}{2(x+3)} + \frac{x-2}{2(x^2+1)}$$

7. $\frac{1}{x^3+1} = \frac{1}{(x+1)(x^2-x+1)} = \frac{A}{(x+1)} + \frac{Bx+c}{(x^2-x+1)}$ .... (i)

multiplying by $(x+1)(x^2-x+1)$ we get

$1 = A(x^2-x+1) + (Bx+c)(x+1)$ .... (ii)

$1 = A(x^2-x+1) + B(x^2+x) + C(x+1)$

$1 = A(x^2-x+1) + B(x^2+x) + C(x+1)$ .... (iii)

put $x = -1$ in (ii)

$(-1)^2 = A((-1)^2 - (-1) + 1) + (B(-1)+c)(-1+1)$

$1 = A(1+1+1)$

$1 = 3A$

$A = \frac{1}{3}$

Now, comparing coefficients of equation (iii)

x^2; $A + B = 0$

as A = $\frac{1}{3}$

$\frac{1}{3} + B = 0$

$B = 0 - \frac{1}{3}$

$B = -\frac{1}{3}$

$Const$; $A + C = 1$

as A = $\frac{1}{3}$

$\frac{1}{3} + C = 1$

$C = 1 - \frac{1}{3}$

$C = \frac{3-1}{3}$

$C = \frac{2}{3}$

put the values in (i) we get

$$\frac{1}{x^3+1} = \frac{\frac{1}{3}}{(x+1)} + \frac{-\frac{1}{3}x+\frac{2}{3}}{(x^2-x+1)}$$

$$= \frac{\frac{1}{3}}{(x+1)} + \frac{\frac{-x+2}{3}}{(x^2-x+1)}$$

$$= \frac{1}{3(x+1)} + \frac{-x+2}{3(x^2-x+1)}$$

8. $\frac{x^2+1}{x^3+1} = \frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{A}{(x+1)} + \frac{Bx+c}{(x^2-x+1)}$ .... (i)

multiplying by $(x+1)(x^2-x+1)$ we get

$x^2+1 = A(x^2-x+1)+(Bx+c)(x+1)$ .... (ii)

$x^2+1 = A(x^2-x+1)+B(x^2+x)+C(x+1)$

$x^2+1 = A(x^2-x+1)+B(x^2+x)+C(x+1)$ .... (iii)

put $x=-1$ in (ii)

$(-1)^2+1 = A((-1)^2-(-1)+1)+(B(-1)+c)(-1+1)$

$2 = A(1+1+1)$

$2 = 3A$

$A = \frac{2}{3}$

Now, comparing coefficients of equation (iii)

$x^2;$ $A+B = 1$

as A = $\frac{2}{3}$

$\frac{2}{3}+B = 1$

$B = 1-\frac{2}{3}$

$B = 1-\frac{2}{3}$

$B = \frac{3-2}{3}$

$B = \frac{1}{3}$

$const;$ $A+C = 1$

as A = $\frac{2}{3}$

$\frac{2}{3}+C = 1$

$C = 1-\frac{2}{3}$

$C = \frac{3-2}{3}$

$C = \frac{1}{3}$

put the values in (i) we get

$\frac{x^2+1}{x^3+1} = \frac{\frac{2}{3}}{(x+1)} + \frac{\frac{1}{3}x+\frac{1}{3}}{(x^2-x+1)}$

$= \frac{\frac{2}{3}}{(x+1)} + \frac{\frac{x+1}{3}}{(x^2-x+1)}$

$= \frac{2}{3(x+1)} + \frac{x+1}{3(x^2-x+1)}$

Exercise 4.4

Resolve into partial fractions.

1. $\frac{x^3}{(x^2+4)^2} = \frac{Ax+B}{(x^2+4)} + \frac{Cx+D}{(x^2+4)^2}$ .... (i)

multiplying by $(x^2+4)^2$ we get

$x^3 = (Ax+B)(x^2+4) + (Cx+D)$ .... (ii)

$x^3 = Ax(x^2+4) + B(x^2+4) + Cx + D$

$x^3 = A(x^3+4x) + B(x^2+4) + Cx + D$ .... (iii)

Now, comparing coefficients of equation (iii)

x^3; $A = 1$

x^2; $B = 0$

x; $4A + C = 0$

as A = 1

$4 + C = 0$

$C = -4$

$const$; $4B + D = 0$

as B = 0

$0 + D = 0$

$D = 0$

put the values in (i) we get

$\frac{x^3}{(x^2+4)^2} = \frac{x}{(x^2+4)} + \frac{-4x}{(x^2+4)^2}$

2. $\frac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2+1)} + \frac{Dx+E}{(x^2+1)^2}$ .... (i)

multiplying by $(x+1)(x^2+1)^2$ we get

$x^4 + 3x^2 + x + 1 = A(x^2+1)^2 + (Bx+C)(x^2+1)(x+1) + (Dx+E)(x+1)$ .... (ii)

$x^4 + 3x^2 + x + 1 = A(x^4+2x^2+1) + (Bx+C)(x^3+x^2+x+1) + (Dx+E)(x+1)$

$x^4 + 3x^2 + x + 1 = A(x^4+2x^2+1) + Bx(x^3+x^2+x+1) + C(x^3+x^2+x+1) + Dx(x+1)$
$+E(x+1)$

$x^4 + 3x^2 + x + 1 = A(x^4+2x^2+1) + B(x^4+x^3+x^2+x) + C(x^3+x^2+x+1) + D(x^2+x)$
$+E(x+1)$ .... (iii)

Put $x = -1$ in equation (ii)

$(-1)^4 + 3(-1)^2 + (-1) + 1 = A((-1)^2+1)^2$

$1 + 3 - 1 + 1 = A(1+1)^2$

$4 = A(2)^2$

$4 = 4A$

$A = 1$

Now, comparing coefficients of equation (iii)

x^4; $A + B = 1$

As A = 1

$1 + B = 1$

$B = 0$

$x^3; \quad B + C = 0$

As B = 0

$0 + C = 0$

$C = 0$

$x^2; \quad 2A + B + C + D = 3$

As A = 1, B = 0, C = 0

$2 + 0 + 0 + D = 3$

$2 + D = 3$

$D = 1$

$x; \quad B + C + D + E = 1$

As B = 0, C = 0, D = 1

$0 + 0 + 1 + E = 1$

$1 + E = 1$

$E = 0$

put the values in (i) we get

$$\frac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2} = \frac{1}{(x+1)} + \frac{0+0}{(x^2+1)} + \frac{x+0}{(x^2+1)^2}$$

$$\frac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2} = \frac{1}{(x+1)} + \frac{x}{(x^2+1)^2}$$

3. $\frac{x^2}{(x+1)(x^2+1)^2} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2+1)} + \frac{Dx+E}{(x^2+1)^2}$ .... (i)

multiplying by $(x+1)(x^2+1)^2$ we get

$x^2 = A(x^2+1)^2 + (Bx+C)(x^2+1)(x+1) + (Dx+E)(x+1)$ .... (ii)

$x^2 = A(x^4+2x^2+1) + (Bx+C)(x^3+x^2+x+1) + (Dx+E)(x+1)$

$x^2 = A(x^4+2x^2+1) + Bx(x^3+x^2+x+1) + C(x^3+x^2+x+1) + Dx(x+1) + E(x+1)$

$x^2 = A(x^4+2x^2+1) + B(x^4+x^3+x^2+x) + C(x^3+x^2+x+1) + D(x^2+x) + E(x+1)$ ...... (iii)

Put $x = -1$ in equation (ii)

$(-1)^2 = A((-1)^2+1)^2$

$1 = A(1+1)^2$

$1 = A(2)^2$

$1 = 4A$

$A = \frac{1}{4}$

Now, comparing coefficients of equation (iii)

$x^4; \quad A + B = 0$

As A = $\frac{1}{4}$

$\frac{1}{4} + B = 0$

$B = -\frac{1}{4}$

$x^3; \quad B + C = 0$

As B = $-\frac{1}{4}$

$$-\frac{1}{4} + C = 0$$

$$C = \frac{1}{4}$$

x^2; $2A + B + C + D = 1$

As A = $\frac{1}{4}$, B = $-\frac{1}{4}$, C = $\frac{1}{4}$

$$\frac{1}{2} - \frac{1}{4} + \frac{1}{4} + D = 1$$

$$\frac{1}{2} + D = 1$$

$$D = 1 - \frac{1}{2}$$

$$D = \frac{1}{2}$$

x; $B + C + D + E = 0$

As B = $-\frac{1}{4}$, C = $\frac{1}{4}$, D = $\frac{1}{2}$

$$-\frac{1}{4} + \frac{1}{4} + \frac{1}{2} + E = 0$$

$$\frac{1}{2} + E = 0$$

$$E = -\frac{1}{2}$$

put the values in (i) we get

$$\frac{x^2}{(x+1)(x^2+1)^2} = \frac{\frac{1}{4}}{(x+1)} + \frac{-\frac{1}{4}x + \frac{1}{4}}{(x^2+1)} + \frac{\frac{1}{2}x - \frac{1}{2}}{(x^2+1)^2}$$

$$\frac{x^2}{(x+1)(x^2+1)^2} = \frac{\frac{1}{4}}{(x+1)} + \frac{\frac{-x+1}{4}}{(x^2+1)} + \frac{\frac{x-1}{2}}{(x^2+1)^2}$$

$$\frac{x^2}{(x+1)(x^2+1)^2} = \frac{1}{4(x+1)} + \frac{-x+1}{4(x^2+1)} + \frac{x-1}{2(x^2+1)^2}$$

4. $\frac{x^2}{(x-1)(x^2+1)^2} = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+1)} + \frac{Dx+E}{(x^2+1)^2}$ .... (i)

multiplying by $(x-1)(x^2+1)^2$ we get

$x^2 = A(x^2+1)^2 + (Bx+C)(x^2+1)(x-1) + (Dx+E)(x-1)$ .... (ii)

$x^2 = A(x^4+2x^2+1) + (Bx+C)(x^3-x^2+x-1) + (Dx+E)(x-1)$

$x^2 = A(x^4+2x^2+1) + Bx(x^3-x^2+x-1) + C(x^3-x^2+x-1) + Dx(x-1) + E(x-1)$

$x^2 = A(x^4+2x^2+1) + B(x^4-x^3+x^2-x) + C(x^3-x^2+x-1) + D(x^2-x) + E(x-1)$ ...... (iii)

Put $x = 1$ in equation (i)

$$(1)^2 = A((1)^2+1)^2$$

$$1 = A(1+1)^2$$

$$1 = A(2)^2$$

$$1 = 4A$$

$$A = \frac{1}{4}$$

Now, comparing coefficients of equation (iii)

x^4; $A + B = 0$

As A = $\frac{1}{4}$

$\frac{1}{4} + B \quad = 0$

$B \quad = -\frac{1}{4}$

$x^3; \quad -B + C = 0$

As B = $-\frac{1}{4}$

$\frac{1}{4} + C \quad = 0$

$C \quad = -\frac{1}{4}$

$x^2; \quad 2A + B - C + D = 1$

As A = $\frac{1}{4}$, B = $-\frac{1}{4}$, C = $-\frac{1}{4}$

$\frac{1}{2} - \frac{1}{4} + \frac{1}{4} + D \quad = 1$

$\frac{1}{2} + D \quad = 1$

$D \quad = 1 - \frac{1}{2}$

$D \quad = \frac{1}{2}$

$x; \quad -B + C - D + E = 0$

As B = $-\frac{1}{4}$, C = $-\frac{1}{4}$, D = $\frac{1}{2}$

$+\frac{1}{4} - \frac{1}{4} - \frac{1}{2} + E \quad = 0$

$-\frac{1}{2} + E \quad = 0$

$E \quad = \frac{1}{2}$

put the values in (i) we get

$$\frac{x^2}{(x-1)(x^2+1)^2} = \frac{\frac{1}{4}}{(x-1)} + \frac{-\frac{1}{4}x - \frac{1}{4}}{(x^2+1)} + \frac{\frac{1}{2}x + \frac{1}{2}}{(x^2+1)^2}$$

$$\frac{x^2}{(x-1)(x^2+1)^2} = \frac{\frac{1}{4}}{(x-1)} + \frac{\frac{-x-1}{4}}{(x^2+1)} + \frac{\frac{x+1}{2}}{(x^2+1)^2}$$

$$\frac{x^2}{(x-1)(x^2+1)^2} = \frac{1}{4(x-1)} - \frac{x+1}{4(x^2+1)} + \frac{x+1}{2(x^2+1)^2}$$

5. $\frac{x^4}{(x^2+2)^2} = \frac{x^4}{x^4+4x+4}$

By long division

$$\frac{x^4}{x^4+4x+4} = 1 + \frac{-4x^2-4}{x^4+4x+4} \qquad = 1 - \frac{4x^2+4}{(x^2+2)^2} \text{ .... (i)}$$

$$\frac{4x^2+4}{(x^2+2)^2} = \frac{Ax+B}{(x^2+2)} + \frac{Cx+D}{(x^2+2)^2} \text{ .... (ii)}$$

multiplying by $(x^2+2)^2$ we get

$4x^2 + 4 \quad = (Ax + B)(x^2 + 2) + (Cx + D)$

$4x^2 + 4 \quad = (Ax + B)(x^2 + 2) + (Cx + D)$

$4x^2 + 4 \quad = Ax(x^2 + 2) + B(x^2 + 2) + Cx + D$

$4x^2 + 4 \quad = A(x^3 + 2x) + B(x^2 + 2) + Cx + D$ .... (iii)

Now, comparing coefficients of equation (iii)

$x^3; \quad A \quad = 0$

x^2; $B = 4$

x; $2A + C = 0$

As A = 0

$0 + C = 0$

$C = 0$

$const$; $2B + D = 4$

As B = 4

$8 + D = 4$

$D = -4$

put the values in (ii) we get

$$\frac{4x^2+4}{(x^2+2)^2} = \frac{0+4}{(x^2+2)} + \frac{0-4}{(x^2+2)^2}$$

$$\frac{4x^2+4}{(x^2+2)^2} = \frac{4}{(x^2+2)} - \frac{4}{(x^2+2)^2}$$

put the values in (i) we get

$$\frac{x^4}{(x^2+2)^2} = 1 - \left[\frac{4}{(x^2+2)} - \frac{4}{(x^2+2)^2}\right]$$

$$= 1 - \frac{4}{(x^2+2)} + \frac{4}{(x^2+2)^2}$$

6. $\frac{x^5}{(x^2+1)^2} = \frac{x^5}{x^4+2x+1}$

By long division

$$\frac{x^5}{x^4+2x+1} = x + \frac{-2x^3-x}{x^4+2x+1} = x - \frac{2x^3+x}{(x^2+1)^2} \quad \text{.......... (i)}$$

$$\frac{2x^3+x}{(x^2+1)^2} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+1)^2} \quad \text{.......................... (ii)}$$

multiplying by $(x^2+1)^2$ we get

$2x^3 + x = (Ax + B)(x^2 + 1) + (Cx + D)$

$2x^3 + x = (Ax + B)(x^2 + 1) + (Cx + D)$

$2x^3 + x = Ax(x^2 + 1) + B(x^2 + 1) + Cx + D$

$2x^3 + x = A(x^3 + x) + B(x^2 + 1) + Cx + D$ .... (iii)

Now, comparing coefficients of equation (iii)

x^3; $A = 2$

x^2; $B = 0$

x; $A + C = 1$

As A = 2

$2 + C = 1$

$C = -1$

$const$; $B + D = 0$

As B = 0

$0 + D = 0$

$D = 0$

put the values in (ii) we get

$$\frac{2x^3+x}{(x^2+1)^2} = \frac{2x+0}{(x^2+1)} + \frac{-x+0}{(x^2+1)^2}$$

$$\frac{2x^3+x}{(x^2+1)^2} = \frac{2x}{(x^2+1)} - \frac{x}{(x^2+1)^2}$$

put the values in (i) we get

$$\frac{2x^3+x}{(x^2+1)^2} = x - \left[\frac{2x}{(x^2+1)} - \frac{x}{(x^2+1)^2}\right]$$

$$= x - \frac{2x}{(x^2+1)} + \frac{x}{(x^2+1)^2}$$

Exercise 5.1

N = The set of natural numbers = $\{1, 2, 3, 4, \cdots\}$

W = The set of whole numbers = $\{0, 1, 2, 3, 4, \cdots\}$

Z = The set of all integers = $\{0, \pm1, \pm2, \pm3, \cdots\}$

E = The set of all even integers = $\{0, \pm2, \pm4, \cdots\}$

O = The set of all odd integers = $\{\pm1, \pm3, \pm5, \cdots\}$

P = The set of prime numbers = $\{2, 3, 5, 7, 11, 13, 17, \cdots\}$

Q = The set of all rational numbers = $\{x \mid x = \frac{m}{n}$, where $m, n \in Z$ and $n \neq 0\}$

Q' = The set of all irrational numbers = $\{x \mid x \neq \frac{m}{n}$, where $m, n \in Z$ and $n \neq 0\}$

R = The set of all real numbers = $Q \cup Q'$

Q. 1: If $X = \{1, 4, 7, 9\}$ and $Y = \{2, 4, 5, 9\}$

Then find:

(i) $X \cup Y = \{1, 4, 7, 9\} \cup \{2, 4, 5, 9\}$
$= \{1, 2, 4, 5, 7, 9\}$

(ii) $X \cap Y = \{1, 4, 7, 9\} \cap \{2, 4, 5, 9\}$
$= \{4, 9\}$

(iii) $Y \cup X = \{2, 4, 5, 9\} \cup \{1, 4, 7, 9\}$
$= \{1, 2, 4, 5, 7, 9\}$

(iv) $Y \cap X = \{2, 4, 5, 9\} \cap \{1, 4, 7, 9\}$
$= \{4, 9\}$

Q. 2: If $X =$ *Set of prime numbers less than or equal to* 17

and $Y =$ *Set of first* 12 *natural numbers, then find the following*

$X = \{2, 3, 5, 7, 11, 13, 17\}$

$Y = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

(i) $X \cup Y = \{2, 3, 5, 7, 11, 13, 17\} \cup \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 17\}$
$= Y \cup \{13, 17\}$

(ii) $X \cap Y = \{2, 3, 5, 7, 11, 13, 17\} \cap \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
$= \{2, 3, 5, 7, 11\}$

(iii) $Y \cup X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} \cup \{2, 3, 5, 7, 11, 13, 17\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 17\}$
$= Y \cup \{13, 17\}$

(iv) $Y \cap X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} \cap \{2, 3, 5, 7, 11, 13, 17\}$
$= \{2, 3, 5, 7, 11\}$

Q. 3: **If** $\boldsymbol{X = \emptyset, Y = Z^+, T = O^+}$**, then find:**

$X = \{\ \}$

$Y = \{1, 2, 3, 4, 5, \dots\}$

$T = \{1, 3, 5, \dots\}$

(i) $X \cup Y$ $= \{\ \} \cup \{1, 2, 3, 4, 5, \dots\}$

$= \{1, 2, 3, 4, 5, \dots\}$

$= Y$

(ii) $X \cup T$ $= \{\ \} \cup \{1, 3, 5, \dots\}$

$= \{1, 3, 5, \dots\}$

$= T$

(iii) $Y \cup T$ $= \{1, 2, 3, 4, 5, \dots\} \cup \{1, 3, 5, \dots\}$

$= \{1, 2, 3, 4, 5, \dots\}$

$= Y$

(iv) $X \cap Y$ $= \{\ \} \cap \{1, 2, 3, 4, 5, \dots\}$

$= \{\ \}$

(v) $X \cap T$ $= \{\ \} \cap \{1, 3, 5, \dots\}$

$= \{\ \}$

(vi) $Y \cap T$ $= \{1, 2, 3, 4, 5, \dots\} \cap \{1, 3, 5, \dots\}$

$= \{1, 3, 5, \dots\}$

$= T$

Q. 4: **If** $\boldsymbol{U = \{x | x \in N \wedge 3 < x \leq 25\}}$

$\boldsymbol{X = \{x | x \text{ is prime} \wedge 8 < x < 25\}}$

and $\boldsymbol{Y = \{x | x \in W \wedge 4 \leq x \leq 17\}}$

Find the value of:

So,

$U = \{4, 5, 6, \dots \dots, 25\}$

$X = \{11, 13, 17, 19, 23\}$

$Y = \{4, 5, 6, \dots \dots, 17\}$

(i) $(X \cup Y)'$

$(X \cup Y)'$ $= U - (X \cup Y)$

$= \{4, 5, 6, \dots \dots, 25\} - (\{11, 13, 17, 19, 23\} \cup \{4, 5, 6, \dots \dots, 17\})$

$= \{4, 5, 6, \dots \dots, 25\} - \{4, 5, 6, \dots \dots, 17, 19, 23\}$

$= \{18, 20, 21, 22, 24, 25\}$

(ii) $X' \cap Y'$

$X' \cap Y' = (U - X) \cap (U - Y)$

$= (\{4, 5, 6, \dots \dots, 25\} - \{11, 13, 17, 19, 23\}) \cap (\{4, 5, 6, \dots \dots, 25\} - \{4, 5, 6, \dots \dots, 17\})$

$= \{4, 5, 6, \dots, 10, 11, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25\} \cap \{18, 19, 20, 21, 22, 23, 24, 25\}$

$= \{18, 20, 21, 22, 24, 25\}$

(iii) $(X \cap Y)'$

$(X \cap Y)'$ $= U - (X \cap Y)$

$= \{4, 5, 6, \dots \dots, 25\} - (\{11, 13, 17, 19, 23\} \cap \{4, 5, 6, \dots \dots, 17\})$

$= \{4, 5, 6, \dots \dots, 25\} - \{11, 13, 17\}$
$= \{4, 5, 6, \dots 10, 12, 14, 15, 16, 18, 19, 20, .., 25\}$

(iv) $X' \cup Y'$

$X' \cup Y' = (U - X) \cup (U - Y)$
$= (\{4, 5, 6, \dots \dots, 25\} - \{11, 13, 17, 19, 23\}) \cup (\{4, 5, 6, \dots \dots, 25\} - \{4, 5, 6, \dots \dots., 17\})$
$= \{4, 5, 6, \dots, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25\} \cup \{18, 19, 20, 21, 22, 23, 24, 25\}$
$= \{4, 5, 6, \dots 10, 12, 14, 15, 16, 18, 19, 20, .., 25\}$

Q. 5: If $X = \{2, 4, 6, \dots, 20\}$ and $Y = \{4, 8, 12, \dots, 24\}$ then find the following:

(i) $X - Y = \{2, 4, 6, \dots, 20\} - \{4, 8, 12, \dots, 24\}$
$= \{2, 6, 10, 14, 18\}$

(ii) $Y - X = \{4, 8, 12, \dots, 24\} - \{2, 4, 6, \dots, 20\}$
$= \{24\}$

Q. 6: If $A = N$ and $B = W$ then find the following:

$A = \{1, 2, 3, \dots \dots\}$
$B = \{0, 1, 2, \dots \dots\}$

(i) $A - B = \{1, 2, 3, \dots \dots\} - \{0, 1, 2, \dots \dots\}$
$= \{\ \}$

(ii) $B - A = \{0, 1, 2, \dots \dots\} - \{1, 2, 3, \dots \dots\}$
$= \{0\}$

Exercise 5.2

Q. 1: If $X = \{1, 3, 5, 7, \dots, 19\}$, $Y = \{0, 2, 4, 6, \dots, 20\}$ and $Z = \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$,
Then find the following:

(i) $X \cup (Y \cup Z)$ $= \{1, 3, 5, 7, \dots, 19\} \cup (\{0, 2, 4, 6, \dots, 20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$
$= \{1, 3, 5, 7, \dots, 19\} \cup \{0, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 23\}$
$= \{0, 1, 2, 3, 4, \dots, 20, 23\}$

(ii) $(X \cup Y) \cup Z$ $= (\{1, 3, 5, 7, \dots, 19\} \cup \{0, 2, 4, 6, \dots, 20\}) \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$
$= \{0, 1, 3, 4, \dots, 20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$
$= \{0, 1, 2, 3, \dots, 20, 23\}$

(iii) $X \cap (Y \cap Z)$ $= \{1, 3, 5, 7, \dots, 19\} \cap (\{0, 2, 4, 6, \dots, 20\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$
$= \{1, 3, 5, 7, \dots, 19\} \cap \{2\}$
$= \{\ \}$

(iv) $(X \cap Y) \cap Z$ $= (\{1, 3, 5, 7, \dots, 19\} \cap \{0, 2, 4, 6, \dots, 20\}) \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$
$= \{\ \} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$
$= \{\ \}$

(v) $X \cup (Y \cap Z)$ $= \{1, 3, 5, 7, \dots, 19\} \cup (\{0, 2, 4, 6, \dots, 20\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$
$= \{1, 3, 5, 7, \dots, 19\} \cup \{2\}$
$= \{1, 2, 3, 5, 7, \dots, 19\}$

(vi) $(X \cup Y) \cap (X \cup Z) =$
$(\{1, 3, 5, 7, \dots, 19\} \cup \{0, 2, 4, 6, \dots, 20\}) \cap (\{1, 3, 5, 7, \dots, 19\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$
$= \{0, 1, 2, 3, \dots, 20\} \cap \{1, 2, 3, 5, 7, \dots, 19, 23\}$
$= \{1, 2, 3, 5, 7, \dots, 19\}$

(vii) $X \cap (Y \cup Z)$ $= \{1, 3, 5, 7, \dots, 19\} \cap (\{0, 2, 4, 6, \dots, 20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$
$= \{1, 3, 5, 7, \dots, 19\} \cap \{0, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 23\}$
$= \{3, 5, 7, 11, 13, 17, 19\}$

(viii) $(X \cap Y) \cup (X \cap Z) =$
$(\{1, 3, 5, 7, \dots, 19\} \cap \{0, 2, 4, 6, \dots, 20\}) \cup (\{1, 3, 5, 7, \dots, 19\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$
$= \{\ \} \cup \{3, 5, 7, 11, 13, 17, 19\}$
$= \{3, 5, 7, 11, 13, 17, 19\}$

Q. 2: If $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6, 8\}$, $C = \{1, 4, 8\}$
Prove the following identities

(i) $A \cap B = B \cap A$

$L.H.S = A \cap B$
$= \{1, 2, 3, 4, 5, 6\} \cap \{2, 4, 6, 8\}$
$= \{2, 4, 6\}$

$R.H.S = B \cap A$
$= \{2, 4, 6, 8\} \cap \{1, 2, 3, 4, 5, 6\}$
$= \{2, 4, 6\}$

(ii) $A \cup B = B \cup A$

$L.H.S = A \cup B$

$= \{1, 2, 3, 4, 5, 6\} \cup \{2, 4, 6, 8\}$

$= \{1, 2, 3, 4, 5, 6, 8\}$

$R.H.S = B \cup A$

$= \{2, 4, 6, 8\} \cup \{1, 2, 3, 4, 5, 6\}$

$= \{1, 2, 3, 4, 5, 6, 8\}$

(iii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$L.H.S = A \cap (B \cup C)$

$= \{1, 2, 3, 4, 5, 6\} \cap (\{2, 4, 6, 8\} \cup \{1, 4, 8\})$

$= \{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 4, 6, 8\}$

$= \{1, 2, 4, 6\}$

$R.H.S = (A \cap B) \cup (A \cap C)$

$= (\{1, 2, 3, 4, 5, 6\} \cap \{2, 4, 6, 8\}) \cup (\{1, 2, 3, 4, 5, 6\} \cap \{1, 4, 8\})$

$= \{2, 4, 6\} \cup \{1, 4\}$

$= \{1, 2, 4, 6\}$

(iv) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$L.H.S = A \cup (B \cap C)$

$= \{1, 2, 3, 4, 5, 6\} \cup (\{2, 4, 6, 8\} \cap \{1, 4, 8\})$

$= \{1, 2, 3, 4, 5, 6\} \cup \{4, 8\}$

$= \{1, 2, 3, 4, 5, 6, 8\}$

$R.H.S = (A \cup B) \cap (A \cup C)$

$= (\{1, 2, 3, 4, 5, 6\} \cup \{2, 4, 6, 8\}) \cap (\{1, 2, 3, 4, 5, 6\} \cup \{1, 4, 8\})$

$= \{1, 2, 3, 4, 5, 6, 8\} \cap \{1, 2, 3, 4, 5, 6, 8\}$

$= \{1, 2, 3, 4, 5, 6, 8\}$

Q. 3: If $\boldsymbol{U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}}$, $\boldsymbol{A = \{1, 3, 5, 7, 9\}}$, $\boldsymbol{B = \{2, 3, 5, 7\}}$, then verify the De-Morgan's Laws i.e., $\boldsymbol{(A \cap B)' = A' \cup B'}$ and $\boldsymbol{(A \cup B)' = A' \cap B'}$

$(A \cap B)' = A' \cup B'$

$L.H.S = (A \cap B)' = U - (A \cap B)$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - (\{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 7\})$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{3, 5, 7\}$

$= \{1, 2, 4, 6, 8, 9, 10\}$

$R.H.S = A' \cup B' = (U - A) \cup (U - B)$

$= (\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 3, 5, 7, 9\}) \cup (\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{2, 3, 5, 7\})$

$= \{2, 4, 6, 8, 10\} \cup \{1, 4, 6, 8, 9, 10\}$

$= \{1, 2, 4, 6, 8, 9, 10\}$

So, $L.H.S = R.H.S$

Now

$(A \cup B)' = A' \cap B'$

$L.H.S = (A \cup B)' = U - (A \cup B)$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - (\{1, 3, 5, 7, 9\} \cup \{2, 3, 5, 7\})$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 2, 3, 5, 7, 9\}$

$= \{4, 6, 8, 10\}$

$R.H.S = A' \cap B' \qquad = (U - A) \cap (U - B)$

$= (\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 3, 5, 7, 9\}) \cap (\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{2, 3, 5, 7\})$

$= \{2, 4, 6, 8, 10\} \cap \{1, 4, 6, 8, 9, 10\}$

$= \{4, 6, 8, 10\}$

So, $L.H.S = R.H.S$

Q. 4: If $\boldsymbol{U = \{1, 2, 3, \ldots\ldots, 20\}}$, $\boldsymbol{X = \{1, 3, 7, 9, 15, 18, 20\}}$, $\boldsymbol{Y = \{1, 3, 5, \ldots\ldots, 17\}}$, then show that

(i) $X - Y = X \cap Y'$

$L.H.S = X - Y$

$= \{1, 3, 7, 9, 15, 18, 20\} - \{1, 3, 5, \ldots\ldots, 17\}$

$= \{18, 20\}$

$R.H.S = X \cap Y'$

$= \{1, 3, 7, 9, 15, 18, 20\} \cap (\{1, 2, 3, \ldots\ldots, 20\} - \{1, 3, 5, \ldots\ldots, 17\})$

$= \{1, 3, 7, 9, 15, 18, 20\} \cap \{2, 4, 6, \ldots., 18, 19, 20\}$

$= \{18, 20\}$

So, $L.H.S = R.H.S$

(ii) $Y - X = Y \cap X'$

$L.H.S = Y - X$

$= \{1, 3, 5, \ldots\ldots, 17\} - \{1, 3, 7, 9, 15, 18, 20\}$

$= \{5, 11, 13, 17\}$

$R.H.S = Y \cap X'$

$= \{1, 3, 5, \ldots\ldots, 17\} \cap (\{1, 2, 3, \ldots\ldots, 20\} - \{1, 3, 7, 9, 15, 18, 20\})$

$= \{1, 3, 5, \ldots\ldots, 17\} \cap \{2, 4, 5, 6, 8, 10, 11, 12, 13, 14, 16, 17, 19\}$

$= \{5, 11, 13, 17\}$

So, $L.H.S = R.H.S$

Exercise 5.3

Q. 1: If $U = \{1, 2, 3, 4, \ldots\ldots, 10\}$, $A = \{1, 3, 5, 7, 9\}$ and $B = \{1, 4, 7, 10\}$, Then verify the following questions.

(i) $A - B = A \cap B'$

$L.H.S = A - B$

$= \{1, 3, 5, 7, 9\} - \{1, 4, 7, 10\}$

$= \{1, 3, 5, 9\}$

$R.H.S = A \cap B'$

$= \{1, 3, 5, 7, 9\} \cap (\{1, 2, 3, \ldots\ldots, 10\} - \{1, 4, 7, 10\})$

$= \{1, 3, 5, 7, 9\} \cap \{1, 2, 3, 5, 6, 8, 9\}$

$= \{1, 3, 5, 9\}$

So, $L.H.S = R.H.S$

(ii) $B - A = B \cap A'$

$L.H.S = B - A$

$= \{1, 4, 7, 10\} - \{1, 3, 5, 7, 9\}$

$= \{4, 10\}$

$R.H.S = B \cap A'$

$= \{1, 4, 7, 10\} \cap (\{1, 2, 3, \ldots\ldots, 10\} - \{1, 3, 5, 7, 9\})$

$= \{1, 4, 7, 10\} \cap \{2, 4, 6, 8, 10\}$

$= \{4, 10\}$

So, $L.H.S = R.H.S$

(iii) $(A \cup B)' = A' \cap B'$

$L.H.S = (A \cup B)' \quad = U - (A \cup B)$

$= \{1, 2, 3, \ldots\ldots, 10\} - (\{1, 3, 5, 7, 9\} \cup \{1, 4, 7, 10\})$

$= \{1, 2, 3, \ldots\ldots, 10\} - \{1, 3, 4, 5, 7, 9, 10\}$

$= \{2, 6, 8\}$

$R.H.S = A' \cap B' \quad = (U - A) \cap (U - B)$

$= (\{1, 2, 3, \ldots\ldots, 10\} - \{1, 3, 5, 7, 9\}) \cap (\{1, 2, 3, \ldots\ldots, 10\} - \{1, 4, 7, 10\})$

$= \{2, 4, 6, 8, 10\} \cap \{2, 3, 5, 6, 8, 9\}$

$= \{2, 6, 8\}$

So, $L.H.S = R.H.S$

(iv) $(A \cap B)' = A' \cup B'$

$L.H.S = (A \cap B)' \quad = U - (A \cap B)$

$= \{1, 2, 3, \ldots\ldots, 10\} - (\{1, 3, 5, 7, 9\} \cap \{1, 4, 7, 10\})$

$= \{1, 2, 3, \ldots\ldots, 10\} - \{1, 7\}$

$= \{2, 3, 4, 5, 6, 8, 9, 10\}$

$R.H.S = A' \cup B' \quad = (U - A) \cup (U - B)$

$= (\{1, 2, 3, \ldots\ldots, 10\} - \{1, 3, 5, 7, 9\}) \cup (\{1, 2, 3, \ldots\ldots, 10\} - \{1, 4, 7, 10\})$

$= \{2, 4, 6, 8, 10\} \cup \{2, 3, 5, 6, 8, 9\}$
$= \{2, 3, 4, 5, 6, 8, 9, 10\}$

So, $L.H.S = R.H.S$

(v) $(A - B)' = A' \cup B$

$L.H.S = (A - B)' = U - (A - B)$
$= \{1, 2, 3, \dots\dots, 10\} - (\{1, 3, 5, 7, 9\} - \{1, 4, 7, 10\})$
$= \{1, 2, 3, \dots\dots, 10\} - \{3, 5, 9\}$
$= \{1, 2, 4, 6, 7, 8, 10\}$

$R.H.S = A' \cup B = (U - A) \cup B$
$= (\{1, 2, 3, \dots\dots, 10\} - \{1, 3, 5, 7, 9\}) \cup \{1, 4, 7, 10\}$
$= \{2, 4, 6, 8, 10\} \cup \{1, 4, 7, 10\}$
$= \{1, 2, 4, 6, 7, 8, 10\}$

So, $L.H.S = R.H.S$

(vi) $(B - A)' = B' \cup A$

$L.H.S = (B - A)' = U - (B - A)$
$= \{1, 2, 3, \dots\dots, 10\} - (\{1, 4, 7, 10\} - \{1, 3, 5, 7, 9\})$
$= \{1, 2, 3, \dots\dots, 10\} - \{4, 10\}$
$= \{1, 2, 3, 5, 6, 7, 8, 9\}$

$R.H.S = B' \cup A = (U - B) \cup A$
$= (\{1, 2, 3, \dots\dots, 10\} - \{1, 4, 7, 10\}) \cup \{1, 3, 5, 7, 9\}$
$= \{2, 3, 5, 6, 8, 9\} \cup \{1, 3, 5, 7, 9\}$
$= \{1, 2, 3, 5, 6, 7, 8, 9\}$

So, $L.H.S = R.H.S$

Q. 2: If $U = \{1, 2, 3, 4, \dots\dots, 10\}$, $A = \{1, 3, 5, 7, 9\}$, $B = \{1, 4, 7, 10\}$, $C = \{1, 5, 8, 10\}$ Then verify the following questions.

(i) $(A \cup B) \cup C = A \cup (B \cup C)$

$L.H.S = (A \cup B) \cup C$
$= (\{1, 3, 5, 7, 9\} \cup \{1, 4, 7, 10\}) \cup \{1, 5, 8, 10\}$
$= \{1, 3, 4, 5, 7, 9, 10\} \cup \{1, 5, 8, 10\}$
$= \{1, 3, 4, 5, 7, 8, 9, 10\}$

$R.H.S = A \cup (B \cup C)$
$= \{1, 3, 5, 7, 9\} \cup (\{1, 4, 7, 10\} \cup \{1, 5, 8, 10\})$
$= \{1, 3, 5, 7, 9\} \cup \{1, 4, 5, 7, 8, 10\}$
$= \{1, 3, 4, 5, 7, 8, 9, 10\}$

So, $L.H.S = R.H.S$

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$

$L.H.S = (A \cap B) \cap C$
$= (\{1, 3, 5, 7, 9\} \cap \{1, 4, 7, 10\}) \cap \{1, 5, 8, 10\}$
$= \{1, 7\} \cap \{1, 5, 8, 10\}$
$= \{1\}$

$R.H.S = A \cap (B \cap C)$

$= \{1, 3, 5, 7, 9\} \cap (\{1, 4, 7, 10\} \cap \{1, 5, 8, 10\})$

$= \{1, 3, 5, 7, 9\} \cap \{1, 10\}$

$= \{1\}$

So, $L.H.S = R.H.S$

(iii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$L.H.S = A \cup (B \cap C)$

$= \{1, 3, 5, 7, 9\} \cup (\{1, 4, 7, 10\} \cap \{1, 5, 8, 10\})$

$= \{1, 3, 5, 7, 9\} \cup \{1, 10\}$

$= \{1, 3, 5, 7, 9, 10\}$

$R.H.S = (A \cup B) \cap (A \cup C)$

$= (\{1, 3, 5, 7, 9\} \cup \{1, 4, 7, 10\}) \cap (\{1, 3, 5, 7, 9\} \cup \{1, 5, 8, 10\})$

$= \{1, 3, 4, 5, 7, 9, 10\} \cap \{1, 3, 5, 7, 8, 9, 10\}$

$= \{1, 3, 5, 7, 9, 10\}$

(iv) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$L.H.S = A \cap (B \cup C)$

$= \{1, 3, 5, 7, 9\} \cap (\{1, 4, 7, 10\} \cup \{1, 5, 8, 10\})$

$= \{1, 3, 5, 7, 9\} \cap \{1, 4, 5, 7, 8, 10\}$

$= \{1, 5, 7\}$

$R.H.S = (A \cap B) \cup (A \cap C)$

$= (\{1, 3, 5, 7, 9\} \cap \{1, 4, 7, 10\}) \cup (\{1, 3, 5, 7, 9\} \cap \{1, 5, 8, 10\})$

$= \{1, 7\} \cup \{1, 5\}$

$= \{1, 5, 7\}$

Q. 3: If $U = N$, then verify De-Morgans's laws by using $A = \emptyset, B = P$.

$\boldsymbol{U} = \{1, 2, 3, \dots \dots.\}$

$\boldsymbol{A} = \{\ \}$

$\boldsymbol{B} = \{2, 3, 5, 7, \dots \dots.\}$

$(A \cap B)' = A' \cup B'$

$L.H.S = (A \cap B)' \quad = U - (A \cap B)$

$= \{1, 2, 3, \dots \dots.\} - (\{\ \} \cap \{2, 3, 5, 7, \dots \dots.\})$

$= \{1, 2, 3, \dots \dots.\} - \{\ \}$

$= \{1, 2, 3, \dots \dots.\}$

$R.H.S = A' \cup B' \quad = (U - A) \cup (U - B)$

$= (\{1, 2, 3, \dots \dots.\} - \{\ \}) \cup (\{1, 2, 3, \dots \dots.\} - \{2, 3, 5, 7, \dots \dots.\})$

$= \{1, 2, 3, \dots \dots.\} \cup \{1, 4, 6, 8, 9, 10, 12, \dots \dots\}$

$= \{1, 2, 3, \dots \dots.\}$

So, $L.H.S = R.H.S$

Now

$(A \cup B)' = A' \cap B'$

$L.H.S = (A \cup B)' \quad = U - (A \cup B)$

$= \{1, 2, 3, \dots \dots.\} - (\{\ \} \cup \{2, 3, 5, 7, \dots \dots.\})$

$= \{1, 2, 3, \dots \dots.\} - \{2, 3, 5, 7, \dots \dots.\}$

$= \{1, 4, 6, 8, 9, 10, 12, \dots \dots\}$

$R.H.S = A' \cap B' \quad = (U - A) \cap (U - B)$

$= (\{1, 2, 3, \dots \dots ..\} - \{\ \ \}) \cap (\{1, 2, 3, \dots \dots ..\} - \{2, 3, 5, 7, \dots \dots ..\})$

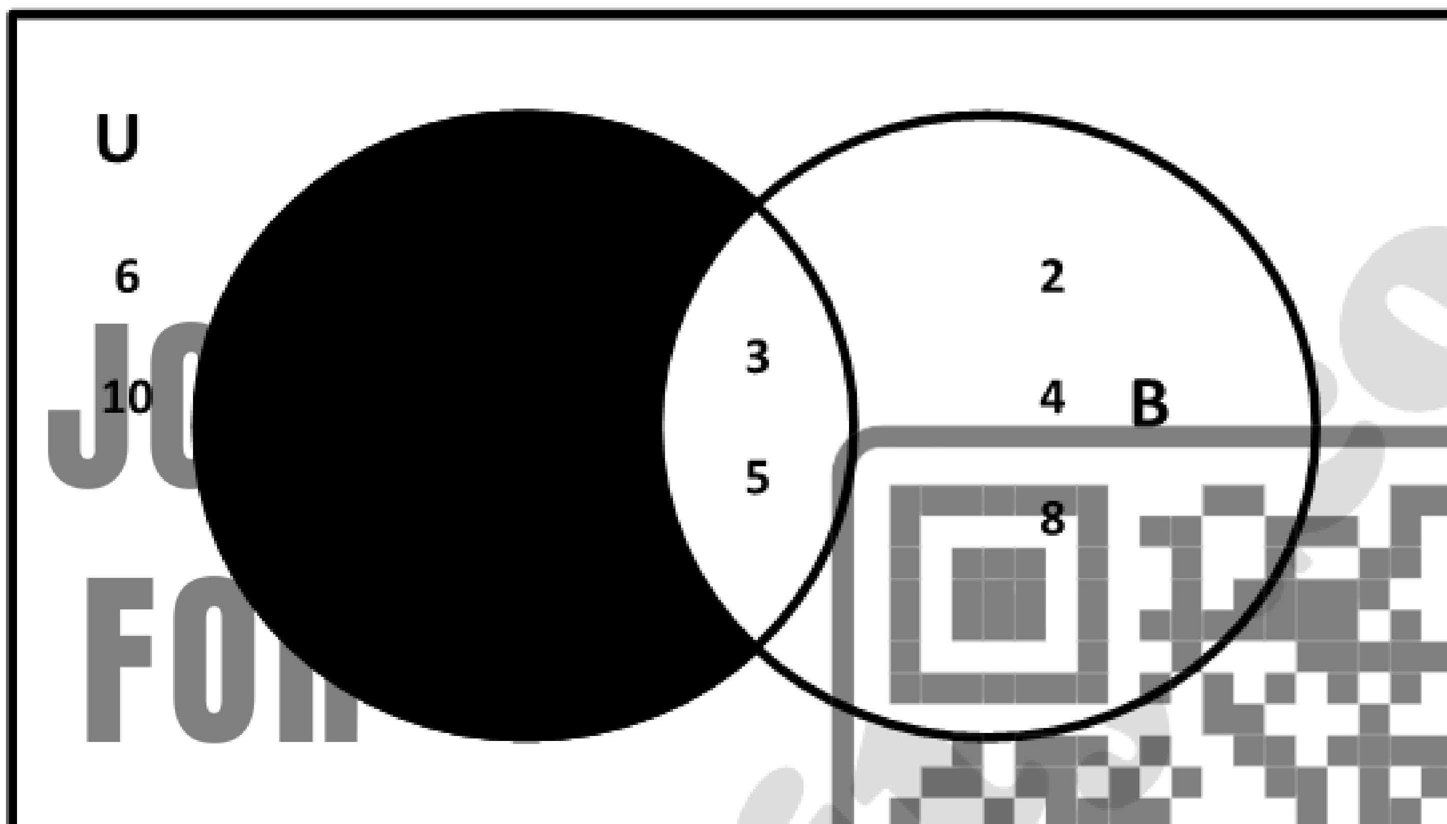
$= \{1, 2, 3, \dots \dots ..\} \cap \{1, 4, 6, 8, 9, 10, 12, \dots \dots\}$

$= \{1, 4, 6, 8, 9, 10, 12, \dots \dots\}$

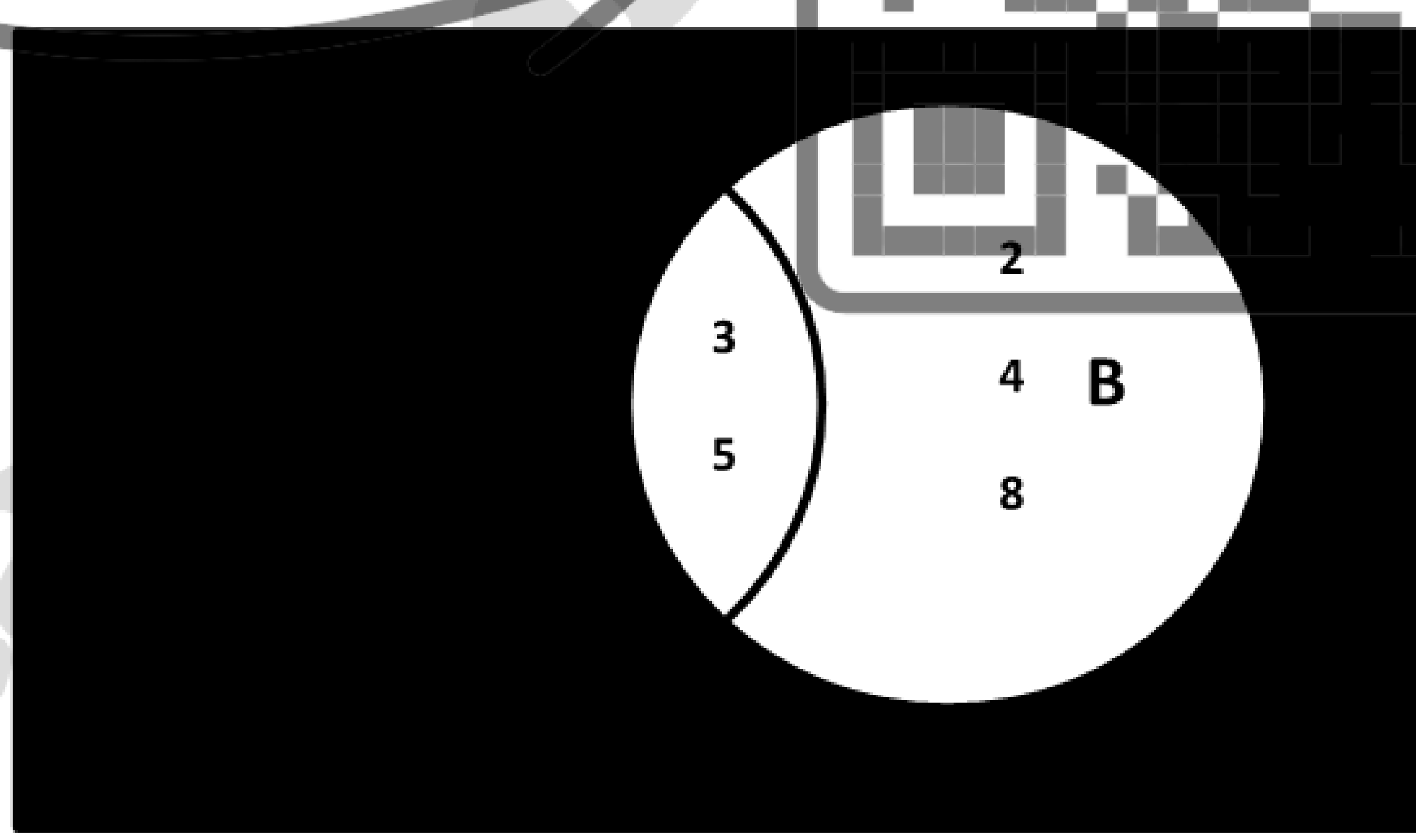
So, $L.H.S = R.H.S$

Q. 4: If $U = \{1, 2, 3, 4, \dots .., 10\}$, $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 3, 4, 5, 8\}$, then prove the following questions by Venn diagram:

(i) $A - B = A \cap B'$



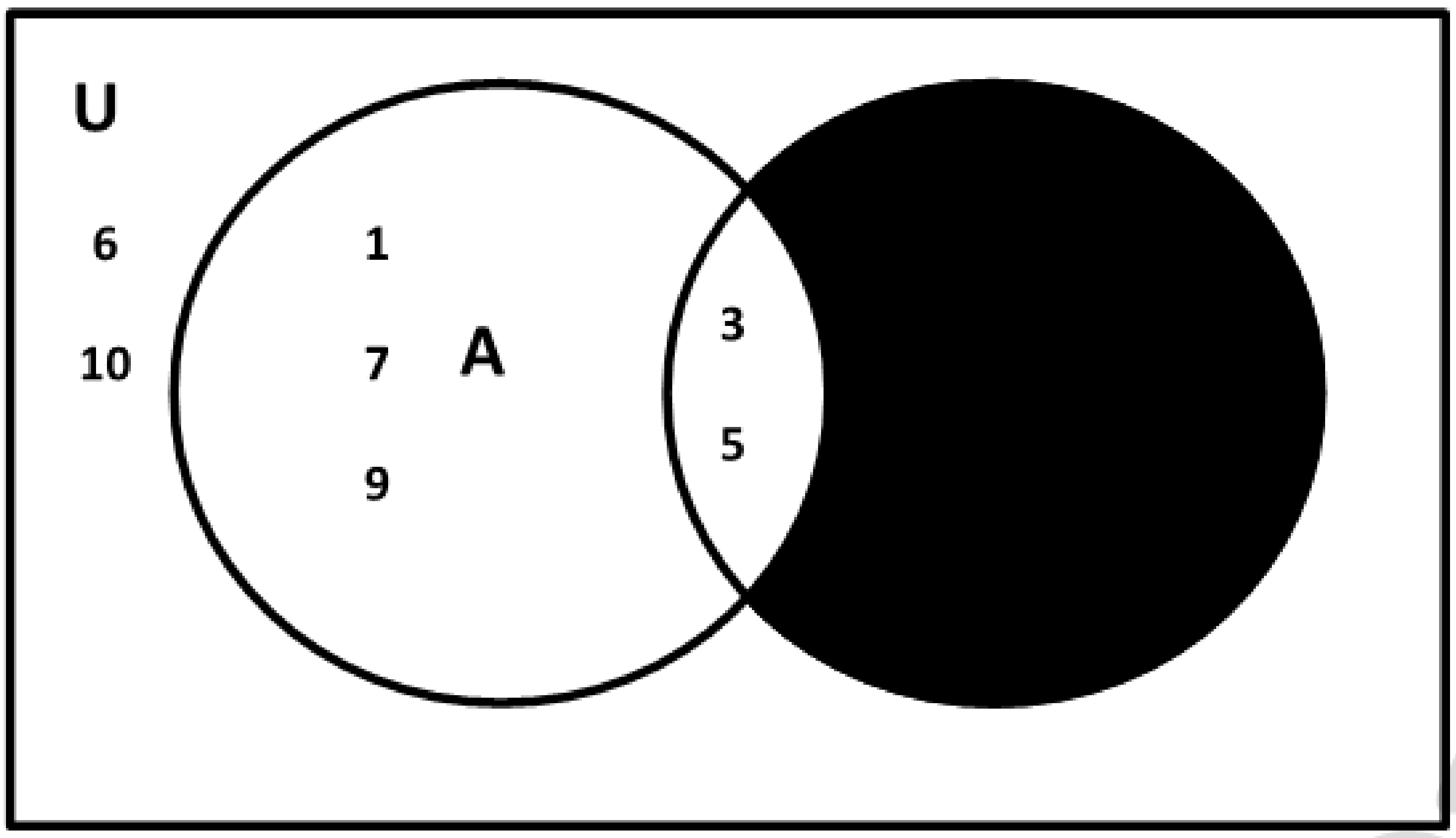
$A - B =$ [redacted]



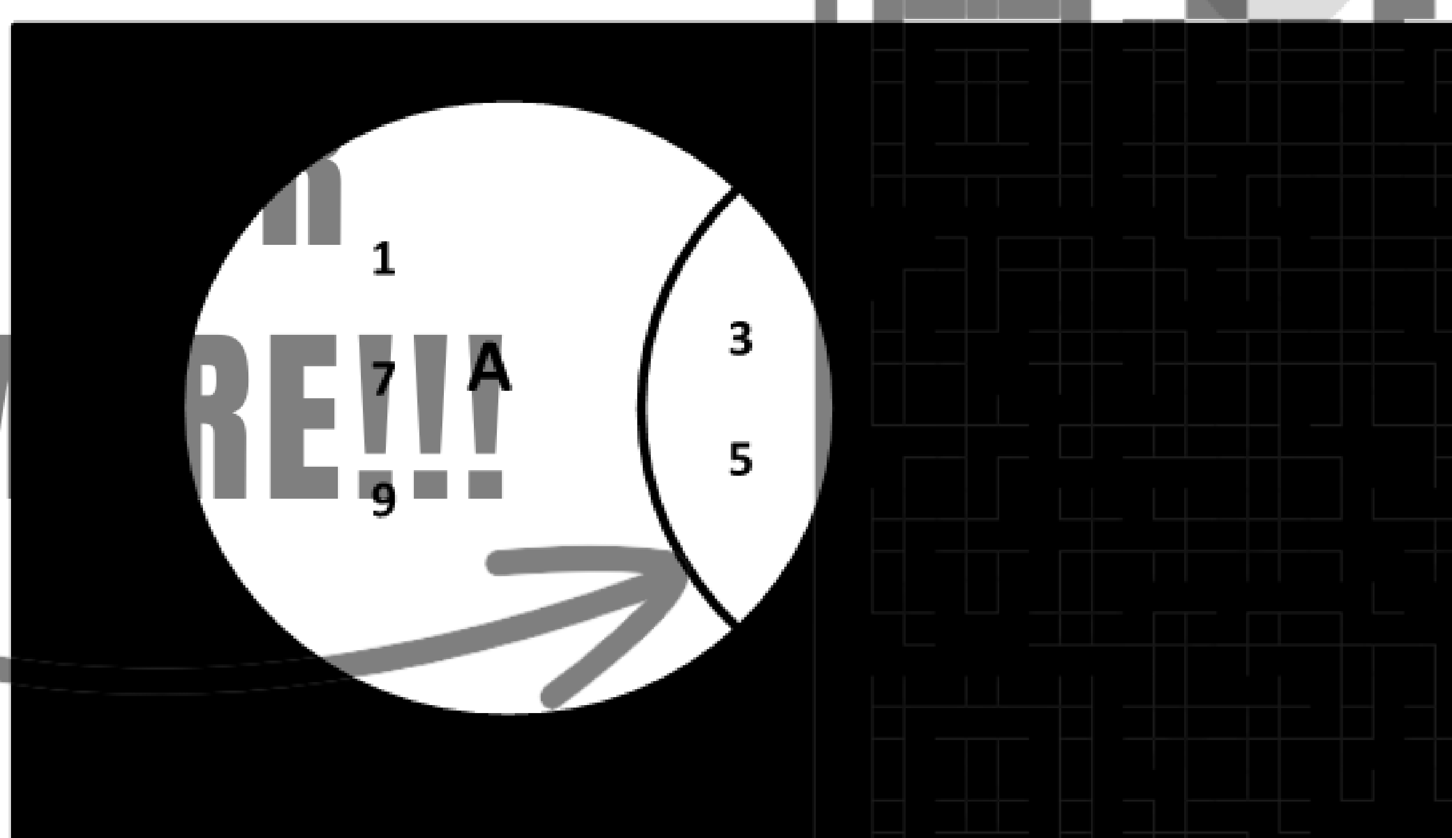
$U - B =$ [redacted]

$A \cap B' =$ [redacted]

(ii) $B - A = B \cap A'$



$B - A =$ [redacted]



$U - A =$ [redacted]

$B \cap A' =$ [redacted]

(iii) $(A \cup B)' = A' \cap B'$

[redacted]

$A \cup B$ = [redacted]

$(A \cup B)'$ = [redacted]

[redacted]

A' = [redacted]

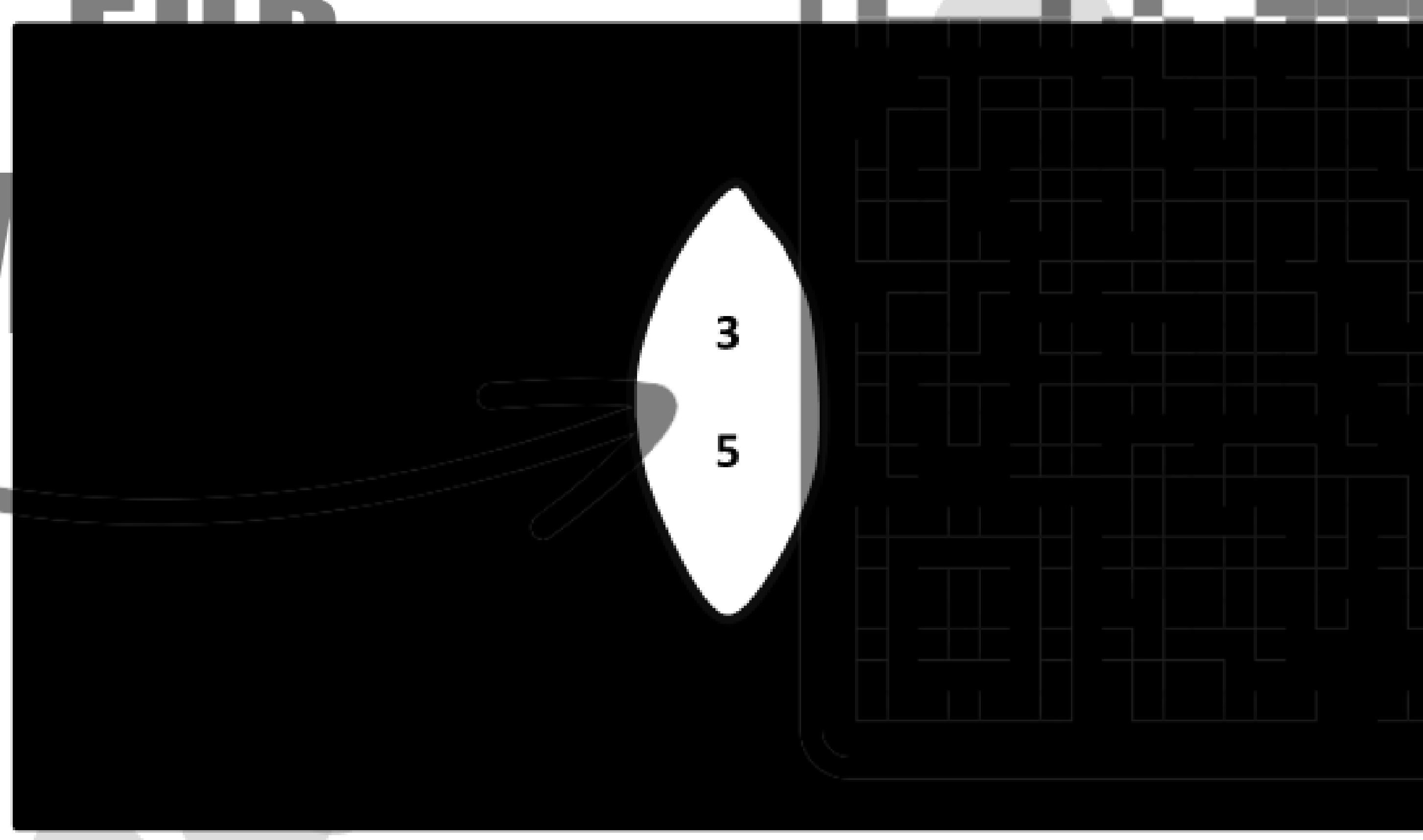
B' = [redacted]

$A' \cap B'$ = [redacted]

(iv) $(A \cap B)' = A' \cup B'$

$A \cap B$ = [redacted]

$(A \cap B)'$ = [redacted]



A' = [redacted]

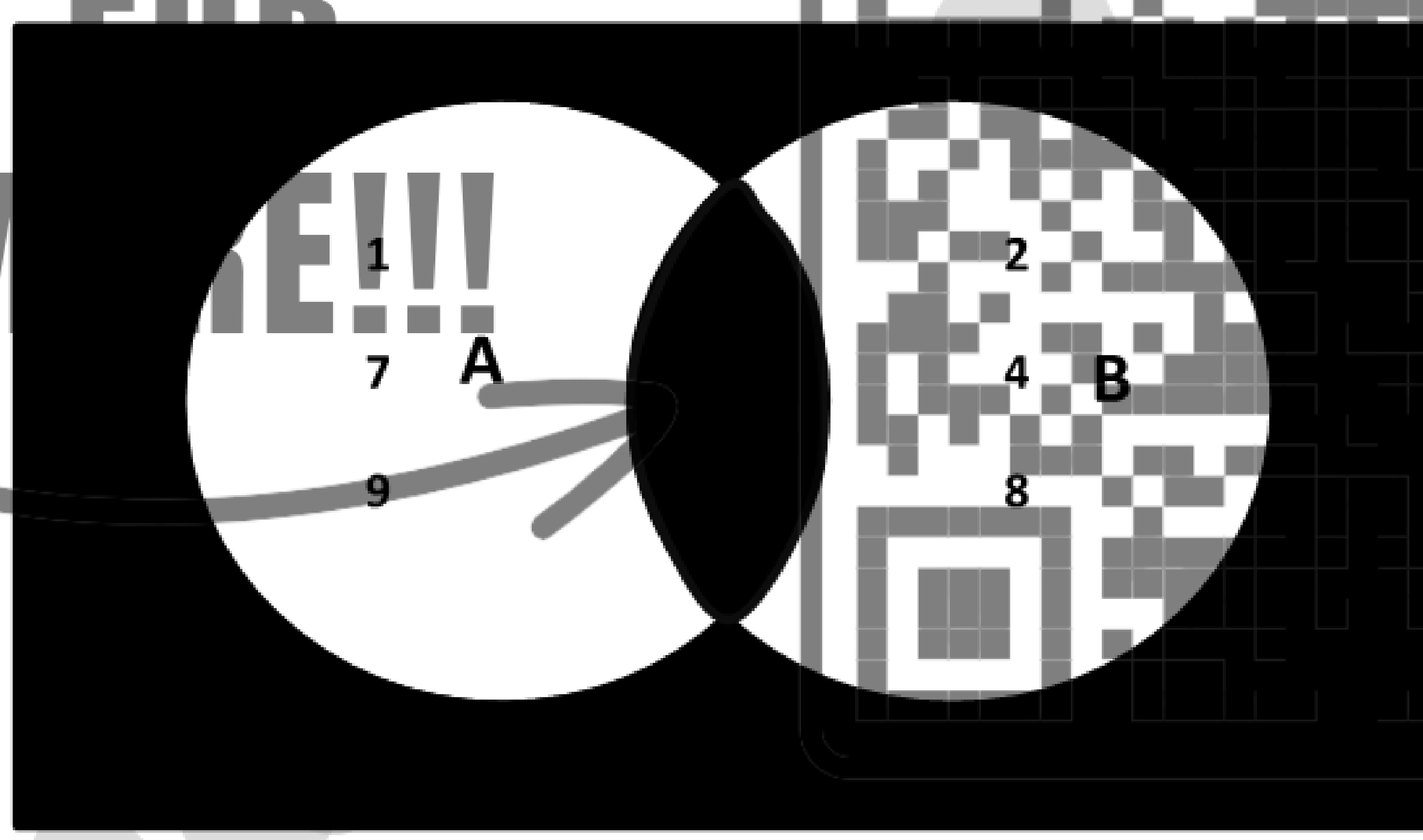
B' = [redacted]

$A' \cup B'$ = [redacted]

(v) $(A - B)' = A' \cup B$

$A - B =$ [redacted]

$(A - B)' =$ [redacted]



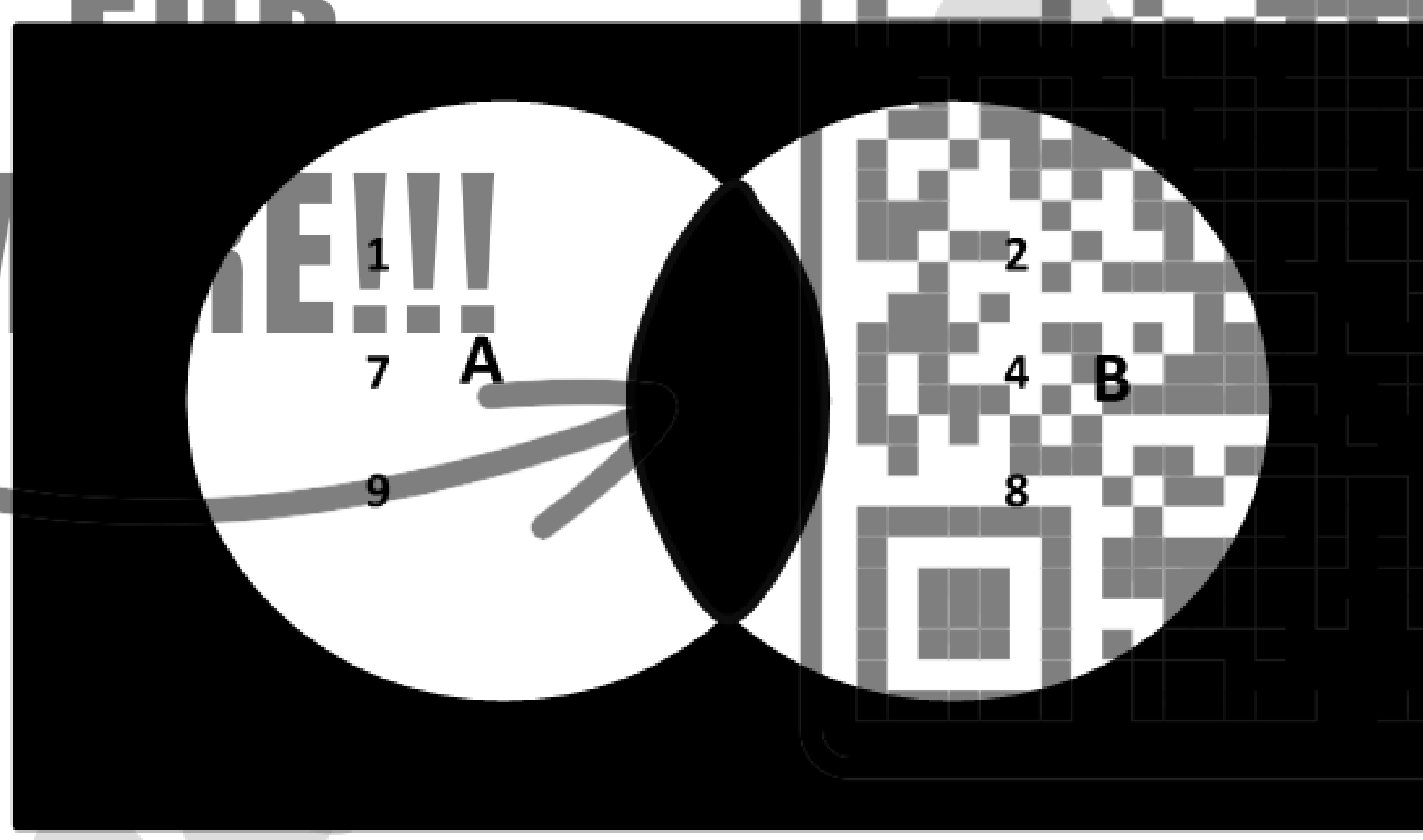
$A' =$ [redacted]

$A' \cup B =$ [redacted]

(vi) $(B - A)' = B' \cup A$

$B - A =$ [redacted]

$(B - A)' =$ [redacted]



$B' =$ [redacted]

$B' \cup A =$ [redacted]

Exercise 5.4

Q. 1: If $A = \{a, b\}$, $B = \{c, d\}$, then find $A \times B$ and $B \times A$.

$A \times B = \{a, b\} \times \{c, d\}$
$= \{(a, c), (a, d), (b, c), (b, d)\}$

$B \times A = \{c, d\} \times \{a, b\}$
$= \{(c, a), (c, b), (d, a), (d, b)\}$

Q. 2: If $A = \{0, 2, 4\}$, $B = \{-1, 3\}$, then find $A \times B, B \times A, A \times A, B \times B$

$A \times B = \{0, 2, 4\} \times \{-1, 3\}$
$= \{(0, -1), (0, 3), (2, -1), (2, 3), (4, -1), (4, 3)\}$

$B \times A = \{-1, 3\} \times \{0, 2, 4\}$
$= \{(-1, 0), (-1, 2), (-1, 4), (3, 0), (3, 2), (3, 4)\}$

$A \times A = \{0, 2, 4\} \times \{0, 2, 4\}$
$= \{(0, 0), (0, 2), (0, 4), (2, 0), (2, 2), (2, 4), (4, 0), (4, 2), (4, 4)\}$

$B \times B = \{-1, 3\} \times \{-1, 3\}$
$= \{(-1, -1), (-1, 3), (3, -1), (3, 3)\}$

Q. 3: Find a and b, if

(i) $(a - 4, b - 2) = (2, 1)$

from above ordered pair we have

$a - 4 = 2$
$a = 6$
$b - 2 = 1$
$b = 3$

(ii) $(2a + 5, 3) = (7, b - 4)$

from above ordered pair we have

$2a + 5 = 7$
$2a = 2$
$a = 1$
$3 = b - 4$
$b - 4 = 3$
$b = 7$

(iii) $(3 - 2a, b - 1) = (a - 7, 2b + 5)$

from above ordered pair we have

$3 - 2a = a - 7$
$-2a - a = -7 - 3$
$-3a = -10$
$a = \frac{10}{3}$
$b - 1 = 2b + 5$
$b - 2b = 5 + 1$

$-b \quad = 6$

$b \quad = -6$

Q. 4: Find the sets X and Y, if $X \times Y = \{(a, a), (b, a), (c, a), (d, a)\}$

As we know first elements (domain) of ordered pairs are related to first set i.e. X and second elements (range) of ordered pairs are related to Y. So,

$X \quad = \{a, b, c, d\}$

$Y \quad = \{a\}$

Q. 5: If $X = \{a, b, c\}$ and $Y = \{d, e\}$, then find the number of elements in

(i) No. of Elements in X $= m = 3$

No. of Elements in Y $= n = 2$

So,

No. of Elements in $X \times Y$ $= m \times n$

$= 3 \times 2$

$= 6$

(ii) No. of Elements in X $= m = 3$

No. of Elements in Y $= n = 2$

So,

No. of Elements in $Y \times X$ $= n \times m$

$= 2 \times 3$

$= 6$

(iii) No. of Elements in X $= m = 3$

So,

No. of Elements in $X \times X$ $= m \times m$

$= 3 \times 3$

$= 9$

Exercise 5.5

Q. 1: If $L = \{a, b, c\}$, $M = \{3, 4\}$, then find two binary relations of $L \times M$ and $M \times L$.

$L \times M = \{a, b, c\} \times \{3, 4\}$

$= \{(a, 3), (a, 4), (b, 3), (b, 4), (c, 3), (c, 4)\}$

$M \times L = \{3, 4\} \times \{a, b, c\}$

$= \{(3, a), (3, b), (3, c), (4, a), (4, b), (4, c)\}$

$R_1 = \{(a, 3), (b, 4), (c, 3)\}$

$R_2 = \{(a, 4), (b, 3), (c, 4)\}$

$R_3 = \{(3, a), (4, a)\}$

$R_4 = \{(3, b), (4, b), (3, c), (4, c)\}$

Q. 2: If $Y = \{-2, 1, 2\}$, then make two binary relations $Y \times Y$. Also find their domain and range.

$Y \times Y = \{-2, 1, 2\} \times \{-2, 1, 2\}$

$= \{(-2, -2), (-2, 1), (-2, 2), (1, -2), (1, 1), (1, 2), (2, -2), (2, 1), (2, 2)\}$

$R_1 = \{(-2, -2), (-2, 1), (1, 2), (2, 2)\}$

$Dom\ R_1 = \{-2, 1, 2\} = L$

$Range\ R_1 = \{-2, 1, 2\}$

$R_2 = \{(-2, 1), (1, 1), (-2, 2)\}$

$Dom\ R_2 = \{-2, 1\}$

$Range\ R_2 = \{1, 2\}$

Q. 3: If $L = \{a, b, c\}$ and $L = \{d, e, f, g\}$, then find two binary relations in each:

(i) $L \times L$

$L \times L = \{a, b, c\} \times \{a, b, c\}$

$= \{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)\}$

$R_1 = \{(a, a), (a, b)\}$

$R_2 = \{(b, c), (c, c)\}$

(ii) $L \times M$

$L \times M = \{a, b, c\} \times \{d, e, f, g\}$

$= \{(a, d), (a, e), (a, f), (a, g), (b, d), (b, e), (b, f), (b, g), (c, d), (c, e), (c, f), (c, g)\}$

$R_1 = \{(a, d), (b, g)\}$

$R_2 = \{(a, f), (b, e), (c, f)\}$

(iii) $M \times M$

$M \times M = \{d, e, f, g\} \times \{d, e, f, g\}$

$= \{(d, d), (d, e), (d, f), (d, g), (e, d), (e, e), (e, f), (e, g), (f, d), (f, e), (f, f), (f, g), (g, d), (g, e), (g, f), (g, g)\}$

$R_1 = \{(d, e), (d, f)\}$

$R_2 = \{(e, e), (f, f), (g, g)\}$

Q. 4: If set M has 5 elements, then find the number of binary relations in M.

No. of Elements in $M = m = 5$

No. of binary relations in $M = 2^{m \times m}$

$= 2^{5\times5}$
$= 2^{25}$

Q. 5: If $L = \{x|x \in N \wedge x \leq 5\}$, $M = \{x|x \in P \wedge x \leq 10\}$, then make the following relations from L to M. Also write the domain and range of each relation.

So, we have from the question

$L = \{1, 2, 3, 4, 5\}$
$M = \{2, 3, 5, 7\}$

$L \times M = \{1, 2, 3, 4, 5\} \times \{2, 3, 5, 7\}$
$= \{(1,2), (1,3), (1,5), (1,7), (2,2), (2,3), (2,5), (2,7), (3,2), (3,3), (3,5), (3,7), (4,2), (4,3), (4,5), (4,7), (5,2), (5,3), (5,5), (5,7)\}$

(i) $R_1 = \{(x, y)|y < x\}$
$= \{(3,2), (4,2), (5,2), (4,3), (5,3)\}$
$Dom\ R_1 = \{3, 4, 5\}$
$Range\ R_1 = \{2, 3\}$

(ii) $R_2 = \{(x, y)|y = x\}$
$= \{(2,2), (3,3), (5,5)\}$
$Dom\ R_2 = \{2, 3, 5\}$
$Range\ R_2 = \{2, 3, 5\}$

(iii) $R_3 = \{(x, y)|x + y = 6\}$
$= \{(1,5), (3,3), (4,2)\}$
$Dom\ R_3 = \{1, 3, 4\}$
$Range\ R_3 = \{2, 3, 5\}$

(iv) $R_4 = \{(x, y)|y - x = 2\}$
$= \{(1,3), (3,5), (5,7)\}$
$Dom\ R_4 = \{1, 3, 5\}$
$Range\ R_4 = \{3, 7\}$

Q. 6: Indicate relations, into function, one-one function, onto function, and bijective function from the following. Also find their domain and range.

(i) $R_1 = \{(1,1), (2,2), (3,3), (4,4)\}$
$Dom\ R_1 = \{1, 2, 3, 4\}$
$Range\ R_1 = \{1, 2, 3, 4\}$

As, we know A relation becomes a function if

$Dom f = A$
and
Every $x \in A$ appears in one and only one ordered pair in f.

So, the given relation is function.

As, All distinct elements of A have distinct images in B so, the given relation is One-One.

Also, every element of set B is an image of at least one element of set A i.e. $Range\ of\ f = B$. So, given relation is also Onto function.

As, the given relation is One-One as well as Onto function so, it is bijective function.

(ii) $R_2 = \{(1,2), (2,1), (3,4), (3,5)\}$

$Dom\ R_2 = \{1, 2, 3\}$
$Range\ R_2 = \{1, 2, 4, 5\}$
As, we know A relation becomes a function if

$Domf = A$

and

Every $x \in A$ appears in one and only one ordered pair in f.

As, we can clearly see the 3 is repeated in 3rd and 4th ordered pair so the given relation is not a function, its only a relation.

(iii) $R_3 = \{(b, a), (c, a), (d, a)\}$
$Dom\ R_3 = \{b, c, d\}$
$Range\ R_3 = \{a\}$
As, we know A relation becomes a function if

$Domf = A$

and

Every $x \in A$ appears in one and only one ordered pair in f.

So, the given relation is a function.
As, it doesn't fulfill any condition of One-One, Onto or into function so the relation is only a function.

(iv) $R_4 = \{(1, 1), (2, 3), (3, 4), (4, 3), (5, 4)\}$
$Dom\ R_4 = \{1, 2, 3, 4, 5\}$
$Range\ R_4 = \{1, 3, 4\}$
As, we know A relation becomes a function if

$Domf = A$

and

Every $x \in A$ appears in one and only one ordered pair in f.

So, the given relation is a function.
As,
It doesn't fulfill condition of One-One.
Every element of set B is an image of at least one element of set A. So, the given relation is an onto function.

(v) $R_5 = \{(a, b), (b, a), (c, d), (d, e)\}$
$Dom\ R_5 = \{a, b, c, d\}$
$Range\ R_5 = \{a, b, d, e\}$
As, we know A relation becomes a function if

$Domf = A$

and

Every $x \in A$ appears in one and only one ordered pair in f.

So, the given relation is a function.
As,
As, All distinct elements of A have distinct images in B so, the given relation is One-One.
It doesn't fulfill condition of Onto function.
So, the given relation is a One-One function.

(vi) $R_6 = \{(1, 2), (2, 3), (1, 3), (3, 4)\}$

$Dom\ R_6$ = $\{1, 2, 3\}$
$Range\ R_6$ = $\{2, 3, 4\}$
As, we know A relation becomes a function if

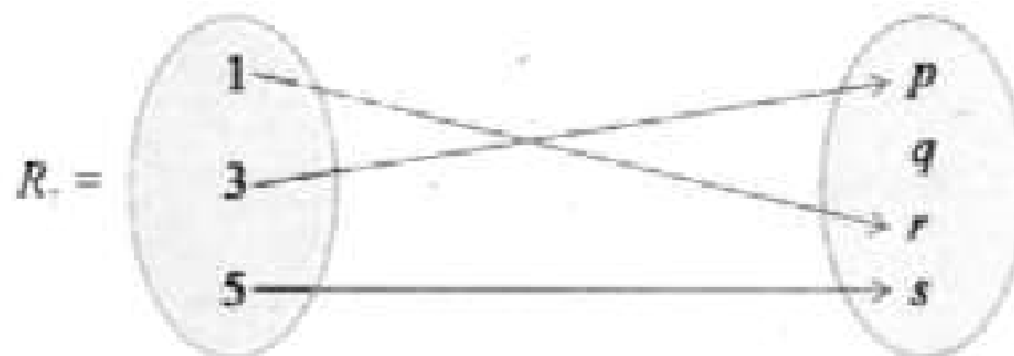
$Domf = A$

and

Every $x \in A$ appears in one and only one ordered pair in f.

As, we can clearly see the 1 is repeated in 1st and 3rd ordered pair so the given relation is not a function, it's only a relation.

(vii)



R_7 = $\{(1, r), (3, p), (5, s)\}$
$Dom\ R_7$ = $\{1, 3, 5\}$
$Range\ R_7$ = $\{p, r, s\}$
As, we know A relation becomes a function if

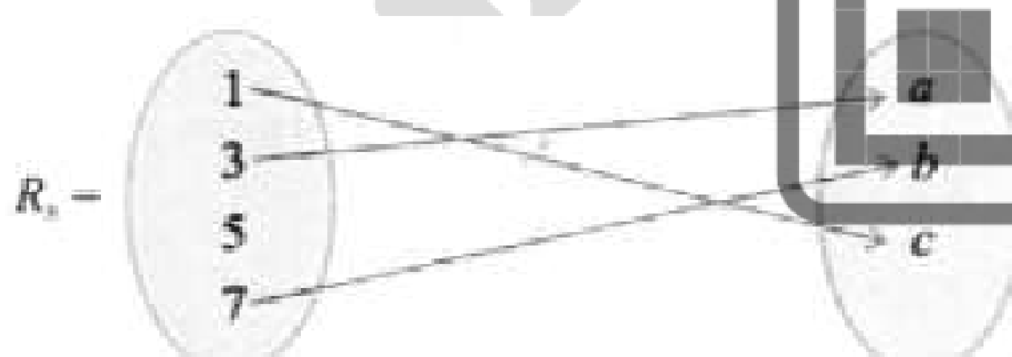
$Domf = A$

and

Every $x \in A$ appears in one and only one ordered pair in f.

As, All distinct elements of A have distinct images in B so, the given relation is One-One.
It doesn't fulfill condition of Onto function.
So, the given relation is a One-One function.

(viii)



R_7 = $\{(1, c), (3, a), (7, b)\}$
$Dom\ R_8$ = $\{1, 3, 7\}$
$Range\ R_8$ = $\{a, b, c\}$
As, we know A relation becomes a function if

$Domf = A$

and

Every $x \in A$ appears in one and only one ordered pair in f.

But $Domf \neq A$, So the given relation is not a function. it's only a relation.

Exercise 6.1

1. **The following data shows the number of members in various families. Construct frequency distribution. Also find cumulative frequencies.**

9,11, 4, 5, 6, 8, 4, 3, 7, 8, 5, 5, 8, 3, 4, 9, 12, 8, 9, 10, 6, 7, 7, 11, 4, 4, 8, 4, 3, 2, 7, 9, 10, 9, 7, 6, 9, 5, 7.

Solution:

Frequency distribution of number of family members.

Number of members	Tally marks	Frequency	Cumulative Frequency
2	I	1	1
3	I I I	3	1+ 3= 4
4	~~IIII~~ I	6	4 + 6 = 10
5	I I I I	4	10+4=14
6	I I I	3	14+3 = 17
7	~~IIII~~ I	6	17+6=23
8	~~IIII~~	5	23+5 = 28
9	~~IIII~~ I	6	28+6=34
10	I I	2	34+2 = 36
11	I I	2	36+2=38
12	I	1	38+1 = 39
	Total	**39**	

2. **The following data has been obtained after weighing 40 students of class V. Make a frequency distribution taking class interval size as 5. Also find the class boundaries and midpoints.**

34, 26, 33, 32, 24, 21, 37, 40, 41, 28, 31, 33, 34, 37, 23, 27, 31, 31, 36, 29, 35, 36,

37, 38, 22, 27, 28, 29, 31, 35, 35, 40, 21, 32, 33, 27, 29, 30, 23.

Also make a less than cumulative frequency distribution. (Hint: Make classes 20-24,25-29….).

Solution:

Frequency Distribution				
Class Limits	**Tally marks**	**Frequency**	**Midpoint**	**Class Boundaries**
20 - 24	卌 |	6	22	19.5 – 24.5
25 - 29	卌 卌	10	27	24.5 – 29.5
30 - 34	卌 卌 ||	12	32	29.5 – 34.5
35 - 39	卌 ||||	9	37	34.5 – 39.5
40 - 44	|||	3	42	39.5 – 44.5
		40		

Less than Cumulative Frequency Distribution

Class Boundaries	Frequency f	Cumulative Frequency	Class Boundaries	Cumulative Frequency
14.5 - 19.5	0	0	Less than 19.5	0
19.5 - 24.5	6	0+6=6	Less than 24.5	6
24.5 - 29.5	10	6 + 10 = 16	Less than 29.5	16
29.5 - 34.5	12	16 + 12 = 28	Less than 34.5	28
34.5 - 39.5	9	28 + 9 = 37	Less than 39.5	37
39.5 - 44.5	3	37 + 3 = 40	Less than 44.5	40

3. From the following data representing the salaries of 30 teachers of a school. Make a frequency distribution taking class interval size of Rs.100, 450, 500, 550, 580, 670, 1200, 1150, 1120, 950, 1130, 1230, 890, 780, 760, 670, 880, 890, 1050, 980, 970, 1020, 1130, 1220, 760, 690, 710, 750, 1120, 760, 1240.

(Hint: Make classes 450-549, 550-649,….).

Solution:

Class Limits	Tally Marks	Frequency
450 - 549	aa	2
550 - 649	aa	2
650- 749	D	4
750 - 849	E	5
850 - 949	C	3
950 - 1049	D	4
1050 - 1149	E	5
1150 - 1249	E	5
	Total =	30

Q4. The following data shows the daily load shedding duration in hours, in 30 localities of a certain city. Make a frequency distribution of the load shedding duration taking 2 hours as class interval size and answer the following questions.

6, 12, 5, 7, 3, 3, 6, 10,2, 14, 11, 12, 8, 6, 8, 9, 7, 11, 6, 9, 12, 13, 10, 14, 7, 6, 10, 11, 14,12,

a) Find the most frequent load shedding hours?

b) Find the least load shedding intervals?

(Hint: Make classes 2-3,4-5,6-7....)

Solution:

Frequency Distribution Table

Class Limits	Talley marks	Frequency
2 - 3	B	2
4 - 5	I	1
6 - 7	E D	9
8 - 9	E	5
10 - 11	E A	6
12- 13	E	5
14- 15	C	3
	Total =	31

(a) Find the most frequent load shedding hours.

6 - 7

(b) Find the least load shedding intervals.

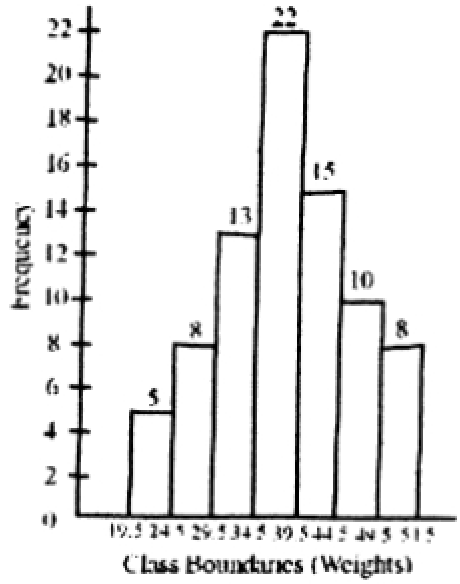
4 – 5

Q5. Construct a Histogram and frequency Polygon for the following data showing weights of students in kg.

Weights	Frequency / No. of students
20-24	5
25-29	8
30-34	13
35-39	22
40-44	15
45-49	10
50-54	8

Solution:

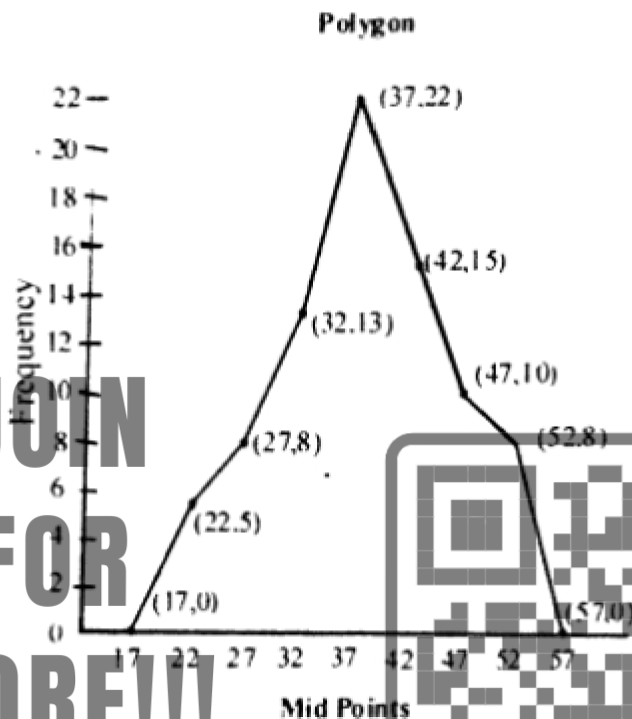
Class Boundaries	Frequency
19.5 - 24.5	5
24.5 - 29.5	8
29.5 - 34.5	13
34.5 - 39.5	22
39.5 - 44.5	15
44.5 - 49.5	10
49.5 - 54.5	8



Class Limits	Mid Points	Frequency
15 – 19	17	0
20 – 24	22	5
25 – 29	27	8
30 – 34	32	13
35 – 39	37	22
40 – 44	42	15
45 – 49	47	10
50 – 54	52	8
55 - 59	57	0

Note:

Two additional groups with same size of class interval are taken. One before the very first group and second after the very last group. These two groups will have frequency “0”



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Measures of Central Tendency:

A specific value of the variable around which the majority of the observations tend to concentrate, this representative shows the tendency or behavior of the distribution of the variable under study. This value is called average or the central value. The measures or techniques that are used to determine this central value are called Measures of Central Tendency.

The following measures of central tendency will be discussed in this section:

1. Arithmetic mean **2.** Median

3. Mode **4.** Geometric mean

5. Harmonic mean **6.** Quartiles

Arithmetic Mean:

Arithmetic Mean (or simply called Mean) is a measure that determines a value (observation) of the variable under study by dividing the sum of all values (observations) of the variable by their number of observations. We denote Arithmetic mean by $\overline{X}$. In symbols we define:

Arithmetic mean of n observations = $\overline{X} = \frac{\sum X}{n} = \frac{\text{Sum of all values of observation}}{\text{No. of observation}}$

Computation of Arithmetic Mean

There are two types of data, ungrouped and grouped. We, therefore have different methods to determine Mean for the two types of data.

Ungrouped Data:

For ungrouped data we use three approaches to find mean. These are as follows.

(i) Direct Method (By Definition)

The formula under this method is given by:

$$\overline{X} = \frac{\sum X}{n} = \frac{\text{Sum of all observation}}{\text{No. of observation}}$$

(ii) Indirect, Short Cut or Coding Methods

There are two approaches under Indirect Method. These are used to find mean when data set consist of large values or large number of values. The purpose is to simplify the computation of Mean. These approaches exist in theory but are not used in practice as many Statistical software are available now to handle large data. However, a student should have knowledge of these two approaches. These are:

(i) using an Assumed or Provisional mean

(ii) using a Provisional mean and changing scale of the variable.

Deviation is defined as difference of any value of the variable from any constant 'A'. For example, we say,

Deviation from mean of X = $(x_i - \overline{X})$ for i = l, 2, ……..n

Deviation from any constant A = $(x_i - A)$ for i = 1, 2, ……n

The Formulae used under indirect methods are:

(i) $\overline{X} = A + \frac{\sum_{i=1}^{n} D_i}{n}$ **(ii)** $\overline{X} = A + \frac{\sum_{i=1}^{n} u_i}{n} \times h$

Where

$D_i = (x_i - A)$, A is any assumed value of X called Assumed or Provisional mean.

$u_i = \frac{(x_i - A)}{h}$, "h" is the class interval size for unequal intervals.

Grouped Data:

A data in the form of frequency distribution is called grouped data. For the grouped data we define formulae under Direct and Indirect methods as given below:

(a) Using Direct method

$$\overline{X} = \frac{\sum fX}{\sum f}$$

Using Indirect method,

(i) $\overline{X} = A + \frac{\sum fD}{\sum f}$ (ii) $\overline{X} = A + \frac{\sum fu}{\sum f} \times h$

where 'X=x_i' denotes the midpoint of a class or group if class intervals are given and 'h' is the class interval size.

(b) Median:

Median is the middle most observation in an arranged data set. It divides the data set into two equal parts. '$\tilde{X}$' is used to represent median. We determine Median by using the following formulae:

Ungrouped data

Case-I:

When the number of observations is odd of a set of data arranged in order of magnitude the median (middle most observation) is located by the formula given below:

$$\text{Median} = \text{size of} \left(\frac{n+1}{2}\right)^{th} \text{observation}$$

Case-2:

When the number of observations is even of a set of data arranged in order of magnitude the median is the arithmetic mean of the two middle observations. That is, median is average of

$\frac{n}{2}$ and $\left(\frac{n}{2}+1\right)^{th}$ values.

$$\text{Median} = \frac{1}{2}\left[\text{size of}\left(\frac{n}{2}th + \frac{n+1}{2}th\right)\text{observation}\right]$$

Grouped Data (Discrete)

The following steps are involved in determining median for grouped data (discrete):

(i) Make cumulative frequency column.

(ii) Determine the median observation using cumulative frequency, i.e., the class containing $\left(\frac{n}{2}\right)^{th}$ observation.

Grouped Data (Continuous):

The following steps are involved in determining median for grouped data (continuous):

(i) Determine class boundaries.

(ii) Make cumulative frequency column.

Determine the median class using cumulative frequency, i.e., the class containing $\left(\frac{n}{2}\right)^{th}$ observation.

Use the formula:

$$\text{Median} = l + \frac{h}{f}\left\{\frac{n}{2} - c\right\}$$

Where

l = lower class boundary of the median class,

h = class interval size of the median class,

f = frequency of the median class,

c = cumulative frequency of the class preceding the median class.

Mode:

Mode is defined as the most frequent occurring observation in the data. It is the observation that occur maximum number of times in given data. The following formula is used to determine Mode:

(i) Ungrouped data and Discrete Grouped data

Mode = the most frequent observation

(ii) Grouped Data (Continuous)

The following steps are involved in determining mode for grouped data:

- Find the group that has the maximum frequency.
- Use the formula

$$\text{Mode} = l + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$$

Where

l = lower class boundary of the modal class or group,

h = class interval size of the modal class,

f_m = frequency of the modal class,

f_1 = frequency of the class preceding the modal class

f_2 = frequency of the class succeeding the modal class.

Geometric Mean:

Geometric mean of a variable X is the n^{th} positive root of the product of the $x_1, x_2, x_3,$

........, x_n observations. In symbols we write,

$$G.M = (x_1 . x_2 . x_3 . \ldots\ldots . x_n)^{1/n}$$

The above formula can also be written by using logarithm.

For Ungrouped data

$$G.M = \text{Anti log}\left(\frac{\sum \log X}{n}\right)$$

For Grouped data

$$G.M = \text{Anti log}\left(\frac{\sum f \log X}{\sum f}\right)$$

Harmonic Mean:

Harmonic mean refers to the value obtained by reciprocating the mean of the reciprocal of $x_1, x_2, x_3, \ldots\ldots x_n$ observations. In symbols, for ungrouped data,

$$H.M. = \frac{n}{\sum \frac{1}{X}}$$

And for grouped data

$$H.M. = \frac{n}{\sum \frac{f}{X}}$$

Properties of Arithmetic Mean:

(i) Mean of a variable with similar observations say constant **k** is the constant **k** itself.

(ii) Mean is affected by change in origin.

(iii) Mean is affected by change in scale.

(iv) Sum of the deviations of the variable X from its mean is always zero.

Calculation of Weighted Mean and Moving Averages:

The Weighted Arithmetic Mean:

The relative importance of a number is called its weight. When numbers x_1, x_2, ... x_n are not equally important, we associate them with certain weights, w_1, w_2, w_3, ..., w_n depending on the importance or significance.

$$\overline{x_w} = \frac{w_1x_1 + w_2x_2 + \ldots + w_nx_n}{w_1 + w_2 + \ldots + w_n} = \frac{\sum wx}{\sum w}$$

is called the weighted arithmetic mean.

Moving Averages:

Moving averages are defined as the successive averages (arithmetic means) which are computed for a sequence of days/months/years at a time. If we want to find 3-days moving average, we find the average of first 3-days, then dropping the first day and add the succeeding day to this group. Place the average of each 3-days against the mid of 3-days. This process continues until all the days, beginning from first to the last, are exhausted.

Exercise 6.2

1. What do you understand by measures of central tendency?

Solution:

The specific value of the variable around which the majority of the observations tend to concentrate is called the central tendency.

2. Define Arithmetic mean, Geometric mean, Harmonic mean, mode and median.

Solution:

(i) Arithmetic Means:

Mean is a measure that determines a value of the variable under study by dividing the sum of all values of the variable by their number of observations.

$$\overline{X} = \frac{\sum x}{n} (\text{for ungrouped data}) \quad \text{and} \quad \overline{X} = \frac{\sum fx}{\sum f} (\text{for grouped data})$$

(ii) Geometric Means:

Geometric mean of a variable x is the nth positive root of the product of the $x_1, x_2, x_3, \ldots\ldots x_n$ observation.

$$G.M = (x_1 \times x_2 \times x_3 . \ldots\ldots x_n)^{1/n}$$

(iii) Harmonic Means:

Harmonic mean refers to the value obtained by reciprocating the mean of the reciprocal of $x_1 . x_2 . x_3 . \ldots\ldots x_n$ observations.

$$\text{H.M.} = \frac{n}{\sum \frac{1}{x}} \quad \text{(for ungrouped data)}$$

and, $$\text{H.M.} = \frac{n}{\sum \frac{f}{x}} \quad \text{(for grouped data)}$$

(iv) Mode:

The most repeated value in an observation is called its mode.

(v) Median:

Median is the middle most observation in an arranged data set. It divides the data set into two equal parts.

3. Find arithmetic mean by direct method for the following set of data:

(i) 12, 14, 17, 20, 24, 29, 35, 45.

(ii) 200, 225, 350, 375, 270, 320, 290.

Solution:

(i) A.M = $\overline{X} = \frac{\sum x}{n} = \frac{12 + 14 + 17 + 20 + 24 + 29 + 35 + 45}{8}$

$= \frac{196}{8} = 24.5$

(ii) A.M = $\overline{X} = \frac{\sum x}{n} = \frac{200 + 225 + 350 + 375 + 270 + 320 + 290}{7}$

$= \frac{2030}{7} = 290$

4. For each of the data in Q. No 3, compute arithmetic mean using indirect method.

Solution:

(i) Take any constant say 24 and take deviations from it (24).

A = 24

X	D = X - A
12	12 - 24 = -12
14	14 - 24 =-10
17	17 - 24 =-7
20	20 - 24 =-4
24	24 - 24 = 0
29	29 -24 = 5
35	35 - 24 = 11
45	45 - 24 = 21
n = 8	$\sum D$ **= 4**

$$\overline{X} = A + \frac{\sum D}{n} = 24 + \frac{4}{8} = 24 + \frac{1}{2} = 24\frac{1}{2} = 24.5$$

(ii) Take any constant 270 and take deviations from it (270).

A = 270

X	D = X - A
200	200 - 270 =-70
225	225 - 270 =-45
350	350 - 270 = 80
375	375 - 270 = 105
270	270 - 270 = 0
320	320 – 270 = 50
290	290 – 270 = 20
n = 7	$\sum D$ **= 140**

$$\overline{X} = A + \frac{\sum D}{n} = 270 + \frac{140}{7} = 270 + 20 = 290$$

5. The marks obtained by students of class XI in mathematics are given below. Compute arithmetic mean by direct and indirect methods.

Classes / Groups	Frequency
0 - 9	2
10 - 19	10
20 - 29	5
30 - 39	9
40 - 49	6
50 - 59	7
60 - 69	1

Solution:

Direct Method:

Classes/groups	Mid-points	f	fx
0 - 9	4.5	2	4.5 × 2 = 9.0
10 - 19	14.5	10	14.5 × 10 =145.0
20 -29	24.5	5	24.5 × 5 = 122.5
30 -39	34.5	9	34.5 × 9 ='310.5
40 -49 .	44.5	6	44.5 × 6 = 267.0
50 - 59	54.5	7	54.5 × 7 = 381.5
60 - 69	64.5	1	64.5 × 1 = 64.5
		n = $\sum f$ = 40	**$\sum fx$=1300**

$$\overline{X} = \frac{\sum fx}{\sum f} = \frac{1300}{40} = 32.5$$

Indirect, short cut method:

Let A =34.5

Classes/ groups	f	Mid-point (x)	D = X - A	U = $\frac{D}{10}$	fD	$f(U)=\frac{f(D)}{10}$
0 - 9	2	4.5	4.5 - 34.5 = -30	-3	-60	-6
10 - 19	10	14.5	14.5 - 34.5 = - 20	-2	-200	-20
20 - 29	5	24.5	24.5 - 34.5 = - 10	-1	-50	-5
30 - 39	9	34.5	34.5 - 34.5 = 0	0	0	0
40 - 49	6	44.5	44.5 - 34.5 = 10	1	60	6
50 - 59	7	54.5	54.5 - 34.5 = 20	2	140	14
60 - 69	1	64.5	64.5 - 34.5 = 30	3	30	3
Total	40				-80	-8

$$\overline{X} = A + \frac{\sum fD}{\sum f} = 34.5 + \frac{(-80)}{40} = 34.5 - 2 = 32.5$$

or

$$\overline{X} = A + \frac{\sum f(U)}{\sum f} \times h = 34.5 + \frac{-8}{40} \times h = 34.5 + \frac{-8}{40} \times 10 = 34.5 - 2 = 32.5$$

6. The following data relates to the ages of children in a school. Compute the mean age by direct and short-cut method taking any provisional mean.

(Hint. Take A = 8)

Class limits	Frequency
4-6	10
7-9	20
10-12	13
13-15	7
Total	50

Also Compute Geometric mean and Harmonic mean.

Solution:

Class Limits	Mid points (x)	f	fx
4 - 6	5	10	5 × 10 = 50
7 - 9	8	20	8 × 20 = 160
10 - 12	11	13	11 ×13 = 143
13 - 15	14	7	14 × 7 = 98
Total	$\sum f = 50$		$\sum fx = 451$

A.M = $\frac{\sum fx}{\sum f} = \frac{451}{50} = 9.02$

Indirect, short cut method:

Let A = 11

Classes/ groups	f	Mid-point (x)	D = X - A	U = $\frac{D}{3}$	fD	fU = $\frac{fD}{3}$
4 - 6	10	5	5 -11 = - 6	-2	-60	-20
7 - 9	20	8	8 - 11= -3	-1	-60	-20
10 - 12	13	11	11 - 11 = 0	0	0	0
13- 15	7	14	14 - 11 = 3	1	21	7
	50				-99	-33

$$\overline{X} = A + \frac{\sum fD}{\sum f} \qquad or \qquad \overline{X} = A + \frac{\sum f(U)}{\sum f} \times h$$

$$= 11 - \frac{99}{50} \qquad\qquad = 11 - \frac{33}{50} \times 3$$

$$= 11 - 1.98 \qquad\qquad = 11 - \frac{99}{50}$$

$$= 9.02 \qquad\qquad = 11 - 1.98 = 9.02$$

Geometric Mean:

We proceed as follows:

Class limits	f	Mid points x	Log x	f log x
4 - 6	10	5	0.69897	6.9897
7 - 9	20	8	0.90309	18.0618
10 - 12	13	11	1.04139	13.5380
13 - 15	7	14	1.14613	8.02291
$\sum f$ = 50				$\sum f$ log x = 46.61248

$$G.M = \text{Anti log}\left(\frac{\sum f \log X}{\sum f}\right) = \text{Anti log}\left(\frac{46.61248}{50}\right)$$

$$= \text{Anti log}(0.9322496) = 8.553$$

Harmonic Means

Class limits	f	Mid points x	$\frac{f}{x}$
4 - 6	10	5	$\frac{10}{5} = 2.0$
7 - 9	20	8	$\frac{20}{8} = 2.5$
10.- 12	13	11	$\frac{13}{11} = 1.18$
13 - 15	7	14	$\frac{7}{14} = 0.50$
	$\sum f$ = 50		$\sum \frac{f}{x}$=6.18

$$H.M. = \frac{\sum f}{\sum \frac{f}{x}} = \frac{50}{6.18} = 8.09$$

7. The following data shows the number of children in various families. Find mode and median.

9, 11, 4, 5, 6, 8, 4, 3, 7, 8, 5, 5, 8, 3, 4, 9, 12, 8, 9, 10, 6, 7, 7, 11, 4, 4, 8, 4, 3, 2, 7, 9, 10, 9, 7, 6, 9, 5.

Solution:

Writing the observations in ascending order

2, 3, 3, 3, 4, 4, 4, 4, 4, 4, 5, 5, 5, 5, 6, 6, 6, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 9, 9, 9, 9, 9, 9, 10, 10, 11, 11, 12.

Mode:

The most frequent observation = 9, 4

Median:

Number of observations = 38

Therefore, median is the mean of 19th and 20th observation = $\frac{7+7}{2} = 7$

8. Find Modal number of heads for the following distribution showing the number of heads when 5 coins are tossed. Also determine median.

X (number of heads)	Frequency (number of times)
1	3
2	8
3	5
4	3
5	1

Solution:

Mode:

The most frequent observation = 2

Median:

For median, we make cumulative frequency column.

x	Frequency	Cumulative frequency
1	3	3
2	8	3 + 8 = 11
3	5	11 + 5 = 16
4	3	16+3=19
5	1	19 + 1 = 20

Median = the class containing $\left(\frac{n}{2}\right)^{th}$ observation

= the class containing $\left(\frac{20}{2}\right)^{th}$ observation

= the class containing $(10)^{th}$ observation

Median = 2

9. The following frequency distribution the weights of boys in kilogram. Compute mean, median, mode.

Class Intervals	Frequency
1 - 3	2
4 - 6	3
7 - 9	5
10 - 12	4
13 - 15	6
16 - 18	2
19 - 21	1

Solution:

Class Intervals	f	Mid Points (x)	fx	Class Boundaries	Cumulative frequency
1 -3	2	2	4	0.5 - 3.5	2
4 - 6	3	5	15	3.5 - 6.5	2 + 3 = 5
7 - 9	5	8	40	6.5 - 9.5	5 + 5 = 10
10 -12	4	11	44	9.5- 12.5	10 + 4 = 14
13 -15	6	14	84	12.5 - 15.5	14 + 6 = 20
16 - 18	2	17	34	15.5 - 18.5	20 + 2 = 22
19 - 21	1	20	20	18.5 - 21.5	22 + 1 = 23
Total	23		241		

Mean:

$$\text{Mean} = \overline{X} = \frac{\sum fx}{\sum f} = \frac{241}{23} = 10.478$$

Median:

$$\text{Median class} = \text{the class containing } \left(\frac{n}{2}\right)^{th} \text{ observation}$$

$$= \text{the class containing } \left(\frac{23}{2}\right)^{th} \text{ observation}$$

$$= \text{the class containing } (11.5)^{th} \text{ observation}$$

Median class is $9.5 - 12.5$

Here $l = 9.5, c = 10, f = 4, h = 3$

$$\text{Median} = l + \frac{h}{f}\left\{\frac{n}{2} - c\right\}$$

$$= 9.5 + \frac{3}{4}\left(\frac{23}{2} - 10\right) = 9.5 + \frac{3}{4}\left(\frac{3}{2}\right) = 9.5 + \frac{9}{8} = 9.5 + 1.125 = 10.625$$

Mode:

$$\text{Mode} = l + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$$

$$\textit{Here} \quad l = 12.5, f_m = 6, f_1 = 4, f_2 = 2, h = 3$$

$$\therefore \text{Mode} = 12.5 + \frac{6-4}{2(6)-4-2} \times 3 = 12.5 + \frac{2}{6} \times 3 = 12.5 + 1 = 13.5$$

10. A student obtained the following marks at a certain examination: English 73, Urdu 82, Mathematics 80, History 67 and Science 62.

(i) If the weights accorded these marks are 4, 3, 3, 2 and 2, respectively, what is an appropriate average mark?

(ii) What is the average mark if equal weights are used?

Solution:

Marks (X)	Weight (w)	Xw
73	4	73 × 4 = 292
82	3	82 × 3 = 246
80	3	80 × 3 = 240
67	2	67 × 2 = 134
62	2	62 × 2 = 124
$\sum X = 364$	$\sum w = 14$	$\sum Xw = 1036$

$$(i)\ \overline{X_w} = \frac{\sum Xw}{\sum w} = \frac{1036}{14} = 74$$

$$(ii)\ \overline{X} = \frac{\sum X}{\sum n} = \frac{364}{5} = 72.8$$

11. On a vacation trip a family bought 21.3 liters of petrol at 39.90 rupees per liter, 18.7 liters at 42.90 rupees per liter, and 23.5 liters at 40.90 rupees per liter. Find the mean price paid per liter.

Solution:

X	w	Xw
21.3	39.90	(21.3) (39.90) = 849.87
18.7	42.90	(21.3) (39.90) = 849.87
23.5	40.90	(21.3) (39.90) = 849.87
$\sum X = 63.5$		$\sum Xw = 2613.25$

$$\text{Mean Price} = \frac{\sum Xw}{\sum X} = \frac{2613.25}{63.5} = 41.15 \text{ rupees per liter}$$

12. Calculate simple moving average of 3 years from the following data:

Years	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Values	102	108	130	140	158	180	196	210	220	230

Solution:

Years	Value	3 - years	3 - years moving
2001	102	-	-
2002	108	340	340/3 = 113.33
2003	130	378	378/3 = 126.00
2004	140	428	428/3 = 142.67
2005	158	478	478/3 = 159.33
2006	180	534	534/3 = 178.00

2007	196	586	586/3 = 195.33
2008	210	626	626/3 = 208.67
2009	220	660	660/3 = 220.00
2010	230	-	-

13. Determine graphically for the following data and check your answer by using formulae.

(i) Median and Quartiles using cumulative frequency polygon.

(ii) Mode using Histogram

Class Boundaries	Frequency
10-20	2
20-30	5
30-40	9
40-50	6
50-60	4
60-70	1

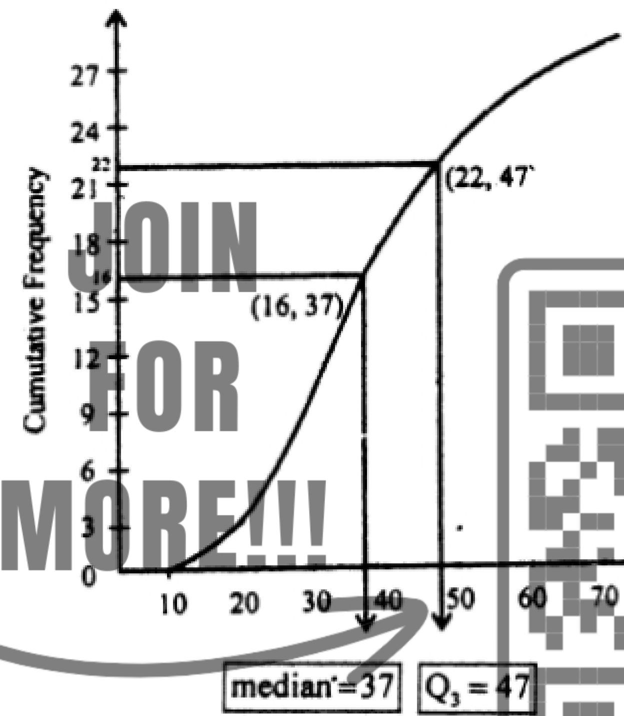
Part (i)

Solution:

Class Boundaries	f	c. f
Less than 10	0	0
Less than 20	2	2
Less than 30	5	7
Less than 40	9	16
Less than 50	6	22

Less than 60	4	26
Less than 70	1	27
	$\sum f = 27$	

Cumulative Frequency Polygon:



Median class Q_3 class

$$\text{Median class} = \left(\frac{n}{2}\right)^{th} \text{ observation} = \left(\frac{27}{2}\right)^{th} = (13.5)^{th} \text{ observation}$$

$$\text{Median} = l + \frac{h}{f}\left\{\frac{n}{2} - c\right\}$$

$$\textit{Here} \quad l = 30, c = 7, f = 9, h = 10, n = 27$$

$$\text{Thus median } x = 30 + \frac{10}{9}\left(\frac{27}{2} - 7\right) = 30 + \frac{10}{9}\left(\frac{13}{2}\right) = 30 + 7.22 = 37.22$$

To find Q_3

we have to find $3\left(\frac{n}{4}\right)^{th}$ observation

$$Q_3 \text{ Class} = 3\left(\frac{n}{4}\right)^{th} \text{ observation} = 3\left(\frac{27}{4}\right)^{th} \text{ observation}$$

$$= 3(6.75)^{th} \text{ observation} = (20.25)^{th} \text{ observation}$$

Q_3 Class is 40-50.

$$\text{Now } Q_3 = l + \frac{h}{f}\left(\frac{3n}{4} - c\right)$$

Here $l = 40, c = 16, f = 6, h = 10, n = 27$

$$Q_3 = 40 + \frac{10}{6}\left(\frac{3\times 27}{4} - 16\right) = 40 + \frac{10}{6}(20.25 - 16)$$

$$= 40 + \frac{10}{6}(4.25) = 40 + 7.08 = 47.08$$

Part (ii)

Solution:

Class Boundaries	Frequency
10-20	2
20-30	5
30-40	9
40-50	6
50-60	4
60-70	1

Histogram:

From the Graph:

Mode = 35.7

Verification of Mode by Formula:

Class Boundaries	Frequency
10-20	2
20-30	$f_1 \rightarrow 5$
30-40	$f_m \rightarrow 9$
40-50	$f_2 \rightarrow 6$
50-60	4
60-70	1

As the group (30 - 40) has the maximum frequency (9).

So, the modal group is (30 – 40).

$$\text{Mode} = l + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$$

$$\textit{Here} \quad l = 30, f_m = 9, f_1 = 5, f_2 = 6, \ h = 10$$

$$\text{Mode} = 30 + \frac{9-5}{2(9)-5-6} \times 10$$

$$= 30 + \frac{4 \times 10}{18-11}$$

$$= 30 + \frac{40}{7}$$

$$= 30 + 5.71$$

$$\text{Mode} = 35.71$$

The result is very clos to the value (35.7) which is obtained from the graph.

Measures of Dispersion:

Statistically, Dispersion means the spread or scatterness of observations in a data set. The spread or scatterness in a data set can *be seen* in two ways:

(i) The spread between two extreme observations in a data set.

(ii) The spread of observations around an average say their arithmetic mean.

The purpose of finding Dispersion is to study the behavior of each unit of population around the average value. This also helps in comparing two sets of data in more detail.

The measures that are used to determine the degree or extent of variation in a data set are called Measures of Dispersion.

We shall discuss only some important absolute measures of dispersion now.

(i) Range:

Range measures the extent of variation between two extreme observations of a data set. It is given by the formula:

Range = $X_{max} - X_{min} = X_m - X_0$

where $X_{max} = X_m$ = the maximum, highest or largest observation.

$X_{min} = X_0$ = the minimum, lowest or smallest observation.

The formula to find range for grouped continuous data is given below:

Range = (Upper class boundary of last group) - (lower class boundary of first group).

(ii) Variance:

Variance is defined as the mean of the squared deviations of x_i (i =1, 2, ……, n) observations from their arithmetic mean. In symbols,

$$\text{Variance of X} = \text{Var}(X) = S^2 = \frac{\sum(X-\overline{X})^2}{n}$$

(iii) Standard Deviation:

Standard deviation is defined as the positive square root of mean of the squared deviations of X_i (i = 1, 2, …., n) observations from their arithmetic mean. In symbols we write.

$$\text{Standard Deviation of X} = \text{S.D}(X) = S = \sqrt{\frac{\sum(X-\overline{X})^2}{n}}$$

Computation of Variance and Standard Deviation:

We use the following formulae to compute Variance and Standard Deviation for Ungrouped and Grouped Data.

Ungrouped Data

The formula of Variance is given by:

$$\text{Var}(X) = S^2 = \frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2$$

And Standard Deviation is given by:

$$\text{S.D}(X) = S = \sqrt{\left[\frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2\right]}$$

Grouped Data:

The formula of Variance is given by:

$$\text{Var}(X) = S^2 = \frac{\sum fX^2}{\sum f} - \left(\frac{\sum fX}{\sum f}\right)^2$$

And Standard Deviation is given by:

$$\text{S.D}(X) = S = \sqrt{\left[\frac{\sum fX^2}{\sum f} - \left(\frac{\sum fX}{\sum f}\right)^2\right]}$$

Exercise 6.3

1. What do you understand by Dispersion?

Solution:

Dispersion:

Dispersion means the spread or scatterness of observations in a data set. By dispersion we mean the extent to which the observations in a sample or in a population are spread out. The main measures of dispersion are range, variance and standard deviation.

2. How do you define measure of dispersion?

Solution:

The measures that are used to determine the degree or extent of variation in a data set are called measure of dispersion.

3. Define Range, Standard deviation and Variance.

Solution:

Range:

Range measures the extent of variation between two extreme observations of a data set. It is given by the formula:

Range = X_{max} - X_{min}

Range = (upper C. B of the last group) - (lower C. B of first group)

Variance:

The mean of the squared deviations of x_i (i = 1, 2, ..., n) observations from their arithmetic mean.

$$\text{Variance} = S^2 = \frac{\sum\left(X-\overline{X}\right)^2}{n}$$

$$= S^2 = \frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2$$

Standard Deviation:

The positive square root of the squared deviations of x_i (i = 1, 2, 3, ..., n) observations from their mean.

$$\text{Standard Deviation} = S = \sqrt{\frac{\sum\left(X-\overline{X}\right)^2}{n}}$$

$$= S = \sqrt{\frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2}$$

4. The salaries of five teachers in Rupees are as follows.

11500, 12400, 15000, 14500, 14800.

Find Range and standard deviation.

Solution:

X = 11500, 12400, 15000, 14500, 14800

Here, X_{max} = 15000, X_{min} = 11500

Range = X_{max} - X_{min}

= 15000 - 11500 = 3500

$$\overline{X} = \frac{\sum X}{n}$$

$$= \frac{11500 + 12400 + 15000 + 14500 + 14800}{5}$$

$$= \frac{68200}{5} = 13640$$

X	$X - \overline{X}$	$(X - \overline{X})^2$
11500	-2140	4579600
12400	-1240	1537600
15000	1360	1849600
14500	860	739600
14800	1160	1345600

$$\sum (X - \overline{X})^2 = 10052000, n = 5$$

$$S.D = S = \sqrt{\frac{\sum (X - \overline{X})^2}{n}} = \sqrt{\frac{10052000}{5}}$$

$$= \sqrt{2010400} = 1417.88$$

5.

a. Find the standard deviation "S" of each set of numbers:

(i) 12, 6, 7, 3, 15, 10, 18, 5

(ii) 9, 3, 8, 8, 9, 8, 9, 18.

b. Calculate variance for the data: 10, 8, 9, 7, 5, 12, 8, 6, 8, 2.

Solution:

(i)

X	$X-\overline{X}$	$\left(X-\overline{X}\right)^2$
12	2.5	6.25
6	-3.5	12.25
7	-2.5	6.25
3	-6.5	42.25
15	5.5	30.25
10	0.5	0.25
18	8.5	72.25
5	-4.5	20.25

$\sum x = 76$ $\qquad$ $\sum\left(X-\overline{X}\right)^2 = 190, n = 8$

$\overline{X} = \frac{76}{8} = 9.5$

$S.D = S = \sqrt{\frac{\sum\left(X-\overline{X}\right)^2}{n}} = \sqrt{\frac{190}{8}}$

$= \sqrt{23.75} = 4.87$

(ii)

X	$X-\overline{X}$	$\left(X-\overline{X}\right)^2$
9	0	0
3	-6	36
8	-1	1
8	-1	1
9	0	0
8	-1	1
9	0	0
18	9	81

$\sum x = 72$ $\qquad$ $\sum\left(X-\overline{X}\right)^2 = 120$

n = 8

$$\overline{X} = \frac{\sum x}{n} = \frac{72}{8} = 9$$

$$S.D = S = \sqrt{\frac{\sum\left(X - \overline{X}\right)^2}{n}} = \sqrt{\frac{120}{8}}$$

$$= \sqrt{15} = 3.87$$

b. Calculate variance for the data: 10, 8, 9, 7, 5, 12, 8, 6, 8, 2.

Solution:

(i)

X	$X - \overline{X}$	$\left(X - \overline{X}\right)^2$
10	2.5	6.25
8	0.5	25
9	1.5	2.25
7	- 0.5	.25
5	- 2.5	6.25
12	4.5	20.25
8	0.5	.25
6	-1.5	2.25
8	0.5	.25
2	- 5.5	30.25

$\sum x = 75$ $\sum\left(X - \overline{X}\right)^2 = 68.5$

n = 10

$$\overline{X} = \frac{\sum x}{n} = \frac{75}{10} = 7.5$$

$$Variance = S^2 = \frac{\sum\left(X - \overline{X}\right)^2}{n}$$

$$= \frac{68.5}{10} = 6.85$$

6. The length of 32 items are given below. Find the mean length and standard deviation of the distribution.

Length	20-22	23-25	26-28	29-31	32-34
Frequency	3	6	12	9	2

Solution:

C.I	f	Mid-point (x)	fx	$X-\overline{X}$	$\left(X-\overline{X}\right)^2$	$f\left(X-\overline{X}\right)^2$
20 – 22	3	21	63	–6	36	108
23 – 25	6	24	144	–3	9	54
26 – 28	12	27	324	0	0	0
29 – 31	9	30	270	3	9	81
32 – 34	2	33	66	6	36	72
	$\sum f$ =n=32		$\sum fx$ = 867		$\sum\left(X-\overline{X}\right)^2$ =90	$\sum f\left(X-\overline{X}\right)^2$ =315

$$\overline{X}=\frac{\sum fx}{n}=\frac{667}{32}=27.093=27\,\text{approx}$$

$$S.D=S^2=\sqrt{\frac{\sum f\left(X-\overline{X}\right)^2}{n}}=\sqrt{\frac{315}{32}}$$

$$=\sqrt{9.84375}=3.137$$

7. **For the following distribution of marks calculate Range.**

Marks in percentage	Frequency/ (No of Students)
33 - 40	28
41 - 50	31
51 - 60	12
61 - 70	9
71 - 75	5

Solution:

C.I	Class Boundaries	f
33 - 40	32.5 – 40.5	28
41 - 50	40.5 – 50.5	32
51 - 60	50.5 – 60.5	12
61 - 70	60.5 – 70.5	9
71 - 75	70.5 – 75.5	5

Here, $X_{max} = 75.5$

$X_{min} = 32.5$

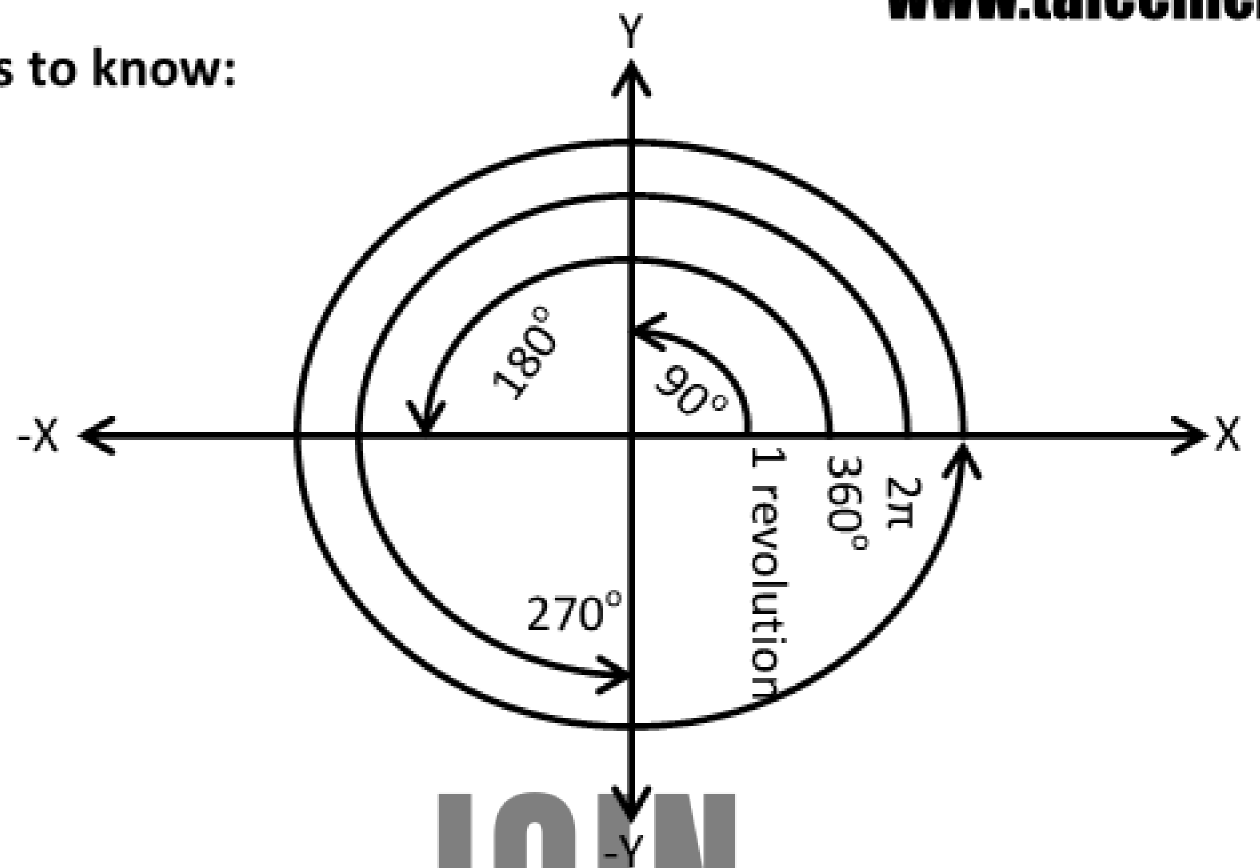
Range $= X_{max} - X_{min}$

$= 75.5 – 32.5$

$= 43$

Exercise 7.1

Things to know:



So,

1 revolution = 2π radian = 360°

and

2π radian = 360°

To get 1 radian divide by 2π

1 radian $= \frac{360^o}{2\pi}$

1 radian $= \frac{180^o}{\pi}$

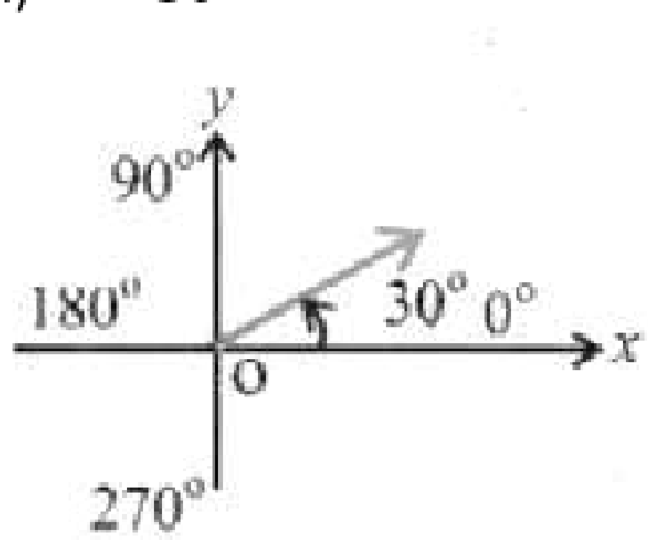
To get 1 degree divide by 360

$\frac{2\pi}{360}$ rad = 1°

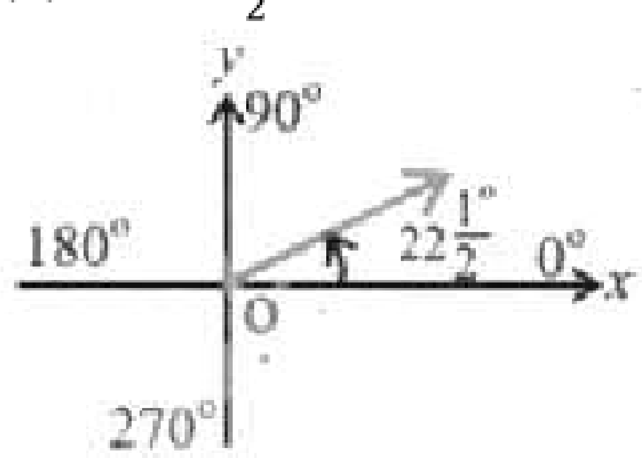
1° $= \frac{\pi}{180}$ rad

Q. 1: Locate the following angles:

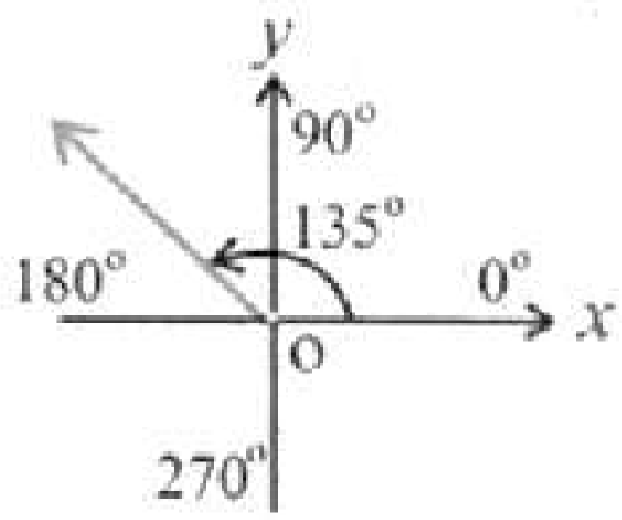
(i) 30^o



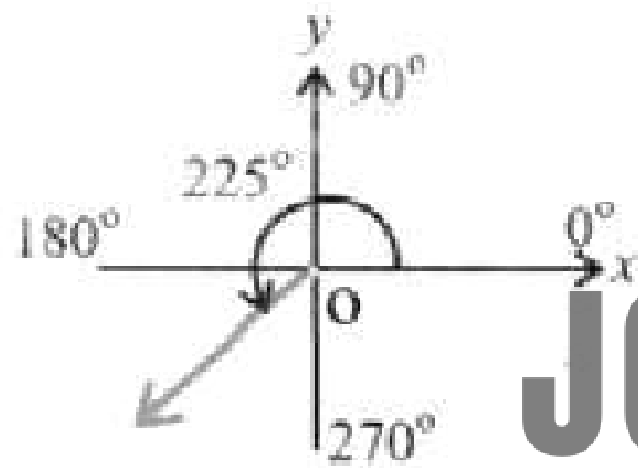
(ii) $22\frac{1}{2}^o$



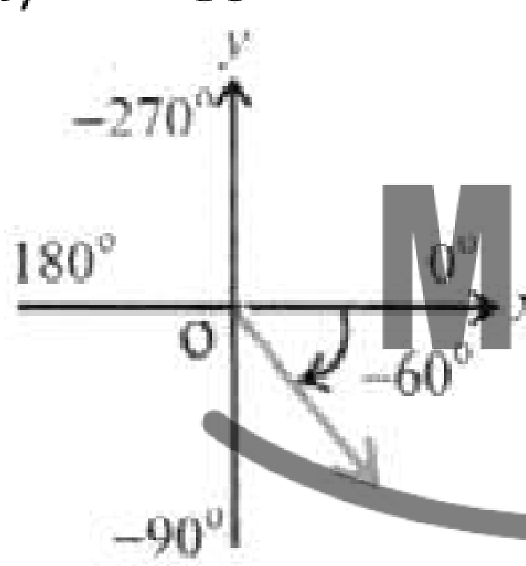
(iii) $22\frac{1}{2}^o$



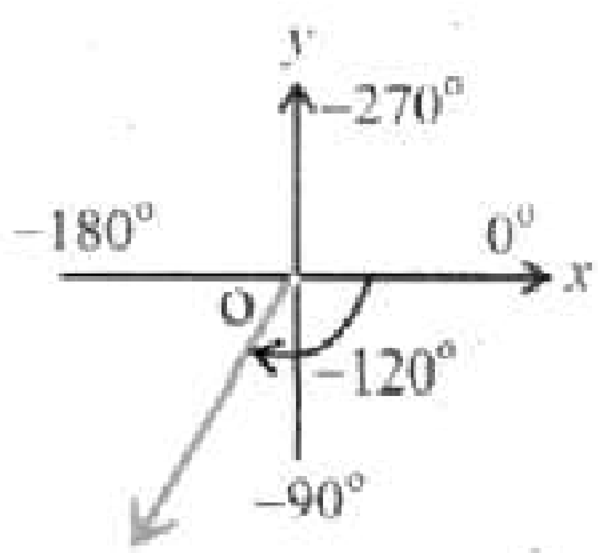
(iv) 225^o



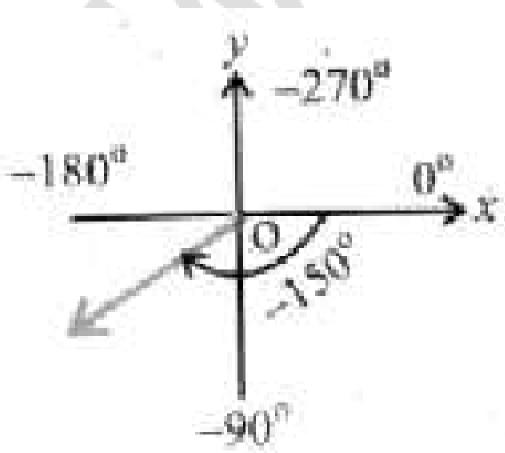
(v) -60^o



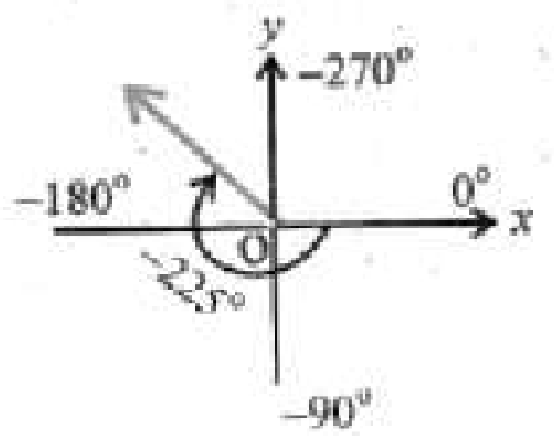
(vi) -120^o



(vii) -150^o



(viii) -225^o



Q. 2: Express the following sexagesimal measures of angles in decimal form.

(i) $45^o30'$ $= 45^o + \left(\frac{30}{60}\right)^o$

$= 45^o + 0.5$

$= 45.5^o$

(ii) $60^o30'30''$ $= 60^o + \left(\frac{30}{60}\right)^o + \left(\frac{30}{3600}\right)^o$

$= 60^o + 0.5 + 0.0083$

$= 60.5083^o$

(iii) $125^o22'50''$ $= 125^o + \left(\frac{22}{60}\right)^o + \left(\frac{50}{3600}\right)^o$

$= 125^o + 0.3667 + 0.0139$

$= 125.3806^o$

Q. 3: Express the following into D° M' S'' form.

(i) 47.36^o $= 47^o + (.36 \times 60)'$

$= 47^o + 21.6'$

$= 47^o + 21' + (.6 \times 60)''$

$= 47^o + 21' + 36''$

$= 47^o21'36''$

(ii) 125.45^o $= 47^o + (.45 \times 60)'$

$= 47^o + 27'$

$= 47^o27'$

(iii) -22.5^o $= -22^o - (.5 \times 60)'$

$= -22^o - 30'$

$= -22^o30'$

(iv) -67.58^o $= -67^o - (.58 \times 60)'$

$= -67^o - 34.8'$

$= -67^o - 34' - (.8 \times 60)''$

$= -67^o - 34' - 48''$

$= -67^o34'48''$

(v) 315.18^o $= 315^o + (.18 \times 60)'$

$= 315^o + 10.8'$

$= 315^o + 10' + (.8 \times 60)''$

$= 315^o + 10' + 48''$

$= 315^o10'48''$

Q. 4: Express the following angles into radians.

(i) 30^o $= 30\left(\frac{\pi}{180}\right)$

$= \frac{\pi}{6}$

(ii) 60^o $= 60\left(\frac{\pi}{180}\right)$

$= \frac{\pi}{3}$

(iii) 135^o $= 135\left(\frac{\pi}{180}\right)$

$= 3\left(\frac{\pi}{4}\right)$

$= \frac{3\pi}{4}$

(iv) 225^o $= 225\left(\frac{\pi}{180}\right)$

$= 5\left(\frac{\pi}{4}\right)$

$= \frac{5\pi}{4}$

(v) -150^o $= -150\left(\frac{\pi}{180}\right)$

$= -5\left(\frac{\pi}{6}\right)$

$= -\frac{5\pi}{6}$

(vi) -225^o $= -225\left(\frac{\pi}{180}\right)$

$= -5\left(\frac{\pi}{4}\right)$

$= -\frac{5\pi}{4}$

(vii) 300^o $= 300\left(\frac{\pi}{180}\right)$

$= 5\left(\frac{\pi}{3}\right)$

$= \frac{5\pi}{3}$

(viii) 315^o $= 315\left(\frac{\pi}{180}\right)$

$= 7\left(\frac{\pi}{4}\right)$

$= \frac{7\pi}{4}$

Q. 5: Convert each of following to degrees.

(i) $\frac{3\pi}{4}$ $= \frac{3\pi}{4}\left(\frac{180}{\pi}\right)$

$= 3(45)$

$= 135^o$

(ii) $\frac{5\pi}{6}$ $= \frac{5\pi}{6}\left(\frac{180}{\pi}\right)$

$= 5(30)$

$= 150^o$

(iii) $\frac{7\pi}{8}$ $= \frac{7\pi}{8}\left(\frac{180}{\pi}\right)$

$= \frac{7}{2}(45)$

$= 157.5^o$

(iv) $\frac{13\pi}{16}$ $= \frac{13\pi}{16}\left(\frac{180}{\pi}\right)$

$= \frac{13}{4}(45)$

$= 146.25^o$

(v) 3 $= 3\left(\frac{180}{\pi}\right)$

$= 171.8869^o$

(vi) 4.5 $= 4.5\left(\frac{180}{\pi}\right)$

$= 257.83^o$

(vii) $\frac{-7\pi}{8}$ $= \frac{-7\pi}{8}\left(\frac{180}{\pi}\right)$

$= \frac{-7}{2}(45)$

$= 157.5^o$

(viii) $-\frac{13\pi}{16}$ $= -\frac{13\pi}{16}\left(\frac{180}{\pi}\right)$

$= -\frac{13}{4}(45)$

$= -146.25^o$

Exercise 7.2

Q. 1: Find θ, when:

(i) $l = 2cm, r = 3.5cm$

As we know

$l = r\theta$

$\theta = \frac{l}{r}$

$\theta = \frac{2}{3.5}$

$\theta = 0.57rad$

(ii) $l = 4.5m, r = 2.5m$

As we know

$l = r\theta$

$\theta = \frac{l}{r}$

$\theta = \frac{4.5}{2.5}$

$\theta = 1.8rad$

Q. 2: Find l, when:

(i) $\theta = 180^0, r = 4.9cm$

$\theta = 180\left(\frac{\pi}{180}\right)$

$= \pi$

As we know

$l = r\theta$

$l = (4.9)(\pi)$

$l = 15.4cm$

(ii) $\theta = 60^0 30', r = 15mm$

$\theta = \left(60 + \frac{30}{60}\right)\left(\frac{\pi}{180}\right)$

$= (60 + 0.5)\left(\frac{\pi}{180}\right)$

$= (60.5)\left(\frac{\pi}{180}\right)$

$= 1.0559rad$

As we know

$l = r\theta$

$l = (15)(1.0559)$

$l = 15.84mm$

Q. 3: Find r, when:

(i) $l = 4cm, \theta = \frac{1}{4}radian$

As we know

$l = r\theta$

$r \quad = \frac{l}{\theta}$

$r \quad = \frac{4}{1/4}$

$r \quad = 16cm$

(ii) $l = 52cm, \theta = 45^o$

$\theta \quad = (45)\left(\frac{\pi}{180}\right)$

$\quad = 0.7854 radian$

As we know

$l \quad = r\theta$

$r \quad = \frac{l}{\theta}$

$r \quad = \frac{52}{0.7854}$

$r \quad = 66.21cm$

Q. 4: In a circle of radius 12m, find the length of an arc which subtends a central angle $\theta = 1.5 radian$.

$\theta = 1.5rad, r = 12m$

As we know

$l \quad = r\theta$

$l \quad = (12)(1.5)$

$l \quad = 18m$

Q. 5: In a circle of radius 10m, find the distance travelled by a point in moving on this circle if the point makes 3.5 revolutions.

$\theta = 3.5 revolutions = 7\pi, r = 10m$

As we know

$l \quad = r\theta$

$l \quad = (10)(7\pi)$

$l \quad = 220m$

Q. 6: What is the circular measure of the angle between the hands of the watch at 3 o'clock?

Full angle of a watch $= 2\pi$

Then,

Angle between two digits $= \frac{2\pi}{12}$

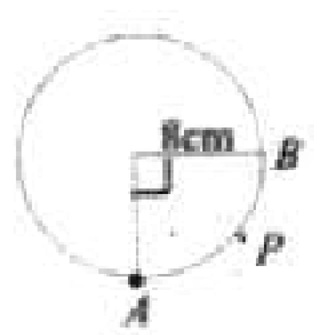
Angle b/w the hands of watch $= \frac{2\pi}{12} \times 3$

$= \frac{\pi}{2} rad$

Q. 7: What is the length of the arc APB?

From figure, we have

$\theta = 90^o = \frac{\pi}{2}, r = 8cm$



As we know

$l \quad = r\theta$

$l \quad = (8)\left(\frac{\pi}{2}\right)$

$l \quad = 12.57cm$

Q. 8: In a circle of radius 12cm, how long an arc subtends a central angle of 84°.

$\theta = 84^o, r = 12cm$

$\theta = (84)\left(\frac{\pi}{180}\right)$

$= 1.4661 radian$

As we know

Area of Arc $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}(12)^2(1.4661)$

$= 105.56 cm^2$

Q. 9: Find the area of the sectors OPR.

(a) $\theta = 60^o, r = 6cm$

$\theta = (60)\left(\frac{\pi}{180}\right)$

$= 1.0472 radian$

As we know

Area of Arc $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}(6)^2(1.0472)$

$= 18.85 cm^2$

(b) $\theta = 45^o, r = 20cm$

$\theta = (45)\left(\frac{\pi}{180}\right)$

$= 0.7854 radian$

As we know

Area of Arc $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}(20)^2(0.7854)$

$= 157.08 cm^2$

Q. 10: Find area of the sector inside a central angle of 20° in a circle of radius 7m.

$\theta = 20^o, r = 7m$

$\theta = (20)\left(\frac{\pi}{180}\right)$

$= 0.3491 radian$

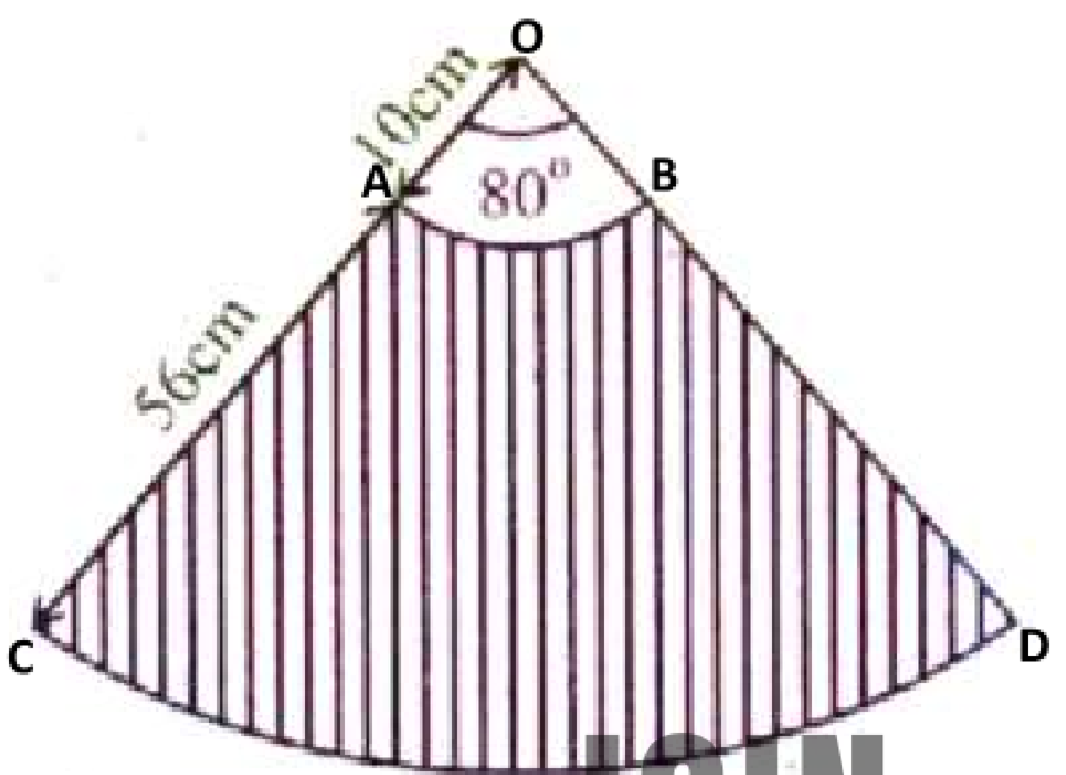
As we know

Area of Arc $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}(7)^2(0.3491)$

$= 8.55m^2$

Q. 11: Sehar is making a skirt. Each panel of this skirt is of the shape shown shaded in the diagram. How much material (cloth) is required for each panel?



We have labeled the diagram as shown in figure.

So, shaded Area ABCD = Area of Arc OCD – Area of Arc OAB ------------------- (i)

As θ is the same for both the Arcs, So

$\theta = (80)\left(\frac{\pi}{180}\right)$

$= 1.3963 radian$

For Arc OCD, we have

r = OA + AC = 10 + 56 = 66 cm

Area of Arc OCD = $\frac{1}{2}r^2\theta$

$= \frac{1}{2}(66)^2(1.3963)$

$= 3041.14cm^2$

For Arc OAB, we have

r = OA = 10 cm

Area of Arc OCD = $\frac{1}{2}r^2\theta$

$= \frac{1}{2}(10)^2(1.3963)$

$= 69.82cm^2$

Now from (i)

shaded Area ABCD = Area of Arc OCD – Area of Arc OAB

= 3041.14 – 69.82

$= 2971.32cm^2$

Q. 12: Find the area of the sector with central angle of $\frac{\pi}{5}$ radian in a circle of radius 10cm.

$\theta = \frac{\pi}{5}, r = 10cm$

As we know

Area of Arc $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}(10)^2\left(\frac{\pi}{5}\right)$

$= 31.42cm^2$

Q. 13: The area of the sector with a central angle θ in a circle of radius 2m is 10 square meter. Find θ in radians.

$Area = 10m^2, r = 2m$

As we know

Area of Arc $= \frac{1}{2}r^2\theta$

$10 = \frac{1}{2}(2)^2\theta$

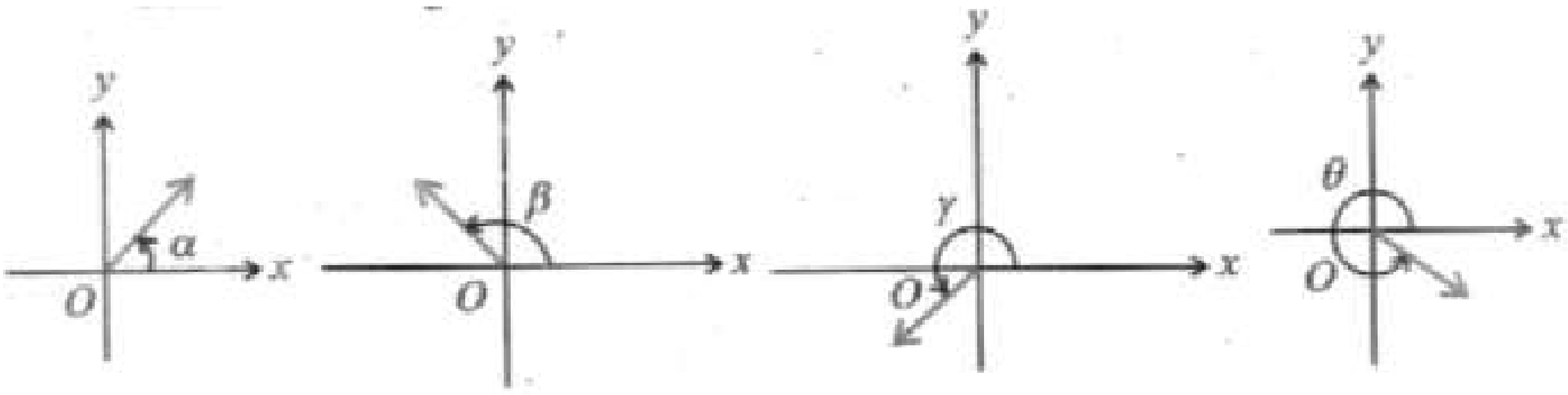
$10 = 2\theta$

$\theta = 5radians$

Exercise 7.3

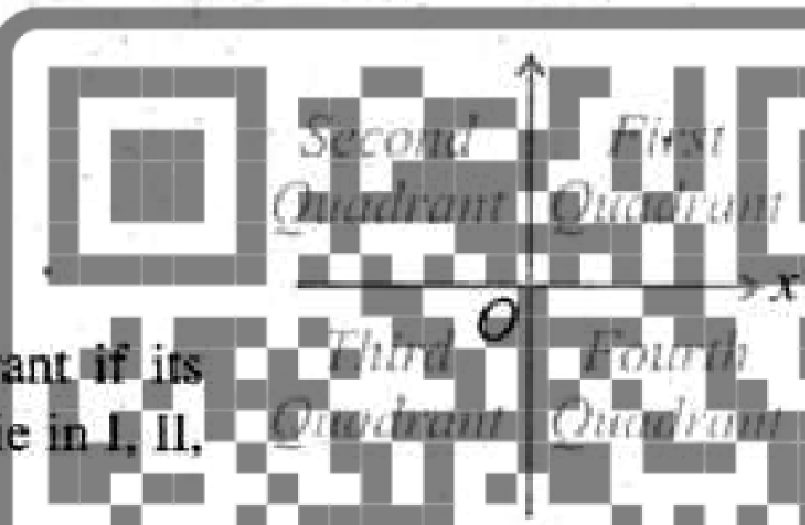
Things To know:

1. Angles in standard position:



2. The Quadrants and Quadrantal Angles:

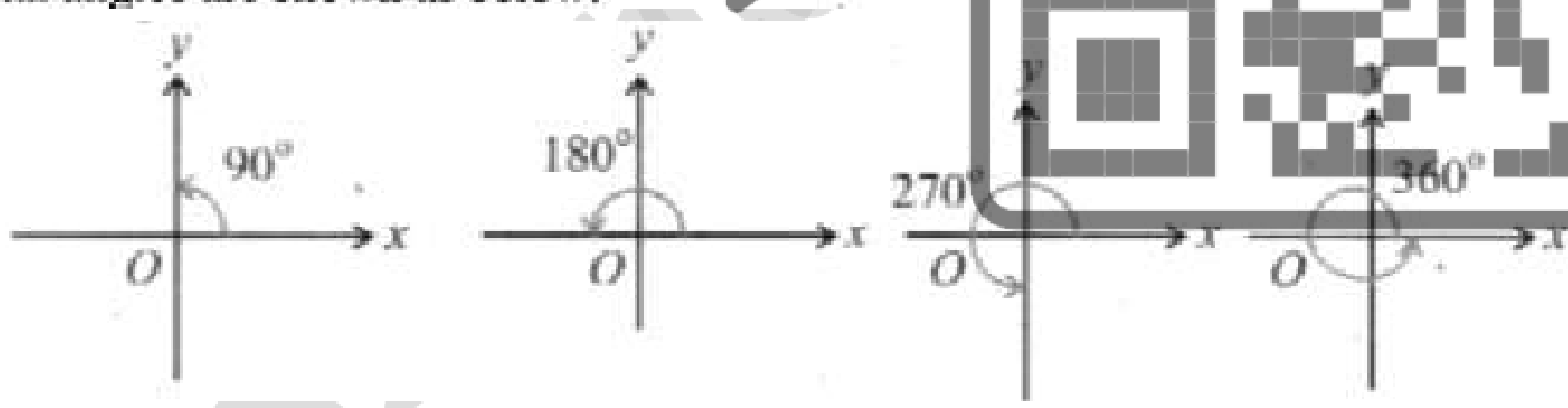
The x-axis and y-axis divides the plane in four regions, called quadrants, when they intersect each other at right angle. The point of intersection is called origin and is denoted by O.



Angles between 0° and 90° are in the first quadrant.
Angles between 90° and 180° are in the second quadrant.
Angles between 180° and 270° are in the third quadrant.
Angles between 270° to 360° are in the fourth quadrant.
An angle in standard position is said to lie in a quadrant if its terminal side lies in that quadrant. Angles α, β, γ and θ lie in I, II, III and IV quadrant respectively in figure 7.3.1.

Quadrantal Angles

If the terminal side of an angle in standard position falls on x-axis or y-axis, then it is called a quadrantal angle *i.e.*, 90°, 180°, 270° and 360° are quadrantal angles. The quadrantal angles are shown as below:



3. Trigonometric ratios and their reciprocals with the help of a unit circle:

There are six fundamental trigonometric ratios called sine, cosine, tangent, cotangent, secant and cosecant. To define these functions we use circular approach which involves the unit circle.

Let θ be a real number, which represents the radian measure of an angle in standard position. Let $P(x, y)$ be any point on the unit circle lying on terminal side of θ as shown in the figure.

We define sine of θ, written as $\sin\theta$ and cosine of θ written as $\cos\theta$, as:

$$\sin\theta = \frac{EP}{OP} = \frac{y}{1} \quad \Rightarrow \quad \sin\theta = y$$

and $$\cos\theta = \frac{OE}{OP} = \frac{x}{1} \quad \Rightarrow \quad \cos\theta = x$$

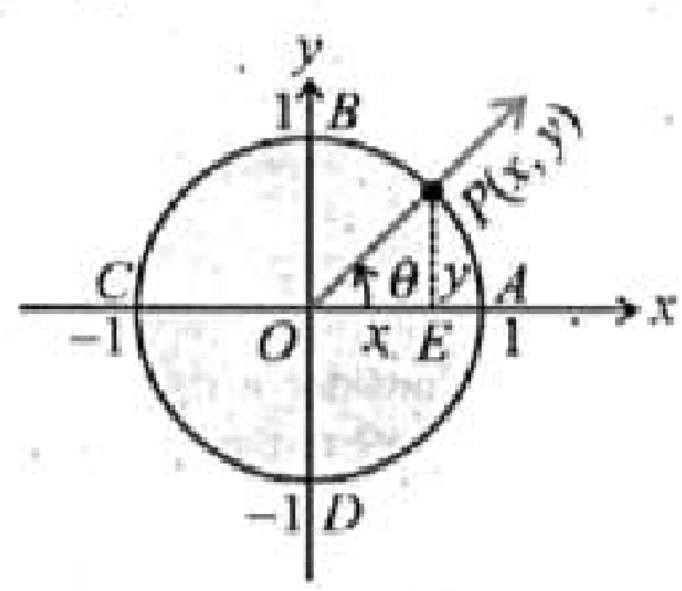


Fig. 7.3.3

$$\tan\theta = \frac{EP}{OE} = \frac{y}{x} \Rightarrow \tan\theta = \frac{y}{x} \quad (x \neq 0)$$

since $y = \sin\theta$ and $x = \cos\theta \Rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta}$

$$\cot\theta = \frac{x}{y} \quad (y \neq 0) \Rightarrow \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sec\theta = \frac{1}{x} (x \neq 0) \quad \text{and} \quad \text{cosec}\,\theta = \frac{1}{y} \quad (y \neq 0)$$

$$= \frac{1}{\cos\theta} \qquad\qquad = \frac{1}{\sin\theta}$$

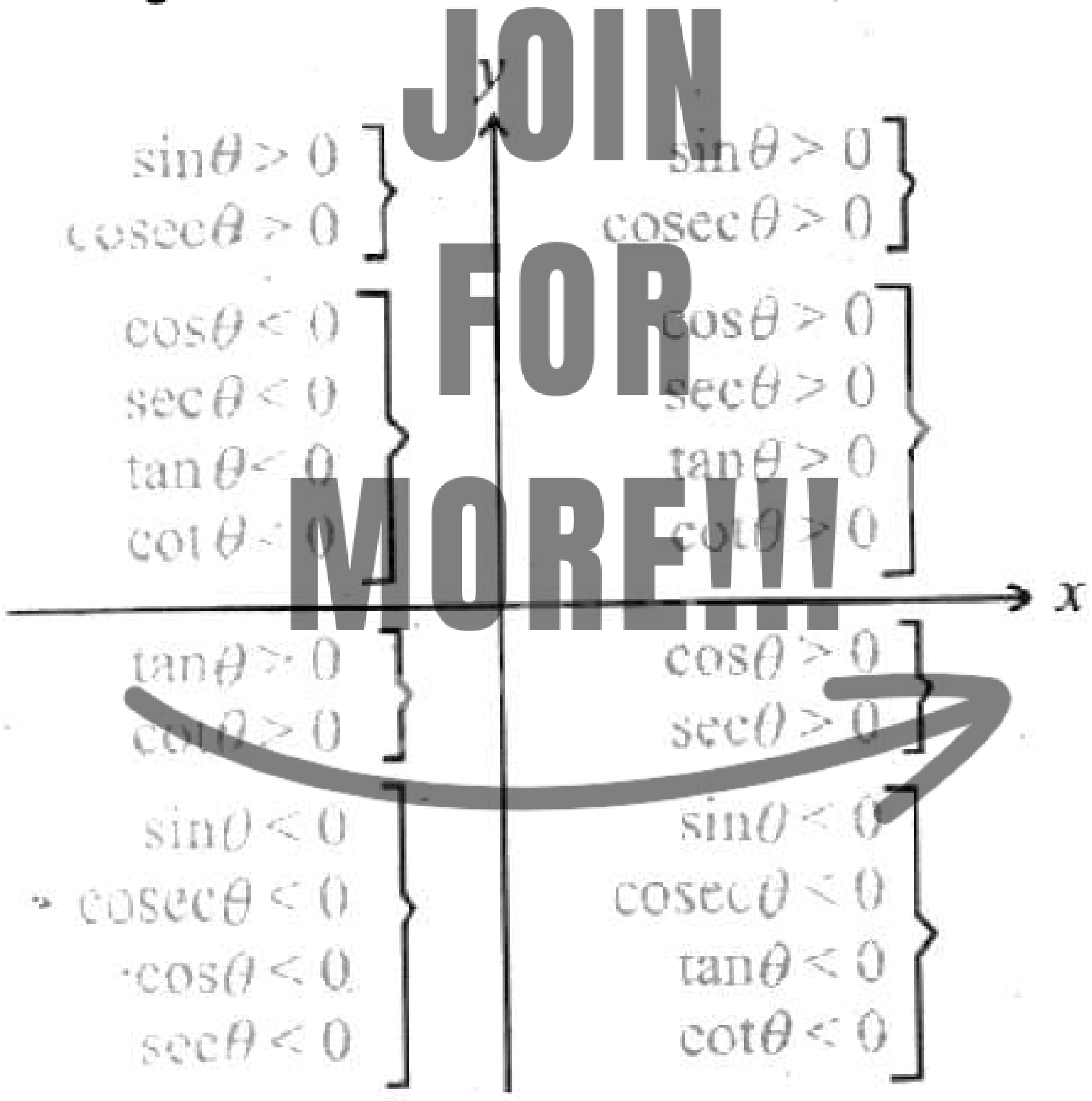
Reciprocal Identities

$$\sin\theta = \frac{1}{\text{cosec}\,\theta} \quad \text{or} \quad \text{cosec}\,\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \quad \text{or} \quad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \quad \text{or} \quad \cot\theta = \frac{1}{\tan\theta}$$

Signs of trigonometric ratios in different Quadrants:

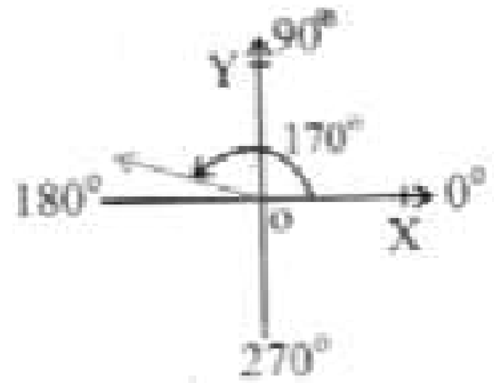


Allied Angles:

$sin(-\Theta) = -sin\Theta$	$cos(-\Theta) = cos\Theta$
$sin(90 - \Theta) = cos\Theta$	$cos(90 - \Theta) = sin\Theta$
$sin(90 + \Theta) = cos\Theta$	$sin(90 + \Theta) = -sin\Theta$
$sin(180 - \Theta) = sin\Theta$	$cos(180 - \Theta) = -cos\Theta$
$sin(180 + \Theta) = -sin\Theta$	$cos(180 + \Theta) = -cos\Theta$

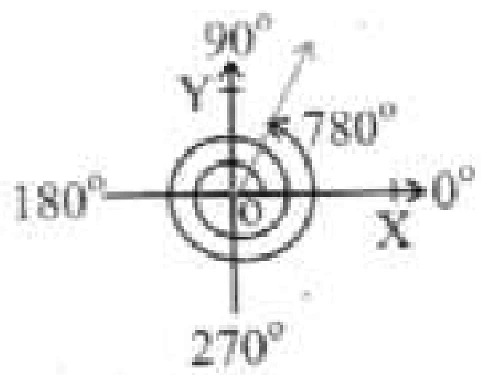
Q. 1: Locate each of the following angles in standard position using protractor or fair free hand guess. Also find a positive and negative angle coterminal with each given angle.

(i) 170°



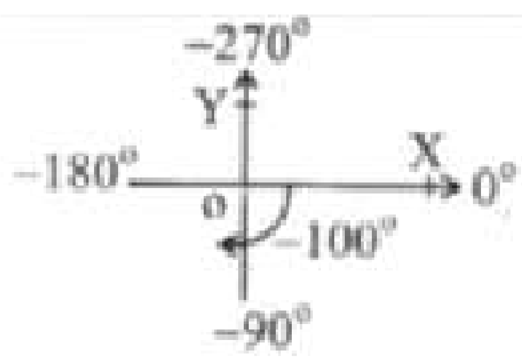
Positive coterminal angle $360^\circ + 170^\circ = 530^\circ$
negative coterminal angle -190°

(ii) 780°



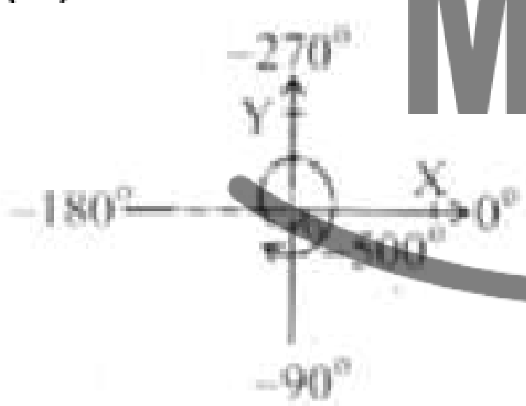
Positive coterminal angle 60°
negative coterminal angle is -300°

(iii) -100°



Positive coterminal angle is 260°
negative coterminal angle $-360^\circ - 100^\circ = -460^\circ$

(iv) -500°



Positive coterminal angle 220°
negative coterminal angle -140°

Q. 2: Identify the closest quadrantal angles between which the following angles lies.

(i) 156°

156° lies between 90° and 180°.

(ii) 318°

318° lies between 270° and 360°.

(iii) 572°

572° lies between 540° and 630°.

(iv) -330°

-330° lies between -270° and -360°. i.e. quadrantal angles are 0° and 90°.

Q. 3: Write the closest quadrantal angles between which the angle lies. Write your answer in radian measure.

(i) $\frac{\pi}{3}$

$\frac{\pi}{3}$ lies between 0 and $\frac{\pi}{2}$

(ii) $\frac{3\pi}{4}$

$\frac{3\pi}{4}$ lies between $\frac{\pi}{2}$ and π

(iii) $\frac{-\pi}{4}$

$\frac{-\pi}{4}$ lies between 0 and $-\frac{\pi}{2}$

(iv) $\frac{-3\pi}{4}$

$\frac{-3\pi}{4}$ lies between $-\frac{\pi}{2}$ and $-\pi$

Q. 4: In which quadrant θ lie when

(i) $sin\theta > 0, tan\theta < 0$

Quadrant II

(ii) $cos\theta < 0, sin\theta < 0$

Quadrant III

(iii) $sec\theta > 0, sin\theta < 0$

Quadrant IV

(iv) $cos\theta < 0, tan\theta < 0$

Quadrant II

(v) $cosec\theta > 0, cos\theta > 0$

Quadrant I

(vi) $sin\theta < 0, sec\theta < 0$

Quadrant III

Q. 5: Fill in the blanks.

(i) $\cos(-150^o) = +\cos150^o$

(ii) $\sin(-310^o) = -\sin310^o$

(iii) $\tan(-210^o) = -\tan210^o$

(iv) $\cot(-45^o) = -\cot45^o$

(v) $\sec(-60^o) = +\sec60^o$

(vi) $cosec(-137^o) = -cosec137^o$

Q. 6: The given point P lies on the terminal side of θ. Find quadrant of θ and all six trigonometric ratios.

(i) (-2, 3)

We have $x = -2$ and $y = 3$, so θ lies in Quadrant II.

$$r = \sqrt{x^2 + y^2}$$
$$= \sqrt{(-2)^2 + (3)^2}$$
$$= \sqrt{4 + 9}$$
$$= \sqrt{13}$$

Thus

$sin\theta = \frac{y}{r} = \frac{3}{\sqrt{13}}$; $cosec\theta = \frac{\sqrt{13}}{3}$

$cos\theta = \frac{x}{r} = \frac{-2}{\sqrt{13}}$; $sec\theta = -\frac{\sqrt{13}}{2}$

$tan\theta = \frac{y}{x} = \frac{-3}{2}$; $cot\theta = -\frac{2}{3}$

(ii) (-3, -4)

We have $x = -3$ and $y = -4$, so θ lies in Quadrant III.

$r = \sqrt{x^2 + y^2}$

$= \sqrt{(-3)^2 + (-4)^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$= 5$

Thus

$sin\theta = \frac{y}{r} = \frac{-4}{5}$; $cosec\theta = \frac{-5}{4}$

$cos\theta = \frac{x}{r} = \frac{-3}{5}$; $sec\theta = -\frac{5}{3}$

$tan\theta = \frac{y}{x} = \frac{4}{3}$; $cot\theta = \frac{3}{4}$

(iii) $(\sqrt{2}, 1)$

We have $x = \sqrt{2}$ and $y = 1$, so θ lies in Quadrant II.

$r = \sqrt{x^2 + y^2}$

$= \sqrt{(\sqrt{2})^2 + (1)^2}$

$= \sqrt{2 + 1}$

$= \sqrt{3}$

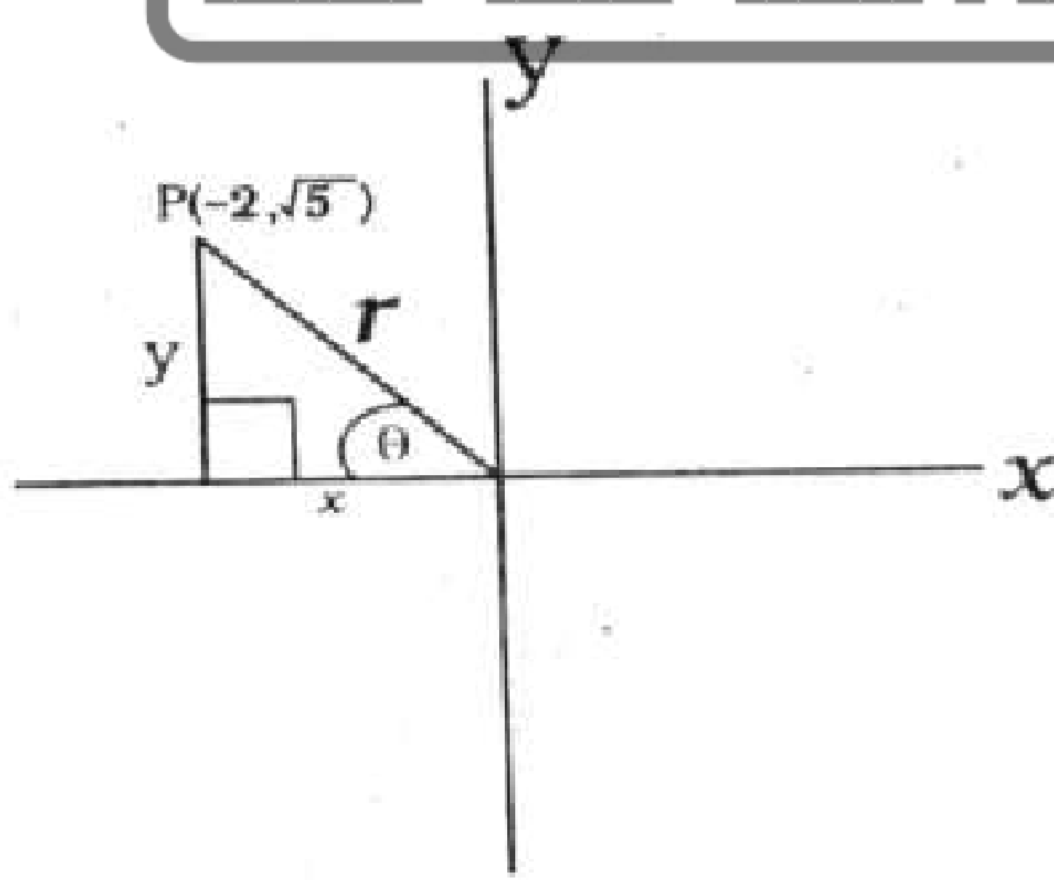
Thus

$sin\theta = \frac{y}{r} = \frac{1}{\sqrt{3}}$; $cosec\theta = \sqrt{3}$

$cos\theta = \frac{x}{r} = \frac{\sqrt{2}}{\sqrt{3}}$; $sec\theta = \frac{\sqrt{3}}{\sqrt{2}}$

$tan\theta = \frac{y}{x} = \frac{1}{\sqrt{2}}$; $cot\theta = \sqrt{2}$

Q. 7: If $cos\theta = \frac{-2}{3}$ and terminal arm of the angle θ is in quadrant II, find the values of remaining trigonometric functions.



In any right triangle XYZ,

$cos\theta = \frac{-2}{3} = \frac{x}{r}$ then, $x = -2$ and $r = 3$

Also,

$sec\theta = \frac{1}{cos\theta} = \frac{-3}{2}$

As we know

$r^2 = x^2 + y^2$

$(3)^2 = (-2)^2 + y^2$

$9 = 4 + y^2$

$5 = y^2$

$y = \pm\sqrt{5}$ so, $y = \sqrt{5}$

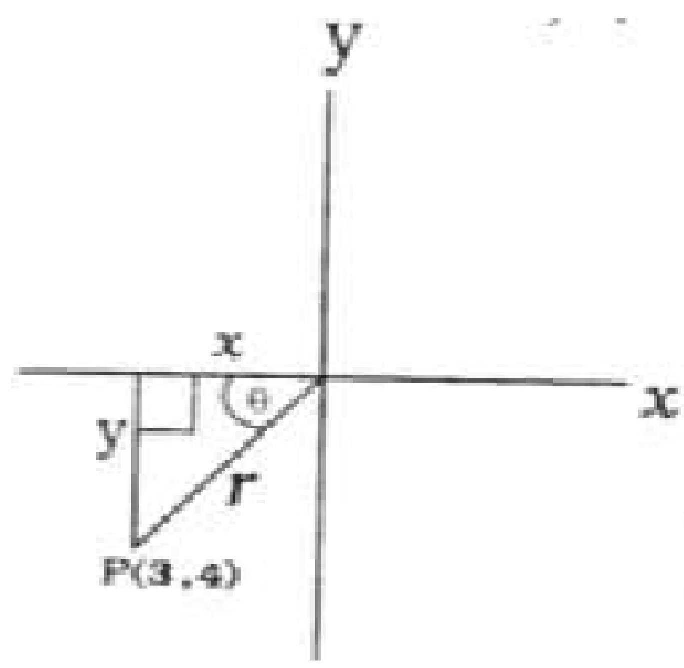
$sin\theta = \frac{y}{r} = \frac{\sqrt{5}}{3}$; $cosec\theta = \frac{3}{\sqrt{5}}$

$tan\theta = \frac{y}{x} = \frac{-\sqrt{5}}{2}$; $cot\theta = \frac{-2}{\sqrt{5}}$

Q. 8: If $tan\theta = \frac{4}{3}$ and $sin\theta < 0$, find the values of other trigonometric functions at θ.

In any right triangle XYZ,

$tan\theta = \frac{4}{3} = \frac{y}{x}$ then, $y = 4$ and $x = 3$



Also,

$cot\theta = \frac{1}{tan\theta} = \frac{3}{4}$

As we know

$r^2 = x^2 + y^2$

$r^2 = (3)^2 + (4)^2$

$r^2 = 9 + 16$

$r^2 = 25$

$r = \pm 5$ so, $r = 5$

As, $sin\theta < 0$ and $tan\theta > 0$. So, the terminal arm of angle lies in Quadrant III.

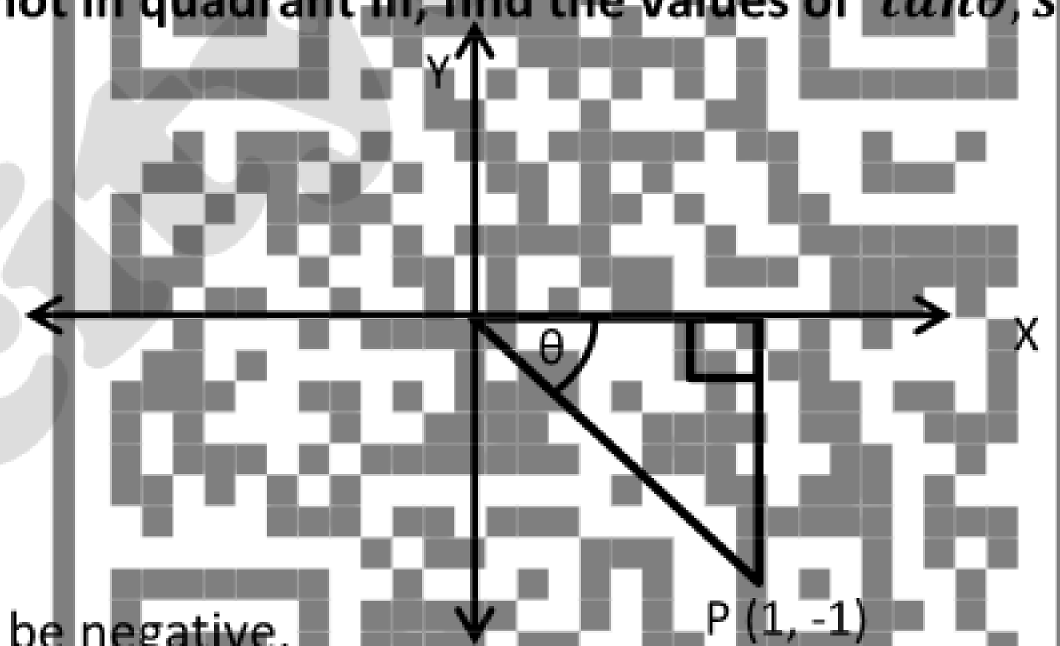
$sin\theta = -\frac{y}{r} = -\frac{4}{5}$; $cosec\theta = \frac{-5}{4}$

$cos\theta = -\frac{x}{r} = \frac{-3}{5}$; $sec\theta = \frac{-5}{3}$

Q. 9: If $sin\theta = \frac{-1}{\sqrt{2}}$ and terminal side of the angle is not in quadrant III, find the values of $tan\theta, sec\theta$ and $cosec\theta$.

In any right triangle XYZ,

$sin\theta = \frac{-1}{\sqrt{2}} = \frac{y}{r}$ then, $y = -1$ and $r = \sqrt{2}$



Also,

$cosec\theta = \frac{1}{sin\theta} = -\sqrt{2}$

As $sin\theta$ is negative in Quadrant III and IV,

therefore in this case θ is in Quadrant IV and $cos\theta$ will be negative.

Now,

$r^2 = x^2 + y^2$

$(\sqrt{2})^2 = x^2 + (-1)^2$

$2 = x^2 + 1$

$1 = x^2$

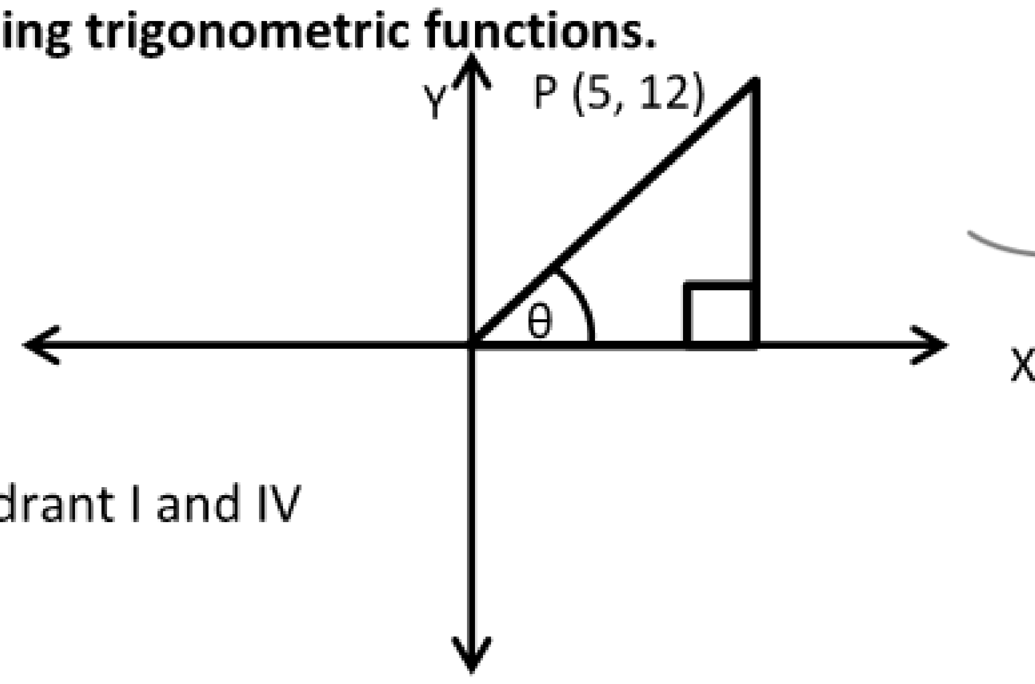
$x = \pm 1$ so, $x = 1$

$sec\theta = \frac{\sqrt{2}}{1} = \sqrt{2}$; $tan\theta = \frac{y}{x} = \frac{-1}{1} = -1$

Q. 10: If $cosec\theta = \frac{13}{12}$ and $sec\theta > 0$, find the remaining trigonometric functions.

In any right triangle XYZ,

$cosec\theta = \frac{13}{12} = \frac{r}{y}$ then, $y = 12$ and $r = 13$



Also,

$sin\theta = \frac{1}{cosec\theta} = \frac{12}{13}$

As $sin\theta > 0$ in Quadrant I and II and $sec\theta > 0$ in Quadrant I and IV

therefore in this case θ is in Quadrant I.

Now,

$r^2 = x^2 + y^2$

$(13)^2 = x^2 + (12)^2$

$169 = x^2 + 144$

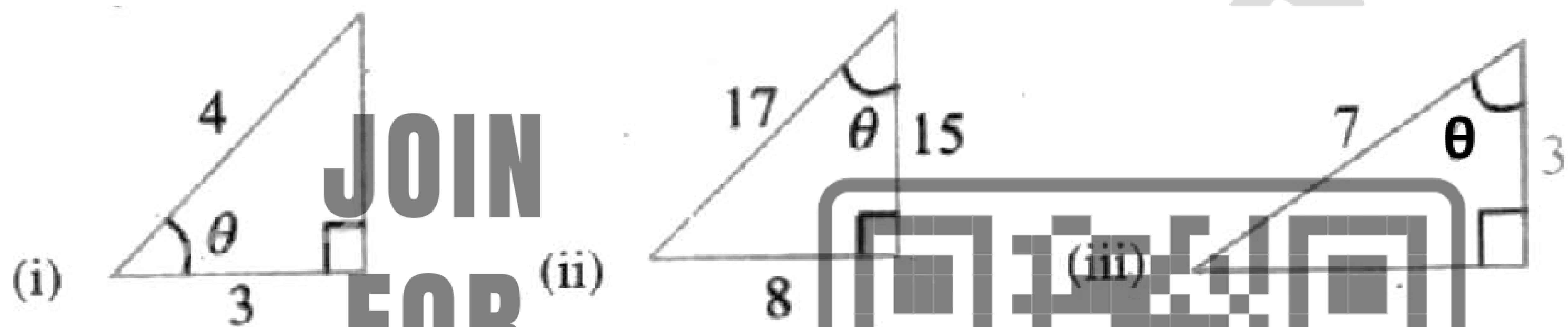
$25 = x^2$

$x = \pm 5$ so, $x = 5$

$cos\theta = \frac{x}{r} = \frac{5}{13}$; $sec\theta = \frac{13}{5}$

$tan\theta = \frac{y}{x} = \frac{12}{5}$; $cot\theta = \frac{5}{12}$

Q. 11: Find the values of trigonometric functions at the indicated angle θ in the right triangle.



(i) From figure we have

$x = 3$ and $r = 4$

Now,

$r^2 = x^2 + y^2$

$(4)^2 = (3)^2 + y^2$

$16 = 9 + y^2$

$7 = y^2$

$y = \pm\sqrt{7}$ so, $y = \sqrt{7}$

$sin\theta = \frac{y}{r} = \frac{\sqrt{7}}{4}$; $cosec\theta = \frac{4}{\sqrt{7}}$

$cos\theta = \frac{x}{r} = \frac{3}{4}$; $sec\theta = \frac{4}{3}$

$tan\theta = \frac{y}{x} = \frac{\sqrt{7}}{3}$; $cot\theta = \frac{3}{\sqrt{7}}$

(ii) From figure we have

$x = 15$, $y = 8$ and $r = 17$

Now,

$sin\theta = \frac{y}{r} = \frac{8}{17}$; $cosec\theta = \frac{17}{8}$

$cos\theta = \frac{x}{r} = \frac{15}{17}$; $sec\theta = \frac{17}{15}$

$tan\theta = \frac{y}{x} = \frac{8}{15}$; $cot\theta = \frac{15}{8}$

(iii) From figure we have

$x = 3$ and $r = 7$

Now,

$r^2 = x^2 + y^2$

$(7)^2 = (3)^2 + y^2$

$49 = 9 + y^2$

$40 = y^2$

$y = \pm 2\sqrt{10}$ so, $y = 2\sqrt{10}$

$sin\theta = \frac{y}{r} = \frac{2\sqrt{10}}{7}$; $cosec\theta = \frac{7}{2\sqrt{10}}$

$cos\theta = \frac{x}{r} = \frac{3}{7}$; $sec\theta = \frac{7}{3}$

$tan\theta = \frac{y}{x} = \frac{2\sqrt{10}}{3}$; $cot\theta = \frac{3}{2\sqrt{10}}$

Q. 12: Find the values of the trigonometric functions. Do not use trigonometric tables or calculator.

(i) $tan30^o$

We know that $2k\pi + \theta = \theta$

$tan30^o = tan\left(2(0)\pi + \frac{\pi}{6}\right) = tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

(ii) $tan330^o$

We know that $2k\pi + \theta = \theta$

$tan330^o = tan\left(2(1)\pi - \frac{\pi}{6}\right) = -tan\frac{\pi}{6} = -\frac{1}{\sqrt{3}}$

(iii) $sec330^o$

We know that $2k\pi + \theta = \theta$

$sec330^o = sec\left(2(1)\pi - \frac{\pi}{6}\right) = sec\frac{\pi}{6} = \frac{2}{\sqrt{3}}$

(iv) $cot\frac{\pi}{4}$

We know that $2k\pi + \theta = \theta$

$cot\frac{\pi}{4} = cot\left(2(0)\pi + \frac{\pi}{4}\right) = cot\frac{\pi}{4} = \frac{1}{1} = 1$

(v) $cos\frac{2\pi}{3}$

$cos\frac{3\pi-\pi}{3} = cos\left(\pi - \frac{\pi}{3}\right) = -cos\frac{\pi}{3} = -\frac{1}{2}$

(vi) $cosec\frac{2\pi}{3}$

$cosec\frac{3\pi-\pi}{3} = cosec\left(\pi - \frac{\pi}{3}\right) = cosec\frac{\pi}{3} = \frac{2}{\sqrt{3}}$

(vii) $cos(-450^o)$

We know that $2k\pi + \theta = \theta$

$cot(-360^o - 90^o) = cos\left(2(-1)\pi - \frac{\pi}{2}\right) = cos\frac{\pi}{2} = 0$

(viii) $tan(-9\pi)$

We know that $2k\pi + \theta = \theta$

$tan(-8\pi - \pi) = tan(2(-4)\pi - \pi) = -tan\pi = 0$

(ix) $cos\left(\frac{-5\pi}{6}\right)$

We know that $2k\pi + \theta = \theta$

$cos\left(-\pi + \frac{\pi}{6}\right) = cos\left(-\pi + \frac{\pi}{6}\right) = -cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

(x) $\sin\frac{7\pi}{6}$

We know that $2k\pi + \theta = \theta$

$$\sin\left(\pi+\frac{\pi}{6}\right) \quad = \sin\left(\pi+\frac{\pi}{6}\right) \quad = -\sin\frac{\pi}{6} \quad = -\frac{1}{2}$$

(xi) $\cot\frac{7\pi}{6}$

We know that $2k\pi + \theta = \theta$

$$\cot\left(\pi+\frac{\pi}{6}\right) \quad = \cot\left(\pi+\frac{\pi}{6}\right) \quad = \cot\frac{\pi}{6} \quad = \frac{1}{\sqrt{3}}$$

(x) $\cos 225^o$

We know that $2k\pi + \theta = \theta$

$$\cos\frac{5\pi}{4} \quad = \cos\left(\pi+\frac{\pi}{4}\right) \quad = -\cos\frac{\pi}{4} \quad = -\frac{1}{\sqrt{2}}$$

Exercise 7.4

Things To know:

$\cos^2\theta + \sin^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

In Problems 1-6, simplify each expressions to a single trigonometric function.

Q. 1: $\frac{\sin^2 x}{\cos^2 x}$

$$\frac{\sin^2 x}{\cos^2 x} = \left(\frac{\sin x}{\cos x}\right)^2 = (\tan x)^2 = \tan^2 x$$

Q. 2: $\tan x \sin x \sec x$

$$\tan x \sin x \sec x = \tan x \sin x \left(\frac{1}{\cos x}\right) = \tan x \tan x = (\tan x)^2 = \tan^2 x$$

Q. 3: $\frac{\tan x}{\sec x}$

$$\frac{\tan x}{\sec x} = \frac{1}{\sec x}.\tan x = \cos x\left(\frac{\sin x}{\cos x}\right) = \sin x$$

Q. 4: $1 - \cos^2 x$

$$1 - \cos^2 x = \cos^2 x + \sin^2 x - \cos^2 x = \sin^2 x$$

Q. 5: $\sec^2 x - 1$

$$\sec^2 x - 1 = \sec^2 x - (\sec^2 x - \tan^2 x) = \sec^2 x - \sec^2 x + \tan^2 x = \tan^2 x$$

Q. 6: $\sin^2 x.\cot^2 x$

$$\sin^2 x.\cot^2 x = \sin^2 x.\frac{\cos^2 x}{\sin^2 x} = \cos^2 x$$

In problems 7-24, verify the identities.

Q. 7: $(1 - \sin\theta)(1 + \sin\theta) = \cos^2 x$

$$L.H.S = (1 - \sin\theta)(1 + \sin\theta) = 1 - \sin^2 x = \cos^2 x = R.H.S$$

Q. 8: $\frac{\boldsymbol{sin\theta+cos\theta}}{\boldsymbol{cos\theta}} = \boldsymbol{1+tan\theta}$

$L.H.S \quad = \frac{sin\theta+cos\theta}{cos\theta}$

$= \frac{sin\theta}{cos\theta} + \frac{cos\theta}{cos\theta}$

$= tan\theta + 1$

$= R.H.S$

Q. 9: $\boldsymbol{(tan\theta + cot\theta)tan\theta = sec^2\theta}$

$L.H.S \quad = \left(\frac{sin\theta}{cos\theta} + \frac{cos\theta}{sin\theta}\right)\frac{sin\theta}{cos\theta}$

$= \left(\frac{sin^2\theta+cos^2\theta}{cos\theta sin\theta}\right)\frac{sin\theta}{cos\theta}$

$= \left(\frac{1}{cos\theta}\right)\frac{1}{cos\theta}$

$= (sec\theta)sec\theta$

$= sec^2\theta$

$= R.H.S$

Q. 10: $\boldsymbol{(cot\theta + cosec\theta)(tan\theta - sin\theta) = sec\theta - cos\theta}$

$L.H.S \quad = \left(\frac{cos\theta}{sin\theta} + \frac{1}{sin\theta}\right)\left(\frac{sin\theta}{cos\theta} - sin\theta\right)$

$= \left(\frac{cos\theta+1}{sin\theta}\right)\left(\frac{sin\theta - sin\theta cos\theta}{cos\theta}\right)$

$= \left(\frac{1+cos\theta}{sin\theta}\right)\left(\frac{sin\theta(1-cos\theta)}{cos\theta}\right)$

$= (1+cos\theta)\left(\frac{1-cos\theta}{cos\theta}\right)$

$= \frac{1-cos^2\theta}{cos\theta}$

$= \frac{1}{cos\theta} - \frac{cos^2\theta}{cos\theta}$

$= sec\theta - cos\theta$

$= R.H.S$

Q. 11: $\frac{\boldsymbol{sin\theta+cos\theta}}{\boldsymbol{tan^2\theta-1}} = \frac{\boldsymbol{cos^2\theta}}{\boldsymbol{sin\theta-cos\theta}}$

$L.H.S \quad = \frac{sin\theta+cos\theta}{tan^2\theta-1}$

$= \frac{sin\theta+cos\theta}{\frac{sin^2\theta}{cos^2\theta}-1}$

$= \frac{sin\theta+cos\theta}{\frac{sin^2\theta-cos^2\theta}{cos^2\theta}}$

$= (cos^2\theta)\left(\frac{sin\theta+cos\theta}{sin^2\theta-cos^2\theta}\right)$

$= (cos^2\theta)\left(\frac{sin\theta+cos\theta}{(sin\theta-cos\theta)(sin\theta+cos\theta)}\right)$

$= \frac{cos^2\theta}{sin\theta-cos\theta}$

$= R.H.S$

Q. 12: $\frac{\boldsymbol{cos^2\theta}}{\boldsymbol{sin\theta}} + \boldsymbol{sin\theta = cosec\theta}$

$L.H.S \quad = \frac{cos^2\theta}{sin\theta} + sin\theta$

$= \frac{cos^2\theta + sin^2\theta}{sin\theta}$

$= \frac{1}{sin\theta}$

$= cosec\theta$

$= R.H.S$

Q. 13: $\boldsymbol{sec\theta - cos\theta = tan\theta sin\theta}$

$L.H.S \quad = sec\theta - cos\theta$

$= \frac{1}{cos\theta} - cos\theta$

$= \frac{1 - cos^2\theta}{cos\theta}$

$= \frac{sin^2\theta}{cos\theta}$

$= \frac{sin\theta}{cos\theta}.sin\theta$

$= tan\theta.sin\theta$

$= R.H.S$

Q. 14: $\boldsymbol{\frac{sin^2\theta}{cos\theta} + cos\theta = sec\theta}$

$L.H.S \quad = \frac{sin^2\theta}{cos\theta} + cos\theta$

$= \frac{sin^2\theta + cos^2\theta}{cos\theta}$

$= \frac{1}{cos\theta}$

$= sec\theta$

$= R.H.S$

Q. 15: $\boldsymbol{tan\theta + cot\theta = sec\theta cosec\theta}$

$L.H.S \quad = tan\theta + cot\theta$

$= \frac{sin\theta}{cos\theta} + \frac{cos\theta}{sin\theta}$

$= \frac{sin^2\theta + cos^2\theta}{cos\theta sin\theta}$

$= \frac{1}{cos\theta sin\theta}$

$= sec\theta.cosec\theta$

$= R.H.S$

Q. 16: $\boldsymbol{(tan\theta + cot\theta)(cos\theta + sin\theta) = sec\theta + cosec\theta}$

$L.H.S \quad = (tan\theta + cot\theta)(cos\theta + sin\theta)$

$= \left(\frac{sin\theta}{cos\theta} + \frac{cos\theta}{sin\theta}\right)(cos\theta + sin\theta)$

$= \left(\frac{sin^2\theta + cos^2\theta}{cos\theta sin\theta}\right)(cos\theta + sin\theta)$

$= \left(\frac{1}{cos\theta sin\theta}\right)(cos\theta + sin\theta)$

$= \frac{cos\theta}{cos\theta sin\theta} + \frac{sin\theta}{cos\theta sin\theta}$

$= \frac{1}{sin\theta} + \frac{1}{cos\theta}$

$= cosec\theta + sec\theta$

$= R.H.S$

Q. 17: $\boldsymbol{\sin\theta(\tan\theta + \cot\theta) = \sec\theta}$

$$L.H.S = (\tan\theta + \cot\theta)\sin\theta$$
$$= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)\sin\theta$$
$$= \left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)\sin\theta$$
$$= \left(\frac{1}{\cos\theta\sin\theta}\right)\sin\theta$$
$$= \frac{1}{\cos\theta}$$
$$= \sec\theta$$
$$= R.H.S$$

Q. 18: $\boldsymbol{\frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta} = 2cosec\theta}$

$$L.H.S = \frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta}$$
$$= \frac{(1+\cos\theta)^2 + \sin^2\theta}{\sin\theta(1+\cos\theta)}$$
$$= \frac{1+\cos^2\theta + 2\cos\theta + \sin^2\theta}{\sin\theta(1+\cos\theta)}$$
$$= \frac{1+2\cos\theta + \cos^2\theta + \sin^2\theta}{\sin\theta(1+\cos\theta)}$$
$$= \frac{1+2\cos\theta + 1}{\sin\theta(1+\cos\theta)}$$
$$= \frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)}$$
$$= \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$$
$$= \frac{2}{\sin\theta}$$
$$= 2cosec\theta$$
$$= R.H.S$$

Q. 19: $\boldsymbol{\frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta} = 2cosec^2\theta}$

$$L.H.S = \frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta}$$
$$= \frac{1+\cos\theta + 1 - \cos\theta}{(1-\cos\theta)(1+\cos\theta)}$$
$$= \frac{2}{1-\cos^2\theta}$$
$$= \frac{2}{\sin^2\theta}$$
$$= 2cosec^2\theta$$
$$= R.H.S$$

Q. 20: $\boldsymbol{\frac{1+\sin\theta}{1-\sin\theta} - \frac{1-\sin\theta}{1+\sin\theta} = 4\tan\theta\sec\theta}$

$$L.H.S = \frac{1+\sin\theta}{1-\sin\theta} - \frac{1-\sin\theta}{1+\sin\theta}$$
$$= \frac{(1+\sin\theta)^2 - (1-\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}$$
$$= \frac{1+\sin^2\theta + 2\sin\theta - 1 - \sin^2\theta + 2\sin\theta}{1-\sin^2\theta}$$
$$= \frac{4\sin\theta}{1-\sin^2\theta}$$

$= \frac{4sin\theta}{cos^2\theta}$

$= \frac{4sin\theta}{cos\theta(cos\theta)}$

$= 4tan\theta sec\theta$

$= R.H.S$

Q. 21: $\boldsymbol{sin^3\theta = sin\theta - sin\theta cos^2\theta}$

$R.H.S$ $= sin\theta - sin\theta cos^2\theta$

$= sin\theta(1 - cos^2\theta)$

$= sin\theta(sin^2\theta)$

$= sin^3\theta$

$= L.H.S$

Q. 22: $\boldsymbol{cos^4\theta - sin^4\theta = cos^2\theta - sin^2\theta}$

$L.H.S$ $= cos^4\theta - sin^4\theta$

$= (cos^2\theta)^2 - (sin^2\theta)^2$

$= (cos^2\theta - sin^2\theta)(cos^2\theta + sin^2\theta)$

$= (cos^2\theta - sin^2\theta)(1)$

$= R.H.S$

Q. 23: $\boldsymbol{\sqrt{\frac{1+cos\theta}{1-cos\theta}} = \frac{sin\theta}{1-cos\theta}}$

$L.H.S$ $= \sqrt{\frac{1+cos\theta}{1-cos\theta}}$

$= \sqrt{\frac{(1+cos\theta)(1-cos\theta)}{(1-cos\theta)(1-cos\theta)}}$

$= \sqrt{\frac{(1-cos^2\theta)}{(1-cos\theta)^2}}$

$= \frac{\sqrt{sin^2\theta}}{\sqrt{(1-cos\theta)^2}}$

$= \frac{sin\theta}{1-cos\theta}$

$= R.H.S$

Q. 23: $\boldsymbol{\sqrt{\frac{sec\theta+1}{sec\theta-1}} = \frac{sec\theta+1}{tan\theta}}$

$L.H.S$ $= \sqrt{\frac{sec\theta+1}{sec\theta-1}}$

$= \sqrt{\frac{(sec\theta+1)(sec\theta+1)}{(sec\theta-1)(sec\theta-1)}}$

$= \sqrt{\frac{(sec\theta+1)^2}{(sec^2\theta-1)}}$

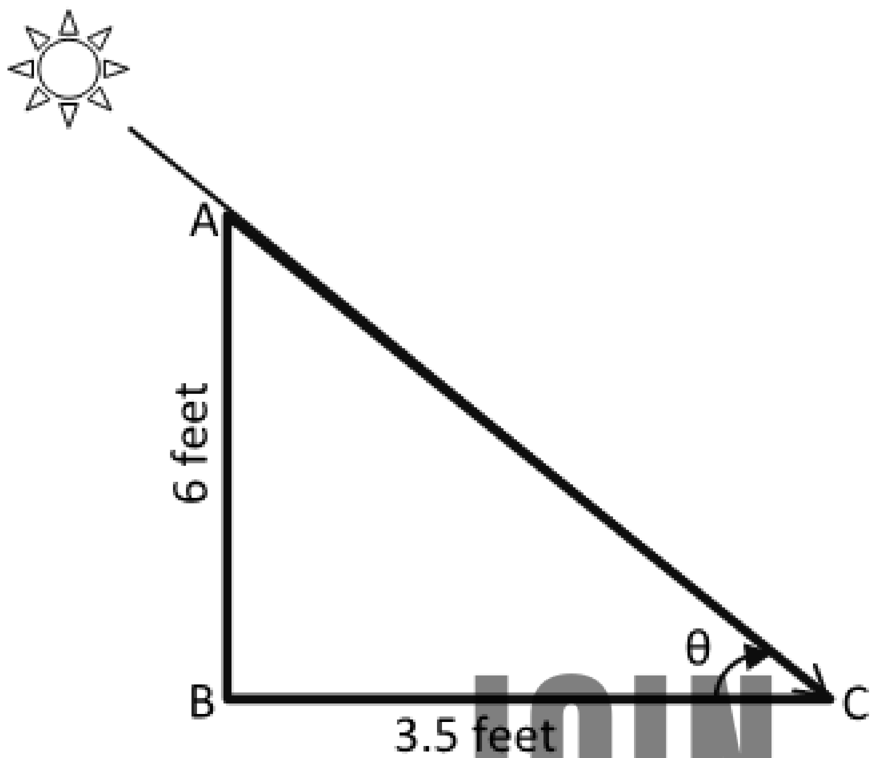
$= \frac{\sqrt{(sec\theta+1)^2}}{\sqrt{tan^2\theta}}$

$= \frac{sec\theta+1}{tan\theta}$

$= R.H.S$

Exercise 7.5

Q. 1: Find the angle of elevation of the sun if a 6 feet man casts a 3.5 feet shadow.



From figure we have

$tan\theta = \frac{AB}{BC}$

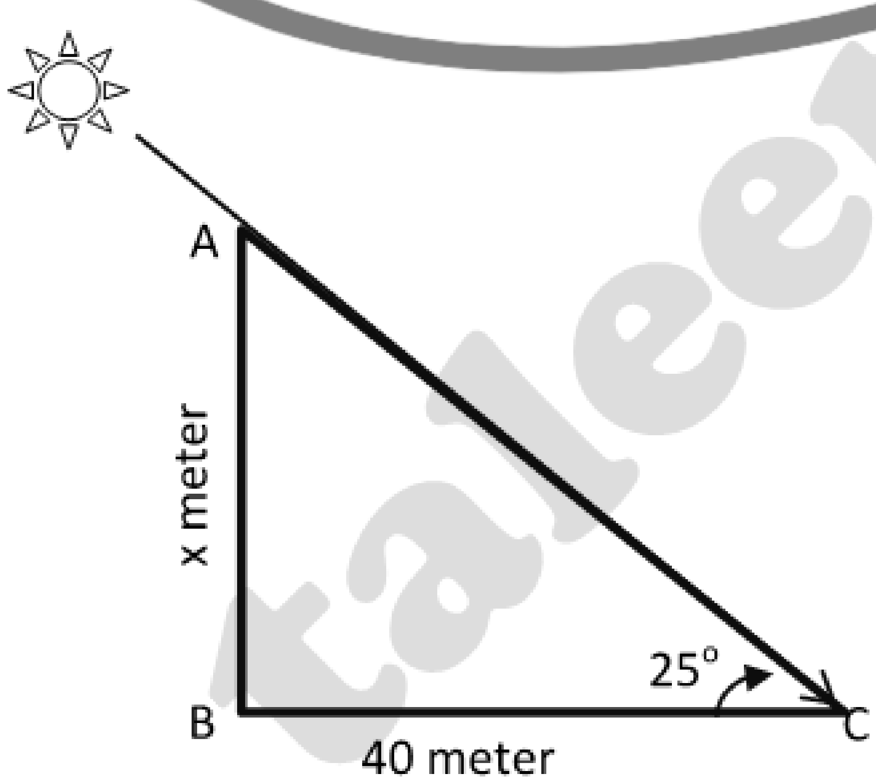
$tan\theta = \frac{6}{3.5}$

$tan\theta = 1.714$

$\theta = tan^{-1}(1.714)$

$\theta = 59.74°$

Q. 2: A tree casts a 40 meter shadow when the angel of elevation of the sun is 25°. Find the height of the tree.



From figure we have

$tan\theta = \frac{AB}{BC}$

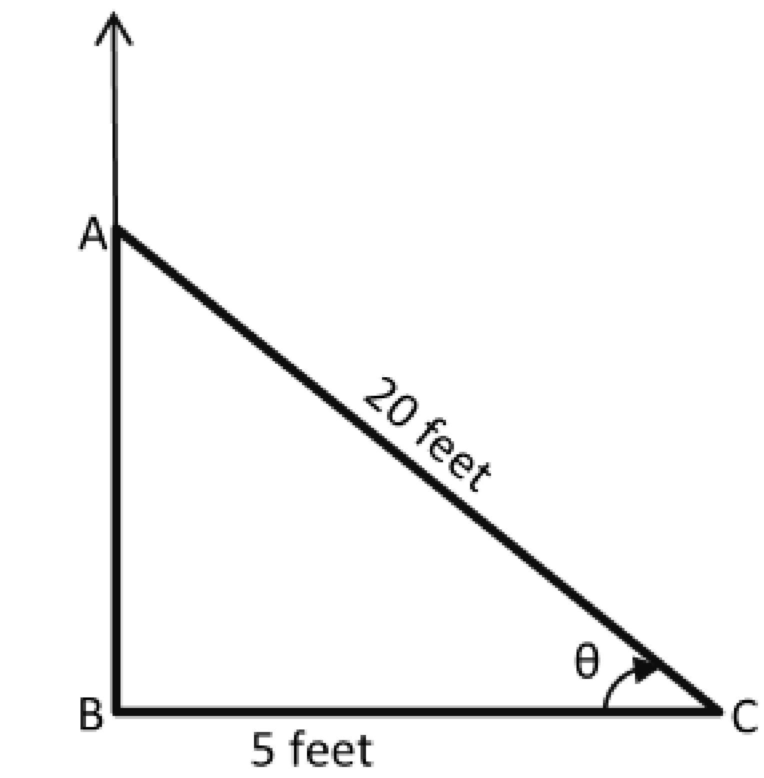
$tan25 = \frac{x}{40}$

$x = 40(tan25)$

$x = 40(0.4663)$

$x = 18.652\ meter$

Q. 3: **A 20 feet long ladder is leaning against a wall. The bottom of the ladder is 5 feet from the base of the wall. Find the acute angle (angle of elevation) the ladder makes with the ground.**



From figure we have

$cos\theta \quad = \frac{BC}{AC}$

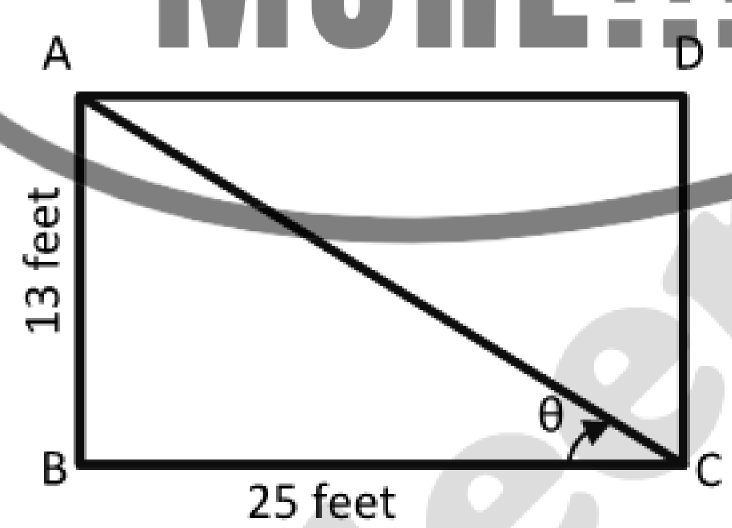
$cos\theta \quad = \frac{5}{20}$

$cos\theta \quad = 0.25$

$\theta \quad = cos^{-1}(0.25)$

$\theta \quad = 75.52°$

Q. 4: **The base of a rectangle is 25 feet and the height of the rectangle is 13 feet. Find the angle that the diagonal of the rectangle makes with the base.**



From figure we have

$tan\theta \quad = \frac{AB}{BC}$

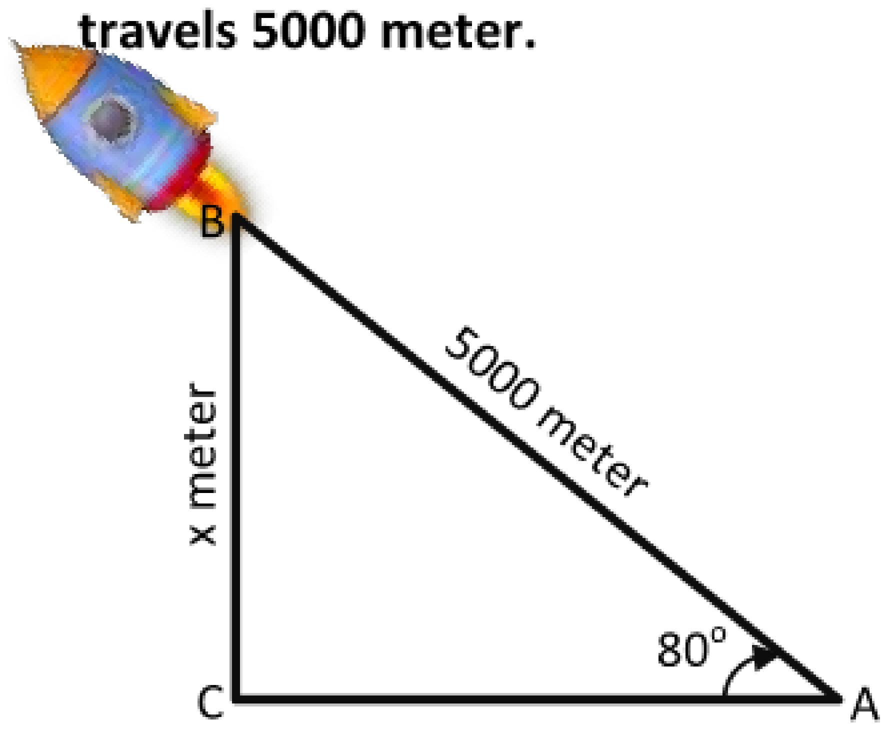
$tan\theta \quad = \frac{13}{25}$

$tan\theta \quad = 0.52$

$\theta \quad = tan^{-1}(0.52)$

$\theta \quad = 27.47°$

Q. 5: **A rocket is launched and climbs at a constant angle of 80°. Find the altitude of the rocket after it travels 5000 meter.**



From figure we have

$sin\theta = \frac{BC}{AB}$

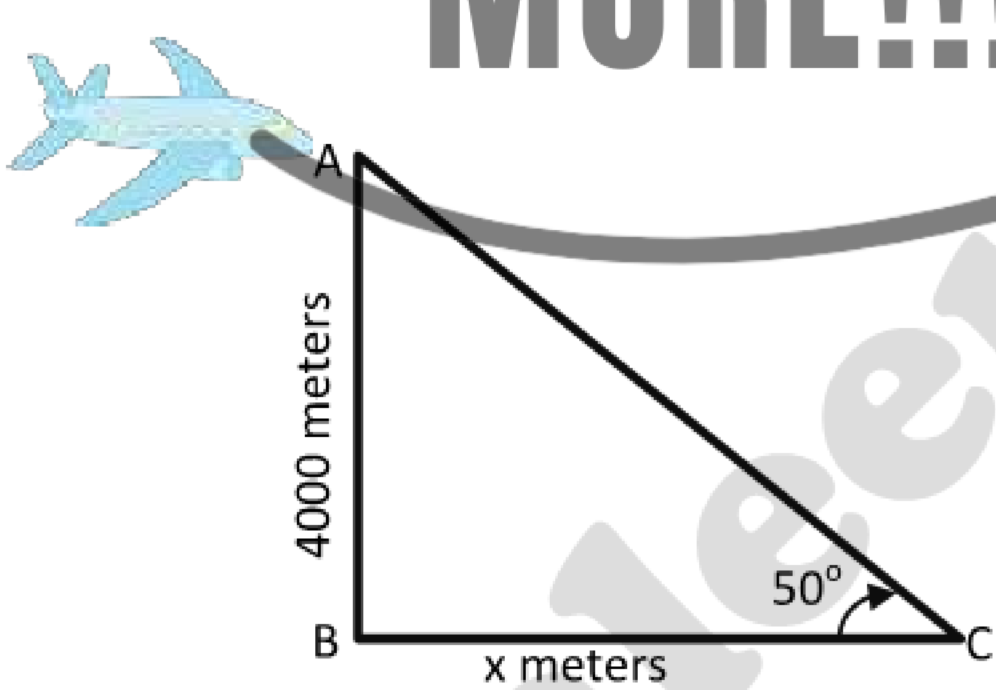
$sin80 = \frac{x}{5000}$

$x = 5000(sin80)$

$x = 5000(0.9848)$

$x = 4924.04\ meter$

Q. 6: **An aeroplane pilot flying at an attitude of 4000m wishes to make an approach to an airport at an angle of 50° with the horizontal. How far from the airport will the plane be when the pilot begins to descent?**



From figure we have

$tan\theta = \frac{AB}{BC}$

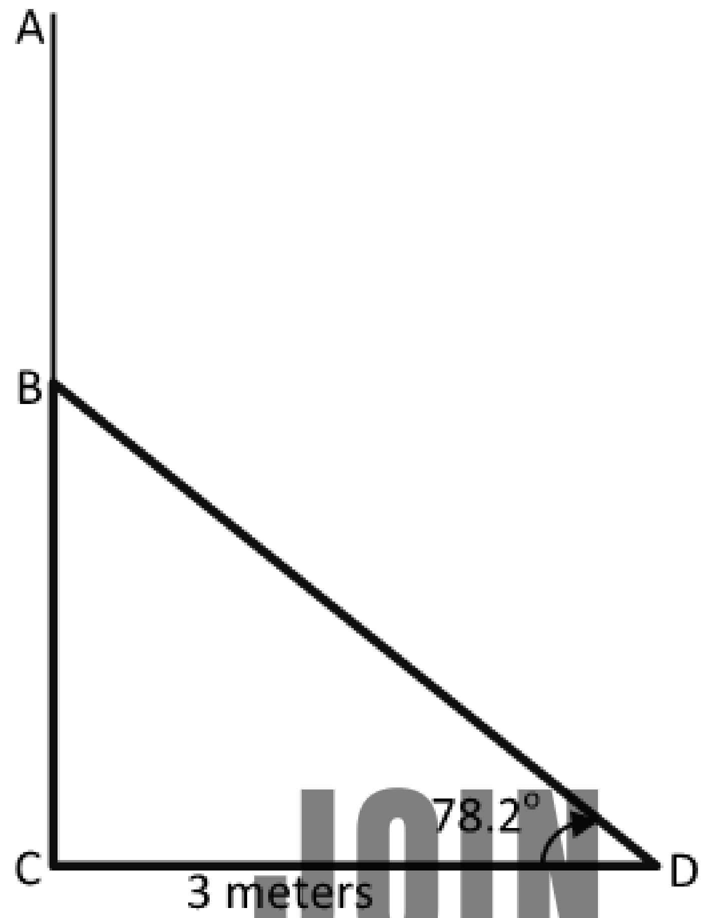
$tan50 = \frac{4000}{x}$

$x = \frac{4000}{tan50}$

$x = \frac{4000}{1.1918}$

$x = 3356.3\ meters$

Q. 6: A guy wire (supporting wire) runs from the middle of a utility pole to the ground. The wire makes an angle of 78.2° with the ground and touch the ground 3 meters from the base of the pole. Find the height of the pole.



From figure we have

$tan\theta = \frac{BC}{CD}$

$tan78.2 = \frac{BC}{3}$

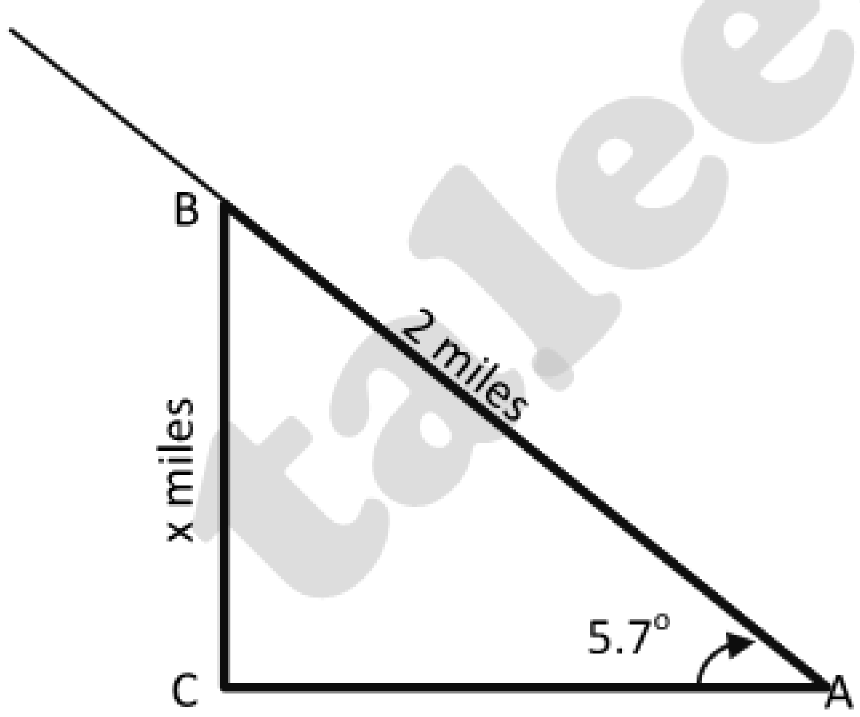
$BC = 3(tan78.2)$

$BC = 3(4.7867)$

$BC = 14.36$

$Height\ of\ pole = 2BC = 2(14.36) = 28.72$ meters

Q. 8: A road is inclined at an angle 5.7°. Suppose that we drive 2 miles up this road starting from sea level. How high above sea level are we?



From figure we have

$sin\theta = \frac{BC}{AB}$

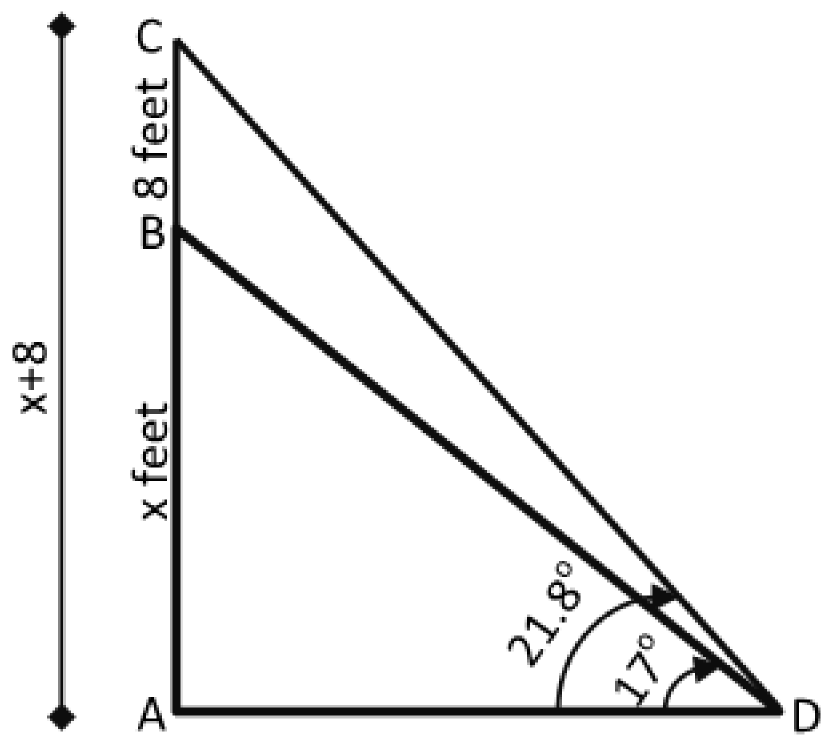
$sin5.7 = \frac{x}{2}$

$x = 2(sin5.7)$

$x = 2(0.0993)$

$x = 0.199\ miles$

Q. 9: A television antenna of 8 feet height is located on the top of a house. From a point on the ground the angle of elevation to the top of the house is 17° and the angle of elevation to the top of the antenna is 21.8°. find the height of the house.



From figure we have

In ΔABD

$tan\theta = \frac{AB}{AD}$

$tan17 = \frac{x}{AD}$

$AD = \frac{x}{tan17}$ ----------- (i)

In ΔACD

$tan\theta = \frac{AC}{AD}$

$tan21.8 = \frac{x+8}{AD}$

$AD = \frac{x+8}{tan21.8}$ ----------- (ii)

Comparing (i) and (ii)

$\frac{x}{tan17} = \frac{x+8}{tan21.8}$

$\frac{x}{0.3057} = \frac{x+8}{0.4}$

$0.4x = 0.3057(x+8)$

$0.4x = 0.3057x + 2.4456$

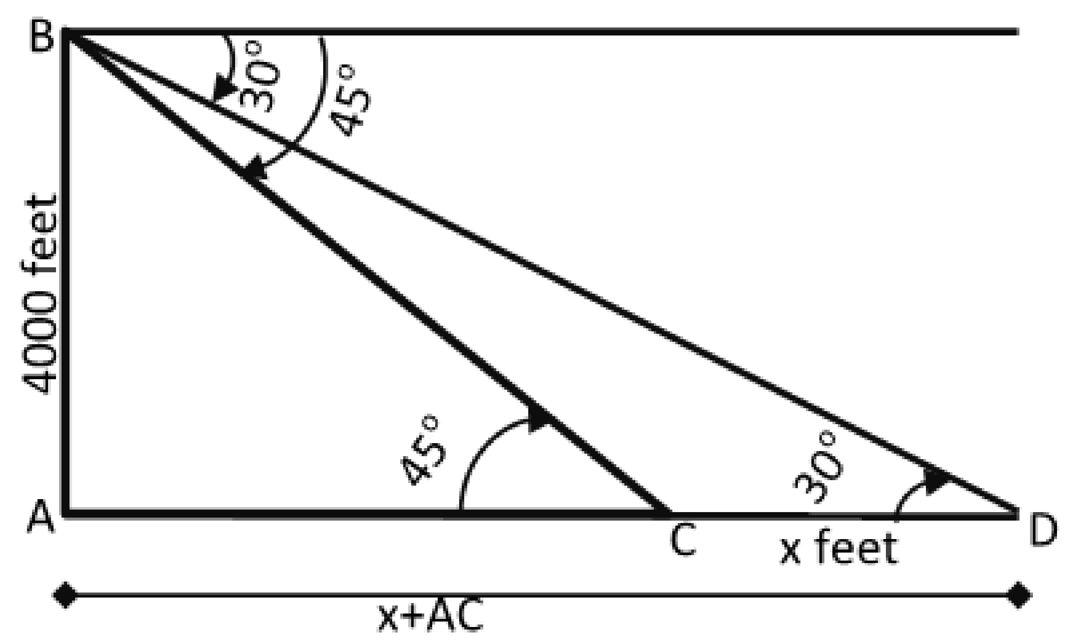
$0.4x - 0.3057x = 2.4456$

$0.0943x = 2.4456$

$x = \frac{2.4456}{0.0943}$

$x = 25.93 feet$

Q. 10: From an observation point, the angles of depression of two boats in line with this point are found to 30^o and 45^o. Find the distance between the two boats if the point of observation is 4000 feet high.



From figure we have

In ΔABC

$tan\theta = \frac{AB}{AC}$

$tan45 = \frac{4000}{AC}$

$AC = \frac{4000}{tan45}$

$AC = 4000$

In ΔABD

$tan\theta = \frac{AB}{AD}$

$tan30 = \frac{4000}{x+AC}$

$0.5774 = \frac{4000}{x+4000}$

$0.5774(x + 4000) = 4000$

$0.5774x + 2309.6 = 4000$

$0.5774x = 4000 - 2309.6$

$0.5774x = 1690.4$

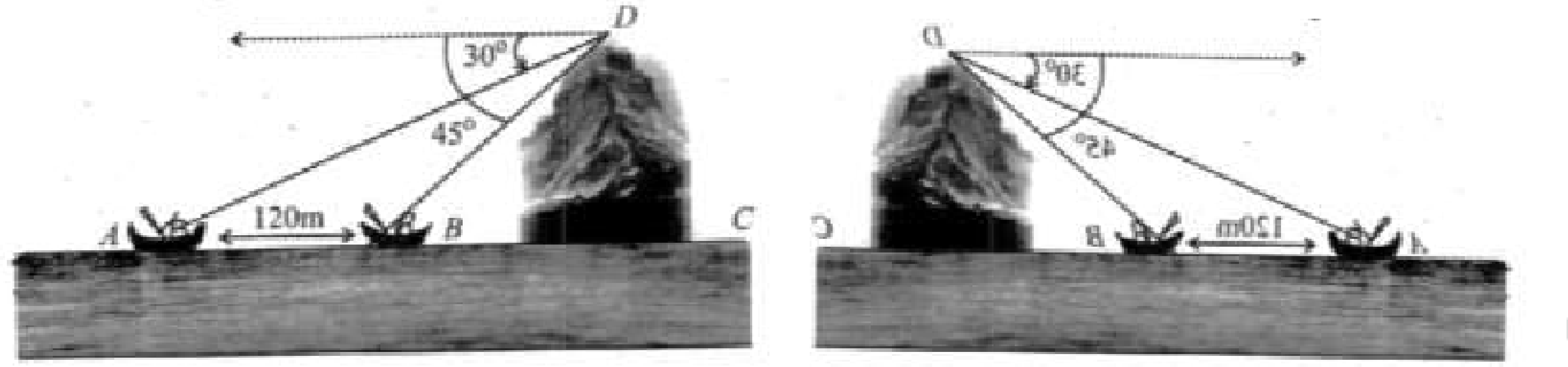
$x = \frac{1690.4}{0.5774}$

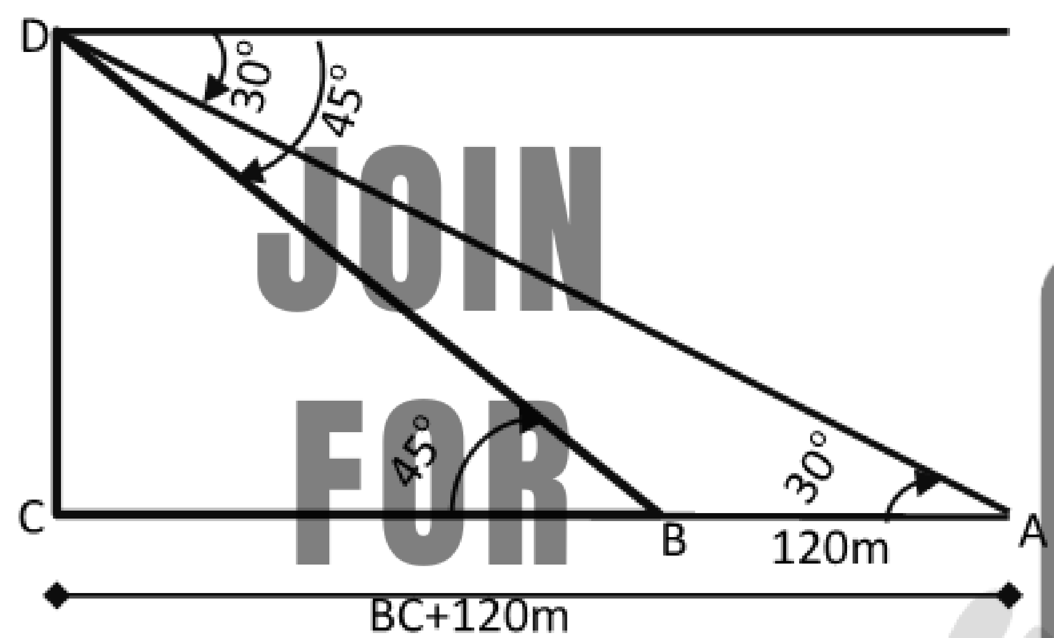
$x = 2927.61 feet$

Q. 11: **Two ships, which are in line with the base of a vertical cliff, are 120 meters apart. The angles of depression from the top of the cliff to the ships are 30° and 45°, as shown in the diagram.**

(a) **Calculate the distance BC**

(b) **Calculate the height CD, of the cliff.**





From figure we have

In ΔBCD

$tan\theta = \frac{CD}{BC}$

$tan45 = \frac{CD}{BC}$

$1 = \frac{CD}{BC}$

$BC = CD$

In ΔACD

$tan\theta = \frac{CD}{AC}$

$tan30 = \frac{CD}{BC+120}$

As, $BC = CD$ So,

$tan30 = \frac{BC}{BC+120}$

$0.5774 = \frac{BC}{BC+120}$

$0.5774(BC + 120) = BC$

$0.5774BC + 69.288 = BC$

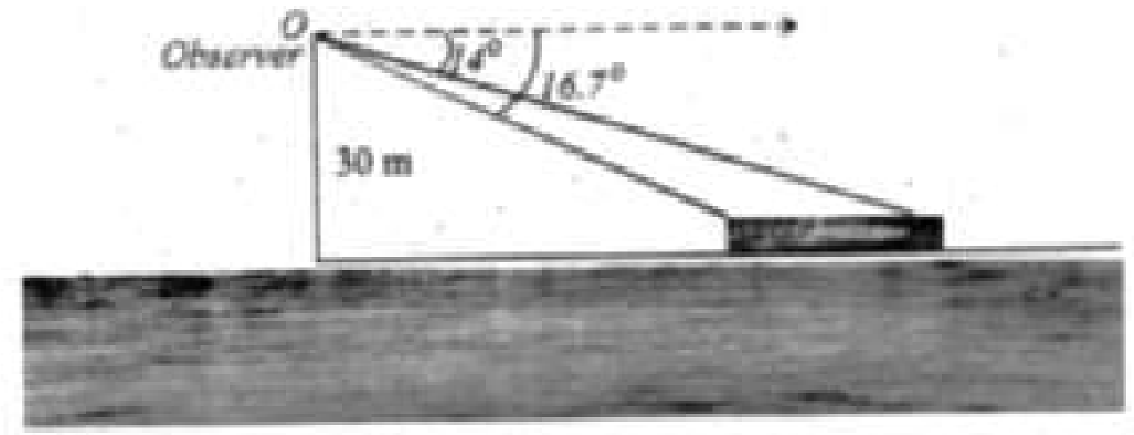
$0.5774BC - BC = -69.288$

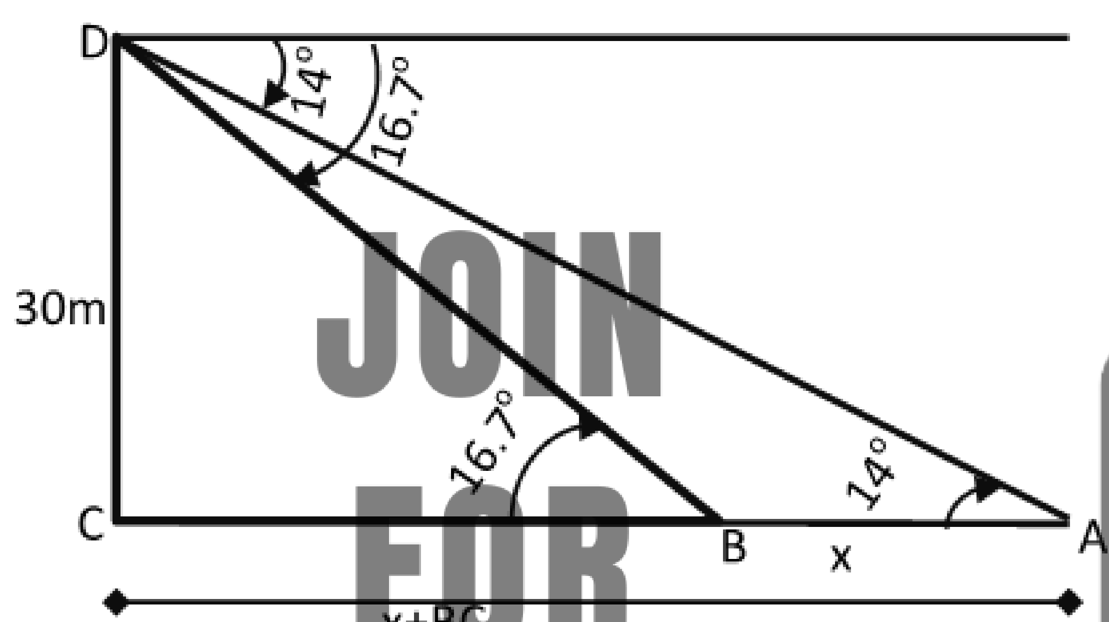
$-0.4226BC = -69.288$

$BC = \frac{-69.288}{-0.4226}$

$BC = 169.93 meters$ Also CD = 169.93 meters.

Q. 12: Suppose that we are standing on a bridge 30 feet above a river watching a log (piece of wood) floating toward us. If the angle with the horizontal to the front of the log is 16.7° and angle with the horizontal to the back of the log is 14°. How long is the log?





From figure we have

In ΔBCD

$tan\theta \quad = \frac{CD}{BC}$

$tan16.7 = \frac{30}{BC}$

$BC \quad = \frac{30}{tan16.7}$

$BC \quad = \frac{30}{0.3}$

$BC \quad = 100m$

In ΔACD

$tan\theta \quad = \frac{CD}{AC}$

$tan14 = \frac{30}{x+BC}$

As, BC = 100 So,

$tan14 = \frac{30}{x+100}$

$0.2493 = \frac{30}{x+100}$

$0.2493(x + 100) = 30$

$0.2493x + 24.93 = 30$

$0.2493x = 30 - 24.93$

$0.2493x = 5.07$

$x \quad = \frac{5.07}{0.2493}$

$x \quad = 20.33\ meter$

2014-2018 کے تمام بورڈز کے پرچہ جات میں سے باب نمبر 8 کے مختصر سوالات اور جوابات

1. Define the collinear and non-collinear points. (3 times) ہم خط نقاط اور غیر ہم خط نقاط سے کیا مراد ہے؟

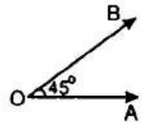
Ans. Two or more points lying on the same straight line are called collinear points otherwise they are called non collinear .

دو یا دو سے زیادہ نقاط پر جو ایک خط پر واقع ہوں۔ ہم خط نقاط کہلاتے ہیں۔ ورنہ غیر ہم خط نقاط کہلاتے ہیں۔

2. Define Acute Angle and represent it by figure. (5 time) (2018) حادہ زاویے کی تعریف کریں اور شکل سے ظاہر کریں۔

Ans. An angle that is less than 90° is called acute angle.

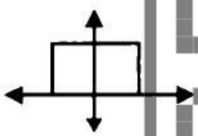
ایسا زاویہ جو کہ 90° سے کم ہو۔ منفرد زاویہ کہلاتے ہیں۔



3. Define supplementary angle and form its figure. (1 time) (2016) سپلیمنٹری زاویے کی تعریف کریں اور شکل بھی بنائیے۔

Ans. The angles whose sum are 180° are called supplementary angles.

ایسے زاویے جن کا مجموعہ 180° ہو سپلیمنٹری زاویے کہلاتے ہیں



4. What is perpendicular? عمود کیا ہے؟

Ans. A line which makes the right angle is called perpendicular.

ایک قطعہ خط جو 90° کا زاویہ بنائے عمود کہلاتا ہے۔

5. what is meant by incentre of the triangle? (3 times) مثلث کے اندرونی مرکز سے کیا مراد ہے؟

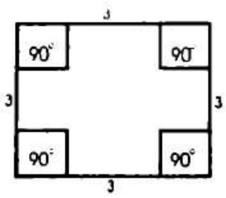
Ans. The point of concurrency of three angles bisectors of triangle is called incentre.

مثلث کے اندر ایسا نقطہ جہاں تینوں زاویوں کے ناصف ملتے ہیں اندرونی مرکز کہلاتا ہے۔

6. Define square and represent it by figure. مربع کی تعریف کریں اور شکل سے اس کو ظاہر کریں۔

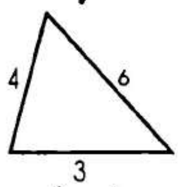
Ans. A quadrilateral having four equal sides and each angle is 90° is called square.

ایسی چوکور جس کے چاروں اضلاع برابر اور ہر زاویہ 90° کا ہو مربع کہلاتی ہے۔



7. Construct a triangle ABC when the lengths of sides are 6 cm , 3 cm and 4 cm respectively.

□ABC بنائیے جب کہ اس کے اضلاع کی لمبائیاں بالترتیب 6 سم، 3 سم اور 4 سم ہوں۔

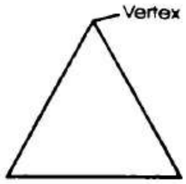


Chord length (وتر کی لمبائی) = $\overline{AB} = 6cm$

8. Define the vertices and represent it by figure. (2 times)(2018) راس کی تعریف کیجئے اور اس کو شکل سے ظاہر کیجئے۔

Ans. A point where two or more straight lines meet. e.g A triangle has three vertices.

ایسا نقطہ جہاں دو یا دو سے زیادہ لائینیں ملتیں ہیں راس کہلاتا ہے مثلاً مثلث میں تین راس ہوتے ہیں۔



9. Define isosceles triangle. **مساوی الساقین مثلث کی تعریف کیجئے۔**

Ans. A triangle having two equal sides is called isosceles triangle.

ایسی مثلث جس کے دو اضلاع برابر ہوں مساوی الساقین مثلث کہلاتی ہے۔

2016

10. Define the right angle. **قائمہ زاویہ کی تعریف کریں۔** **(4 times)(2018)**

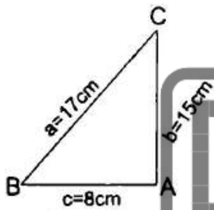
Ans: An angle which is equal to 90 ° is called right angle.

ایسا زاویہ جس کی پیمائش 90° ہو اسے قائمۃ الزاویہ کہتے ہیں۔

11. In a ΔABC if a = 17cm, b = 15cm , c = 8cm Find $m\angle A$ **(2 times)(2018)**

اگر مثلث ABC میں a = 17cm, b = 15cm , c = 8cm ہو تو $m\angle A$ معلوم کریں۔

Ans: a = 17cm, b = 15cm , c = 8cm



Using pythagoras theorem. مسئلہ فیثا غورث کی رو سے

$(BC)^2 = (AB)^2 + (AC)^2 \Rightarrow a^2 = c^2 + b^2$

Putting values of a,b and c

$(17)^2 = (8)^2 + (15)^2 \Rightarrow 289 = 64 + 225 \Rightarrow 289 = 289$

So it is a right angled triangle and $m\angle A = 90^o$

12. Whether the triangle with sides 3cm, 4cm and 5cm is acute, obtuse or right angled?

مثلث کے اضلاع 3 سم، 4 سم اور 5 سم ہیں کیا یہ حادۃ الزاویہ۔ منفرجۃ الزاویہ یا قائمۃ الزاویہ مثلث ہے۔

Ans: Let a = 3cm , b = 4cm , c = 5cm

Then
$a^2 = (3)^2 = 9$

$b^2 = (4)^2 = 16$

$c^2 = (5)^2 = 25$

according to given condition دی گئی صورت کے مطابق

$(5)^2 = (3)^2 + (4)^2$

$25 = 9 + 16$

$(c)^2 = a^2 + b^2$

so according to given measurement it is right angle triangle.

پس دی گئی پیمائشوں کے مطابق یہ ایک قائمۃ الزاویہ مثلث ہے۔

13. Define Projection. **(2 times)(2018)** **ظل کی تعریف کیجئے۔**

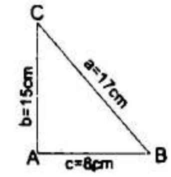
Ans: The projection of a given point on a line segment is the foot the perpendicular ⊥ drawn from the point on that line segment.

کسی نقطہ سے ایک دیئے ہوئے قطعہ خط پر عمود کھینچا جائے تو پایہ عمود کو نقطے کا ظل یا سایہ کہتے ہیں۔

14. In a ΔABC a = 17cm, b =15cm , and c = 8cm then find $m\angle B$

اگر ΔABC میں a = 17cm, b =15cm , اور c = 8cm ہو تو $m\angle B$ معلوم کریں۔

Ans:



$$\cos B = \frac{base}{hypotenuse} = \qquad \cos B = \frac{m\overline{AB}}{m\overline{BC}} \Rightarrow \qquad m\angle B = Cos^{-1}\left(\frac{8}{17}\right)$$

$$= 61.9^0$$

15. Whether the triangle with sides 5cm, 7cm, 8cm is acute, obtuse or right angled triangle.

ایک مثلث کے اضلاع 5 سم، 7 سم اور 8 سم ہیں۔ کیا وہ حادۃ الزاویہ، منفرجۃ الزاویہ یا قائمۃ الزاویہ مثلث ہے؟

Ans: Let a = 5cm , b = 7cm , c = 8cm

Then $a^2 = (5)^2 = 25$

$b^2 = (7)^2 = 49 \Rightarrow \qquad c^2 = (8)^2 = 64$

According ot given condition — دی گئی صورت کے مطابق مسئلہ فیثا غورث لگانے سے

$(8)^2 = (5)^2 + (7)^2 \quad \Rightarrow \quad 64 = 25 + 49 \Rightarrow \quad 64 = 74$

But that is not true. So it is not right angle triangle. So it is an acute triangle.

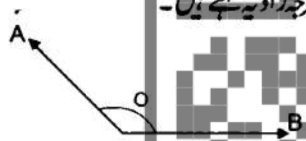
لیکن مسئلہ فیثا غورث کے مطابق درست نہیں ہے۔ اس لیے یہ ایک حادۃ الزاویہ مثلث ہے۔

16. Define obtuse angle. (6 times)(2018)

منفرجہ زاویہ کی تعریف کریں۔

Ans: An angle that is greater than 90 ^{o} and less than 180 ^{o} is known as obtuse angle.

وہ زاویہ جو 90^o سے زیادہ اور 180^o سے کم ہو اسے منفرجہ زاویہ کہتے ہیں۔



17. Define triangle.

مثلث کی تعریف کریں۔

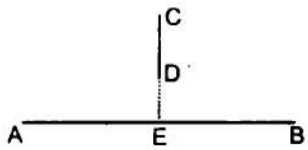
Ans: A close figure having three sides and three angle is known as triangle.

ایک بند شکل جس کے تین زاویے اور تین ضلعے ہوں مثلث کہلاتی ہیں

……2017……

18. Define zero dimension.

Ans: Projection of a vertical line segment $\overline{CD}$ on a line segment $\overline{AB}$ is a point on $\overline{AB}$ which is of zero dimension.



……2018……

19. Define zero dimension.

صفری سمت کی تعریف کریں۔

Ans. Zero dimension: The projection of line segment $\overline{CD}$ on a line segment $\overline{AB}$ is the portion $\overline{EF}$ of latter intercepted between foots of the perpendicular drawn from C & D.However projection of a vertical line segment $\overline{CD}$ on a line segment $\overline{AB}$ is a point on $\overline{AB}$ which is zero dimension.

ii. Sides of triangle are 8cm, 15cm & 17cm wherever it is acute angle, abtuse angle or rightangle triangle.

مثلث کے اضلاع 8 سم، 15 سم اور 17 سم ہیں کیا یہ ایک حادہ زاویہ منفرجہ زاویہ یا قائمۃ الزاویہ مثلث کے اضلاع ہیں۔

Ans. By phythagoras theorem — مسئلہ فیثا غورث کی رو سے

$c^2 = a^2 + b^2 \quad \Rightarrow \quad (17)^2 = (15)^2 + (8)^2 \quad \Rightarrow \quad 289 = 225 + 64 \Rightarrow \quad 289 = 289$

It is right triangle — یہ ایک قائمۃ الزاویہ مثلث ہے۔

2014-2018 کے تمام بورڈز کے پرچہ جات میں سے
باب نمبر 9 کے معروضی سوالات

1. **A complete circle is divided into :** **(9 times)(2017)** مکمل دائرے کو تقسیم کیا جاتا ہے:

(a) 90° (b) 180° (c) 270° (d) 360°

2. **The symbol for a triangle is denoted by :** **(10 times)(2018)** مثلث کو ظاہر کرنے کے لئے علامت ہے:

(A) ∠ (B) Δ (C) ⊥ (D) Θ

3. **Radii of a circle are :** **(7 times)(2018)** ایک ہی دائرے کے رداس ہیں:

(A) all equal تمام برابر (B) double of the diametr قطر سے دوگنا (C) all un-equal تمام غیر برابر (D) half of any chord کسی بھی وتر سے آدھے

4. **Through how many non-collinear points a circle can pass :** دائرہ کتنے غیر خطی نقاط سے گزرتا ہے:
(6 times) (2018)

(a) one ایک (b) two دو (c) three تین (d) none of these

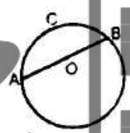
5. **A chord passing through the centre of a circle is called :** **(6 times)(2018)** دائرے کے مرکز سے گزرنے والا وتر کہلاتا ہے:

(a) radius رداس (b) diameter قطر (c) circumference محیط (d) arc قوس

6. **The distance of any point of the circle to its centre is called :** **(10 times)(2018)** دائرے کے کسی نقطے کا اس کے مرکز تک کا فاصلہ کہلاتا ہے:

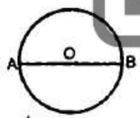
(a) radius رداس (b) diameter قطر (c) chord وتر (d) arc قوس

7. **In the given circular figure, ACB is called :** **(1 time) (2016)** دی گئی دائروی شکل میں ACB کہلاتا/ کہلاتی ہے:



(a) an arc ایک قوس (b) a secant قاطع خط (c) a chord ایک وتر (d) diameter قطر

8. **In the circular figure AOB is called :** **(2 times)** دائروی شکل میں AOB کہلاتا/ کہلاتی ہے:



(a) an arc ایک قوس (b) a secant قاطع خط (c) a chord ایک وتر (d) diameter قطر

9. **The circular region bounded by two radii and the corresponding arc is called** **(3 times)(2018)** دائرے کا وہ رقبہ جو دو رداسوں اور ان کے متعلقہ قوس سے گھرا ہو کہلاتا ہے

(a) Circumference of a circle دائرے کا محیط (b) Sector of a circle دائرے کا سیکٹر (c) diameter of a circle دائرے کا قطر (d) segment of a circle قطعہ دائرہ

2017

10. **Locus of a point in a plane equidistant from a fixed point is called:** **(2 times)(2018)** مستوی کے تمام نقاط کا سیٹ جو معین نقطہ سے برابر فاصلے پر ہوں کہلاتا ہے۔

(a) Radius رداس **(b)** Circle دائرہ **(c)** Circumference محیط **(d)** Diameter قطر

جوابات

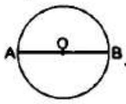
1	2	3	4	5	6	7	8	9	10
D	B	A	C	B	A	A	D	B	B

2014-2018 کے تمام بورڈز کے پرچہ جات میں سے
باب نمبر 9 کے مختصر سوالات اور جوابات

1. What is diameter of a circle? (8 times)(2018)

دائرے کے قطر سے کیا مراد ہے؟

Ans. The chord passing through centre of circle is called diameter. Here AB is diameter of circle



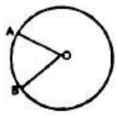
دائرے کے مرکز سے گزرنے والا وتر قطر کہلاتا ہے۔

2. Differentiate between a sector and a segment of a circle. (6 times)(2018)

ایک دائرے کے سیکٹر اور قطعہ میں فرق بیان کیجئے۔

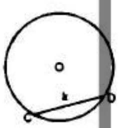
Ans. A sector of a circle is the Area bounded by two radii and the arc intercepted betweenthem e.g. AOB is sector.

دائرے کے دو راسی قطعات اور ان کے متعلقہ قوس سے گھرا ہوا علاقہ دائرے کا سیکٹر کہلاتا ہے



A segment is an area bounded by an arc and a chord.

دائرے کا وہ خط جو اس کی قوس اور متعلقہ وتر نے گھیرا ہو قطعہ خط کہلاتا ہے۔



3. Define circle. (5 times)(2018)

دائرے کی تعریف کیجئے۔

Ans. A circle is locus of a moving point P in plane which is always equidistance from same fixed point O.

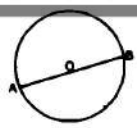
کسی سطح میں متحرک نقطہ P کا وہ راستہ جو ایک معین نقطہ O سے ہمیشہ یکساں فاصلے پر رہے، دائرہ کہلاتا ہے۔

4. Differentiate between a chord and the diameter of a circle . (3 times)(2016)

ایک دائرے کے وتر اور اُس کے قطر میں فرق بیان کیجئے۔

Ans. The straight line joining any two points of the circumference is called a chord . A chord passing through the centre of the circle is called the diameter.

محیط پر دیے ہوئے دو نقاط کو ملانے والا قطعہ خط دائرے کا وتر ہوتا ہے۔



اور رداس کا دگنا قطر کہلاتا ہے۔

5. Differentiate between the following terms and illustrate them by dagrams.

A chord and an arc of a circle (1 time)(2016)

درج ذیل اصطلاحات میں فرق بیان کیجئے اور ان کی بذریعہ اشکال وضاحت کیجئے۔

Ans. An arc ACB of a circle is any portion of its circumfernce A chord AKB of a circle is a straight line joining any two points on the circum ference of a circle .

دائرے کے محیط کا ایک ٹکڑا ACB دائرے کی قوس ہوتی ہے۔ جبکہ محیط پر دیے ہوئے دو نقاط کا ملانے والا قطعہ خط AKB ایک وتر ہوتا ہے۔

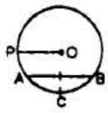
6. Differenatiate between the circumference and arc of a circle (5 times)(2018)

دائرے کے محیط اور قوس میں فرق بیان کیجئے۔

Ans. The boundry traced by moving point P is called circumference of the circle and part of the circumference is called an arc of the circle.

دائرے کی قوس کی لمبائی کو محیط کہتے ہیں

دائرے کے محیط کا ایک حصہ قوس کہلاتا ہے



ACB is Arc

7. Define circum circle. **(11 time) (2018)**

محاصرہ دائرہ کی تعریف کیجئے۔

Ans. A circle which passes through three vertices of triangle is called circum circle.

ایسا دائرہ جو مثلث کے تینوں راسوں میں سے گزرتا ہو محاصرہ دائرہ کہلاتا ہے۔

8. Differentiate between chord and diameter of a circle.

ایک دائرے کا وتر اور اس کے قطر میں فرق بیان کیجئے۔

(2 times)(2018)

Ans. A line joining any two points of circle is called chord. A line which passes through center of circle is called diameter.

ایسی لائن جو دائرے کے دو نقاط کو ملائے وتر کہلاتی ہے۔ ایسی لائن جو دائرے کے مرکز سے گزرے قطر کہلاتی ہے۔

9. Write the formula to find the area of a circle. **(1 time) (2016)**

دائرہ کا رقبہ کا کلیہ لکھئے۔

Ans. (رقبہ A) $= \pi r^2$

10. Differentiate between the escribed and circumscribed circle.

جانبی دائرہ اور محاصرہ دائرہ میں فرق بیان کیجئے۔

Ans. A circle which touches one side of triangle externally and two produced side internally is called escribed circle.

A circle which passes through three vertices of triangle is called circum circle.

ایسا دائرہ جو مثلث کے ایک ضلع کو بیرونی طور پر اور دو بڑھے ہوئے اضلاع کو اندرونی طور پر مس کرے جانبی دائرہ کہلاتا ہے۔

ایسا دائرہ جو مثلث کے تینوں راسوں میں سے گزرتا ہو محاصرہ دائرہ کہلاتا ہے۔

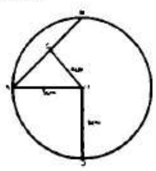
11. What are you meant by radial segment? **(7 times)(2018)**

Ans. A line segment that joins center to any point on circle is called radial segment.

ایسا قطع خط جو دائرے کے مرکز سے کسی نقطہ کو ملائے رداسی قطعہ کہلاتا ہے۔

12. In the adjacent circular figure with certre O and radius 5cm. Find the length of the chord intercepted at 4cm away from the centre of thiscircle.

دیے ہوئے دائرے کی شکل میں مرکز O اور رداس 5 سم ہے۔ اگر ایک وتر مرکز سے 4 سم کے فاصلے پر ہو تو وتر کی لمبائی معلوم کیجئے۔



Ans. By pythagoras theorem

$$H^2 = P^2 + B^2$$

$$5^2 = P^2 + 4^2$$

$$P^2 = 25 - 16 = 9$$

$$P = 3$$

$$\overline{AC} = 3cm$$

Chord length (وتر کی لمبائی) $= \overline{AB} = 6cm$

......2016......

13 Define chord and arc of circle.

دائرے کے وتر اور قوس کی تعریف کیجئے۔

Ans: **Chord**:- A chord of a circle is a line segment joining any two points on the circumference of the circle.

Arc:- Part of circumference of a circle is called an arc of circle.

14 Define Arc. قوس کی تعریف کیجئے۔

(2 times)(2018)

Ans: Part of circumference of a circle is called an arc of circle.

کسی دائرے میں گھومنے والے نقطہ اسی نقطے تک بننے والا راستہ محیط کہلاتا ہے جبکہ محیط کا ایک ٹکڑا دائرے کی قوس کہلاتا ہے۔

15 Define circumference of a circle.

Ans: The boundry traced by moving point P is called circumference of a circle. Mathematically $2\pi r$ is circumference of circle with radius r.

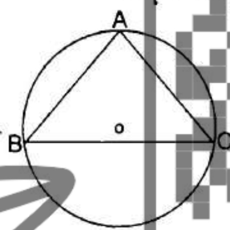
کسی دائرے میں گھومنے والے نقطہ P سے سے اسی نقطہ تک بننے والا راستہ محیط کہلاتا ہے۔ کسی دائرے کا محیط $2\pi r$ ہوتا ہے۔

16 Define circumscribed circle and draw geometric figure.

محاصرہ دائرہ کی تعریف لکھیے اور شکل بنائیے۔

Ans: The circle passing through the vertices of triangle is known as circumcircle. Its radius is called circumradius and centre as circumcentre.

ایسا دائرہ جو مثلث کے تینوں راسوں میں سے گزرتا ہو محاصرہ دائرہ کہلاتا ہے۔



دو یا دو سے زیادہ نقاط جو ایک خط پر واقع ہوں ہم خط نقاط کہلاتے ہیں۔

17 What is meant by non-collinear points? غیر ہم خط نقاط سے کیا مراد ہے؟

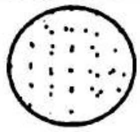
Ans: Two or more points not lying on the same straight line are called non collinear points.

دو یا دو سے زیادہ نقاط جو ایک خط پر واقع نہ ہوں غیر ہم خط نقاط کہلاتے ہیں۔

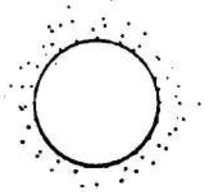
......2017......

1. Differentiate between interior and exterior of a circle.

Ans: Interior of a circle: The points which lie inside the circle called interior of circle.



Exterior of circle: The points which lie outside the circle called exterior of circle.



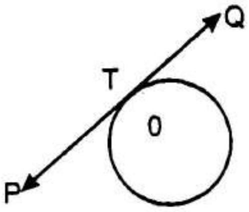
......2018......

1. Define interior of a circle. دائرے کے اندرونہ کی تعریف کریں۔

Ans. Area/region within the circumference of a circle is called interior of a circle.

دائرے کے محیط کا اندر والا رقبہ دائرے کا اندرونہ کہلاتا ہے۔

7. in the adjacent figure of the circle the line $\overline{PTQ}$ is named as — متصلہ دائرے کی شکل میں $\overline{PTQ}$ کو کہا جاتا ہے۔



(a) an arc (b) A chord (c) A tangent (d) A secant

8. Two tangents drawn to a circle from a point outside are. (9 times) (2018) — ایک دائرے کے بیرونی نقطہ سے کھینچے گئے دو مماس لمبائی کے لحاظ سے ...... ہوتے ہیں۔

(a) Half نصف (b) Equal برابر (c) Double دوگنا (d) Triple تین گنا

9. A Tangent Line intersects the circle at : (3 times) — ایک خط مماس دائرے کو کاٹتا ہے۔

(a) Three points تین نقاط پر (b) Two points دو نقاط پر (c) One point ایک نقطہ پر (d) None کوئی نہیں

10. The distance between the centers of two congruent circle touching externally is — دو بیرونی طور مس کرنے والے مساوی مراکز کا فاصلہ ہوتا ہے

(a) of zero lenght رداس سے کم (b) equal radiuse of each cicle دائرہ کا رداس (c) the dimeter of each circle دائرے کا قطر (d) twice the diameter of each circle دائرے کا قطر

2017

11. A line which has only one point in comon with a circle is called: — ایک خط جس کا دائرے کے ساتھ صرف ایک نقطہ مشترک ہو، کہتے ہیں۔

(a) Sine of a circle (sine کا دائرے) (b) Consine of a cricle (Consine کا دائرے) (c) Tangent of a circle (Tangent کا دائرے) (d) Secant of a circle (Secant کا دائرے)

جوابات

1	2	3	4	5	6	7	8	9	10	11
D	D	C	C	C	D	C	B	C	C	C

2014-2018 کے تمام بورڈز کے پرچہ جات میں سے باب نمبر 10 کے مختصر سوالات اور جوابات

1. **What do you mean by tangent of a circle?** **(16 times)(2018)** دائرے کے مماس سے کیا مراد ہے۔

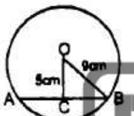
Ans. A tangent to a circle is the straight line which touches the circumference at one point only.
دائرے کا مماس ایک ایسا خط ہے جو دائرے کے محیط کو صرف ایک نقطہ پر مس کرتا ہے

2. **Define the length of tangent** **(5 time) (2018)** مماس کی لمبائی سے کیا مراد ہے۔

Ans. The length of tangent to a circle is measured from the given point to the point of the contact.
۔مماس کی لمبائی کسی دائرے کے بیرونی نقطہ p سے نقطہ مماس تک ہوتی ہے

3. **Calculate the length of a chord which stands at a distance 5cm from th centre of a cricle whose radius is 9cm.**
ایک دائرہ جس کا رداس 9 سم ہے اور اس کے وتر کا مرکز سے فاصلہ 5 سم ہو تو وتر کی لمبائی معلوم کریں۔

Ans. By pyhtagoras theorem. مسئلہ فیثا غورث کی رو سے

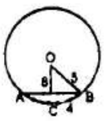


$$(OB)^2 = (OC)^2 + (CB)^2$$
$$(9)^2 = (5)^2 + (CB)^2$$
$$81 - 25 = (mCB)^2$$
$$\sqrt{56} = mCB$$
$$7.48 = mCB$$
$$mAB = 2 \times 7.48 = 14.96cm$$

4. **How many number of tangents are drawn from any point out of the circle ? what are the relation between their length?** **(3 times)**
کسی بیرونی نقطہ سے دائرے پر کتنے مماس کھینچے جاسکتے ہیں اور ان کی لمبائیوں کا آپس میں کیا تعلق ہے؟

Ans. Two tangents are drawn from any point at the circle and they are equal in length.
بیرونی نقطہ دائرے پر دو مماس کھینچے جاتے ہیں۔ اور ان کی لمبائیاں آپس میں برابر ہوتیں ہیں۔

5. **If a circle has chord of length 8 cm and its radius is 5 cm then find the distance of the chord form centre**
اگر دائرے میں ایک قطعہ خط کی لمبائی 8cm ہو اور رداس 5cm ہو تو مرکز سے قطعہ خط کا فاصلہ معلوم کریں



Ans.

$$(OB)^2 = (OC)^2 + (BC)^2$$
$$(5)^2 = (OC)^2 + (4)^2 \Rightarrow 25 - 16 = (OC)^2$$
$$OC = 3cm$$

6. **What is the relation between the tangents drawn at the end points of the diameter of a circle ?** **(3 times)**
دائرے کے قطر کے سروں پر کھینچے گئے مماس کا آپس میں کیا تعلق ہے؟

Ans. The tangents drawn at the end points of the diameter of a circle are parallel .
دائرے کے قطر کے سروں پر کھینچے گئے مماس آپس میں متوازی ہوتے ہیں۔

7. **Define a secant line** **(15 times)(2018)** قاطع خط کی تعریف کیجئے۔

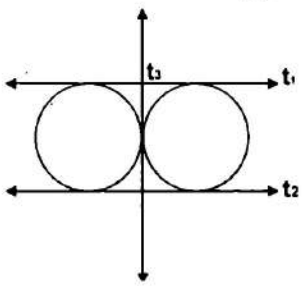
Ans. A secant is a straight line which cuts the circumference of circle in two points distinct points.

8. How many common tangents can be drawn for two touching circles?

دومس کرتے ہوئے دائروں کے کتنے مشترک مماس کھینچے جاسکتے ہیں؟

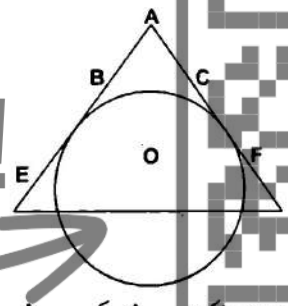
Ans: Three common tangents, can be drawn for two touching circles.

دومس کرتے ہوئے دائروں کے تین مشترک مماس کھینچے جاسکتے ہیں۔



9. Define Escribed circle and draw geometric figure. **جانبی دائرے کی تعریف کریں اور شکل بنائیں۔**

Ans: The circle touching one side of the triangle externally and two produced sides internally is called escribed circle (e.circle)



ایسا دائرہ جو مثلث کے ایک ضلع کو باہر سے اور دو بڑھتے ہوئے اضلاع کو اندر سے مس کرے جانبی دائرہ کہلاتا ہے۔

10. Define the point of contact of a circle. **دائرہ کے نقطہ تماس سے کیا مراد ہے؟**

Ans: Common point of circumference of circle and tagent is known as point of contact of a circle in figure T is point of contact of circle.

دائرے کے محیط اور مماس کا مشترک نقطہ دائرے کا نطقہ تماس کہلاتا ہے۔

……2018……

11. If r = 20cm & $\pi = 3.1416$ find half the perimeter of circle.

اگر r = 20cm اور $\pi = 3.1416$ ہو تو نصف دائرے کا محیط معلوم کریں۔

Ans. Perimeter of circle (نصف دائرے کا محیط) = $2\pi r$

Half perimter of circle (نصف دائرے کا محیط) = $\frac{2\pi r}{2}$

$= \pi r$

$= 3.1416 \times 20$

$= 62.832cm$

2014-2018 کے تمام بورڈز کے پرچہ جات میں سے
باب نمبر 11 کے معروضی سوالات

1. The semi circumference of a circle subtends a central angle of : (9 times)(2018)

دائرے کے نصف محیط کا مرکزی زاویہ ہوتا ہے:

(a) 90° (b) 180° (c) 270° (d) 360°

2. A 4 cm long chord subtends a central angle of 60^{0}. The radial segment of this circle is : (9 times) (2018)

ایک 4 سم لمبائی والا وتر مرکز پر 60° زاویہ بناتا ہے۔ دائرے کا رداس ____ ہوگا۔

(A) 1 (B) 4 (C) 3 (D) 2

3. If an arc of a circle subtends central angle 60° then the corresponding chord of the arc will make the central angle of : (6 times) (2018)

ایک قوس کا مرکزی زاویہ 60° ہے اس کے وتر کا مرکزی زاویہ ____ ہوگا:

(a) 20° (b) 40° (c) 60° (d) 80°

4. An arc subtends a central angle 40° then the corresponding chord will subtend a central angle of (10 times) (2018)

ایک قوس کا مرکزی زاویہ 40° ہے اس کا متعلقہ وتر کا مرکزی زاویہ ____ ہوتا ہے:

(a) 20° (b) 40° (c) 60° (d) 80°

5. The length of a chord and the radial segment of a circle are congruent, then central angle made by the chord is : (6 times)(2018)

ایک دائرے میں وتر اور رداس کی لمبائیاں برابر ہوں تو وتر سے بننے والا مرکزی زاویہ ہوگا:

(a) 30° (b) 45° (c) 60° (d) 75°

6. A pair of chord of a circle subtending two congruent central angle is : (7 times)(2018)

دو متماثل مرکزی زاویے جن دو وتروں سے بنتے ہیں۔ وہ آپس میں ____ ہوں گے۔

(a) congruent متماثل (b) incongruent غیر متماثل (c) overlapping متراکب (d) parallel متوازی

7. Out of two congruent arcs of a circle, if one arc makes a central angle of 30°, then the other arc will subtend the central angle of : (4 times) (2018)

ایک دائرے کی دو متماثل قوسوں میں سے اگر ایک قوس کا مرکزی زاویہ 30° ہو تو دوسری کا مرکزی زاویہ ____ ہوتا ہے:

(a) 15° (b) 30° (c) 45° (d) 60°

8. **The length of a chord and the radiacal segment of a circle are congruent the central angle made by the chord will be : (2 times)**

ایک دائرے کا وتر اور رداس کی لمبائیاں برابر ہیں وتر سے بننے والا مرکزی زاویہ ______ ہوگا۔

(a) 30° (b) 45° (c) 60° (d) 75°

9. **A complete circle is divided into :**

ایک مکمل دائرے کو تقسیم کیا جاتا ہے:

(a) 90° (b) 180° (c) 270° (d) 360°

10. **The semi circumference and the diameter of a circle both subtend a central angle of : (2 time) (2018)**

دائرے کے نصف محیط کا مرکزی زاویہ ہوتا ہے:

(a) 90° (b) 180° (c) 270° (d) 360°

11. **Out of two congruent arcs of a circle if one arc makes a central angle of 30^0 then the other arc will subtend the central angle of (3 times)**

ایک دائرے کی دو متماثل قوسوں میں سے اگر ایک قوس کا مرکزی زاویہ 30^0 ہو تو دوسری کا مرکزی زاویہ ______ ہوتا ہے۔

(a) 30^0 (b) 45^0 (c) 60^0 (d) 30^0

12. **The chord Length of a cricle subtending a central Angle of 180" is always . (2 time) (2016)**

اگر دائرے کا وتر مرکزی زاویہ "180 بنائے تو وتر کی لمبائی ہوگی:

(a). Equal to Radical segment رداس کے برابر
(b). Less than Radical segment رداس سے کم
(c). Double of the Radical segment رداس کا دوگنا
(d). None کوئی نہیں

13. **The arc opposite to incongruent central angles of a circle are always (3 time)(2018)**

ایک دائرے میں دو غیر متماثل مرکزی زاویوں کے سامنے والی قوسیں ہوتی ہیں۔..........

(a) Congruent متماثل (b) Incongruent غیر متماثل (c) Parallel متوازی (d) Perpendicular عمود

......2018......

14. **A 4cm long chord subtands a central angle of 60^0, the radial segment of this circle is ___:**

ایک 4 سم لمبائی والا وتر مرکز پر 60^0 کا زاویہ بناتا ہے، دائرے کا رداس ---- ہوگا۔

(a) 1cm **(b)** 2cm **(c)** 3cm **(d)** 4cm

جوابات

1	2	3	4	5	6	7	8	9	10
A	B	C	B	C	A	B	C	D	A
11	12	13	14						
A	C	B	D						

2014-2018 کے تمام بورڈز کے پرچہ جات میں سے باب نمبر 11 کے مختصر سوالات اور جوابات

1. **Differentiate between "Minor and Major Arc of a Circle " and illusfrate them by diagrams**

ایک دائرے میں صغیرہ قوس اور کبیرہ قوس میں فرق بیان کیجئے۔ اور ان کی بذریعہ اشکال وضاحت کیجئے۔

(3 times)(2018).

Ans. Minro Arc:- An arc smaller than that of the semicircle is called minor Arc.

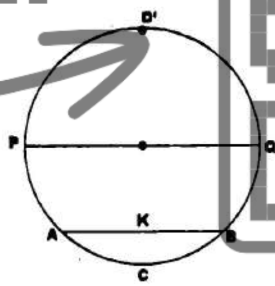
ایسی قوس جو نصف دائرے سے چھوٹی ہو قوس صغیرہ کہلاتی ہے

Major Arc:- An arc greater than semi circle is called major arc.

ایسی قوس جو نصف دائرے سے بڑی ہو قوس کبیرہ کہلاتی ہے

The following figure , the smaller arc $\overline{ACB}$ is minor arc and $\overline{ADB}$ is major arc of circle.

تصویر میں $\overline{ACB}$ قوس صغیرہ اور $\overline{ADB}$ قوس کبیرہ ہے۔



2. **If an arc of a circle substands a central angle of 60^0 what is the central angle of the corresponding chrod of the arc?**

ایک قوس کا مرکزی زاویہ 60^0 ہے۔ اس کے وتر کا مرکزی زاویہ کیا ہوگا؟

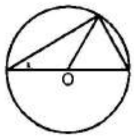
Ans. The central angle of the corresponding chord of the arc will be $60''$

وتر کا مرکزی زاویہ $60''$ ہوگا۔

2014-2018 کے تمام بورڈز کے پرچہ جات میں سے
باب نمبر 12 کے معروضی سوالات

1. In figure, O is the center of circle then the angle x is — شکل میں دائرے کا مرکز O ہے۔تب x ____ ہے:

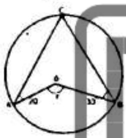
(3 time) (2017)



(a) 15° (b) 30° (c) 45° (d) 60°

2. In the figure, "0" is the central of a angle x is : — دی گئی شکل میں دائرے کا مرکز "0" ہے تب x ____ ہے:

(2 time) (2017)



(a) 50° (b) 75° (c) 100° (d) 125°

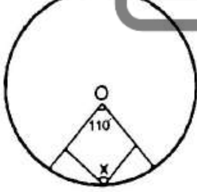
3. A circle passes through the vertices of a right angled triangle ABC with $m\overline{AC} = 3cm, m\overline{BC} = 4cm$ and $m\angle C = 90°$ then radius of the circle is (5 times)(2017)

کسی قائمۃ الزاویہ △ABC میں $m\overline{AC} = 3cm, m\overline{BC} = 4cm$ اور $m\angle C = 90°$ ہو تو مثلث کے راسوں میں گزرنے والے دائرے کا رداس ____ ہے:

(a) 1.5 cm (b) 2.0 cm (c) 2.5 cm (d) 3.5 cm

4. In the given figuere, O is the centre of the cirlce than x is : (10 times)(2017)

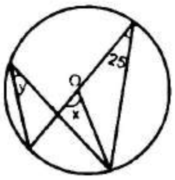
دی گئی شکل میں دائرے کا مرکز O ہے۔تب x ____ ہے:



(a) 55° (b) 110° (c) 220° (d) 125°

5. Given that O is the centre of the circle the angle marked Y will be : (3 times)

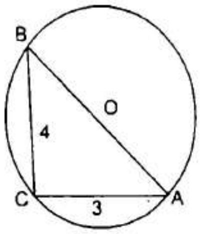
دائرے کے مرکزی نقطہ O معلوم ہو تو نشان زدہ زاویہ Y ہوگا:



(a) 12 1/2° (b) 25° (c) 50° (d) 75°

6. **In the adjacent figure if** $m\overline{AC} = 3cm, m\overline{BC} = 4cm$ $m\angle C = 90°$ **then the radius of circle is :**

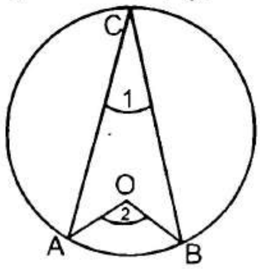
دی گئی شکل میں اگر:
$m\overline{AC} = 3cm, m\overline{BC} = 4cm$
تو دائرے کا رداس ہوگا:



(a) 1.5 cm (b) 2.0 cm (c) 2.5 cm (d) 3.5 cm

7. **In the adjacent circular figure central and inscribed angles on the same arc AB**

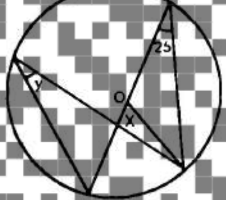
شکل میں AB ایک ہی قوس پر مرکزی اور محصور زاویے بنتے ہیں تب



(a) $m\angle1 = m\angle2$ (b) $m\angle1 = 2m\angle2$ (c) $m\angle2 = 3m\angle1$ (d) $m\angle2 = 2m\angle1$

8. **Given that O is the centre of a circle the marked Y will be**

دائرے کا مرکزی نقطہ O معلوم ہو تو نشان زدہ Y زاویہ ہوگا۔



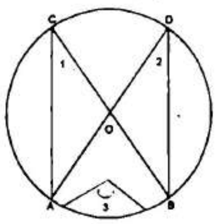
(a) 12 1/2^{0} (b) 25^{0} (c) 50^{0} (d) 75^{0}

9. **In the figure , O is the centre of the circle , then angle x is** ....
(1 time) (2016)

شکل میں دائرے کا مرکز O ہے تب x .... ہے۔

(a) 60^{0} (b) 45^{0} (c) 30^{0} (d) 15^{0}

10. **In given figure , if** $m\angle3 = 75''$ **then** $m\angle1$ **and** $m\angle2$ **will be :**



دی گئی شکل میں اگر $m\angle3 = 75''$ ہو تو $m\angle1$ اور $m\angle2$ ہونگے۔

(a) $75'', 75''$ (b) $75'', 37\frac{1}{2}''$ (c) $37\frac{1}{2}'', 75''$ (d) $37\frac{1}{2}'', 37\frac{1}{2}''$

جوابات

1	2	3	4	5	6	7	8	9	10
B	C	C	D	B	C	D	B	C	D

2014-2018 کے تمام بورڈز کے پرچہ جات میں سے
باب نمبر 12 کے مختصر سوالات اور جوابات

1. Define segment of a circle. (13 times)(2018) **قطعہ دائرہ سے کیا مراد ہے؟**

Ans. A segment is an area bounded by an arc and a chord.

دائرے کا وہ ٹکڑا جو اس کی قوس اور متعلقہ وتر نے گھیرا ہو قطعہ دائرہ کہلاتا ہے۔

2. Define the circumangle. (13 times) (2018) **محاصر زاویہ کسے کہتے ہیں؟**

Ans. The angle of subtended by an arc of the circle at the circumference of a circle is called a circumangle.

دائرے کی ایک قوس جو اس کے محیط پر زاویہ بناتی ہے۔ اس کو محاصر زاویہ کہلاتا ہے۔

3. Define the central Angle and form the figure. (12 times)(2018) **مرکزی زاویے کی تعریف کریں اور شکل بھی بنائیں**

Ans. The angle sublended by an arc at the centre of the circle is called ccentral angle.

ایک قوس دائرہ کے مرکز پر جو زاویہ بناتی ہے۔ اسے مرکزی زاویہ کہتے ہیں۔

4. What do you know about the opposite angles of a cyclic quadrilateral? **کسی دائرہ کی سائیکلک چوکور کے متقابلہ زاویے کے بارے میں آپ کیا جانتے ہیں؟**

Ans. Opposite angles are supplementry.

متقابلہ زاویے سپلیمنٹری ہوتے ہیں۔

5. Define quadrilateral. **چوکور کی تعریف کیجئے۔**

Ans. A four sided closed figure is called quadrilateral whose sides are not equal to each other.

چار اضلاع والی بند شکل چوکور کہلاتی ہے۔ جس کے اضلاع برابر نہیں ہوتے۔

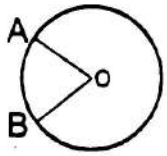
6. Define cyclic quadrilateral. **سائیکلک چوکور کی تعریف کریں۔ (5 times)(2018)**

Ans: ^ quadrilateral is called cyclic when a circle can be drawn through its four vertices.

وہ چوکور جس کے چاروں راسوں سے دائرہ کھینچا جا سکتا ہوا سے سائیکلک چوکور کہتے ہیں۔

7. Define sector of a circle. **دائرے کے سیکٹر کی تعریف کیجئے۔ (3 times)(2018)**

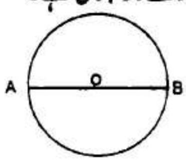
Ans: A sector of a circle is the area bounded by two radii and the arc intercepted between them. In figure AOB is sector.



8. Define chord of a circle. **دائرے کے وتر کی تعریف کیجئے۔ (5 times)(2018)**

Ans: A line joining any two points of circle and not passing through the centre is called chord.

ایسی لائن جو دائرے کے دو نقاط کو ملائے اور درمیان میں سے بھی نہ گزرے وتر کہلاتی ہے۔



9. Define in-centre of a circle. **دائرہ کے محصور مرکز سے کیا مراد ہے؟ (3 times)(2018)**

Ans: Centre of inscribed circle or incircle is called in-centre. محصور دائرے کے مرکز کو محصور مرکز کہا جاتا ہے۔

2014 تا 2018 کے تمام بورڈز کے پرچہ جات میں سے
باب نمبر 13 کے معروضی سوالات

1. **How many common tangents can be drawn for two disjoint circles : (5 times)(2018)** دو غیر متقاطع دائروں کے کتنے مشترک مماس کھینچے جاسکتے ہیں:

(a) 2 (b) 3 (c) 4 (d) 5

(A) ∠ (B) Δ (C) ⊥ (D) Θ

2. **How many tangents can be drawn from a point outside the circle. (9 times)(2018)** دائرے کے باہر نقطہ سے کتنے مماس کھینچے جاسکتے ہیں؟

(A) 1 (B) 2 (C) 3 (D) 4

3. **The length of the diameter of the circle is _____ times the radius of the circle. (7 times) (2018)** ایک دائرے کے قطر کی لمبائی دائرے کے رداس کا _____ گنا ہوتی ہے:

(a) 1 (b) 2 (c) 3 (d) 4

4. **The measure of an external angle of regular octagon is : (5 times)(2018)** ایک منتظم مثمن کے بیرونی زاویوں کی مقدار ہوتی ہے:

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{8}$ (d) $\frac{2\pi}{3}$

5. **How many common tangents can be drawn of two touching circles: (6 times)(2018)** دو مس کرتے ہوئے داروں کے کتنے مشترک مماس بنائے جاسکتے ہیں؟

(a) 2 (b) 3 (c) 4 (d) 5

6. **The tangent and radius of a circle at the point of contact are : (4 times)(2018)** دائرے کا مماس اور رداس ایک دوسرے پر ہوتے ہیں:

(a) parallel متوازی (b) not perpendicular عمود نہیں (c) perpendicular عمود (d) none of these ان میں سے کوئی نہیں

7. **The measure of a external angle of a regular hexagon is : (8 times)(2018)** ایک منتظم مسدس کے بیرونی زاویواں کی مقدار ہوتی ہے

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

8. **The circumference of a circle is called : (6 times)(2016)** دائرے کا محیط کہلاتا ہے:

(a) chord وتر (b) segment قطعہ (c) boundary سرحد (d) none of these ان میں کوئی نہیں

9. **Angle inscribed in a semi-circle is : (13 times)(2018)** نصف دائرے میں محصور زاویہ ہوتا ہے:

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{7}$

10. **Through how many Non-collinear points , a circle can pass.** دائرہ کتنی خطی نقاط سے گزرتا ہے۔

(a) 1 (b) 2 (c) 3 (d) None کوئی نہیں

11. **The lengths of two transverse tangents to pair of circles are.** دو دائروں پر دو معکوس مماس کی لمبائیاں ہوتی ہیں۔

(a) Unequal غیر برابر (b) equal برابر (c) overlapping متراکب (d) 75°

2016

12. A line intersecting a circle is called (5 times)(2018) دائرے کو قطع کرتا خط کہلاتا ہے

(a) tangent عمود (b) secant خط قاطع (c) chord وتر (d) boundary سرحد

2017

13. اگر دو دائروں کے مراکز کے درمیان فاصلہ رداسوں کے مجموعہ کے برابر ہو تو دائرے ہوں گے:

(a) قطع کرتے ہیں Intersect (b) بیرونی طور پر مس کرتے ہیں Touch each other externally (c) قطع نہیں کرتے Do not intersect (d) اندرونی طور پر مس کرتے ہیں Touch each other internally

14. Angle inscribed in a semicircle is:(2 Times)(17) نصف دائرے میں محصور زاویہ ہوتا ہے۔

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

2018

15. A circle passes through the vertices of right angled Δ ABC with $m\overline{AC} = 3cm, m\overline{BC} = 4cm$ and $\Delta\ m\angle C = 90^0$:

کسی قائمۃ الزاویہ مثلث ABC میں $m\overline{AC} = 3cm$ ، $m\overline{BC} = 4cm$ اور $m\angle C = 90^0$ اس مثلث کے راسوں میں سے گزرنے والا دائرے کا رداس ---- ہے۔

(a) 1.5cm (b) 2.0cm (c) 2.5cm (d) 3.5cm

16. The circumference of a circle is called: دائرے کا محیط کہلاتا ہے۔

(a) chord وتر (b) Segment قطعہ (c) boundary سرحد (d) Secant خط قاطع

17. The length of the diameter of a circle is how many times the radius of the circle: ایک دائرے کے قطر کی لمبائی دائرے کے رداس کے کتنے گنا ہوتی ہے:

(a) 4 گنا Times (b) 3 گنا Times (c) 2 گنا Times (d) 1 گنا Times

جوابات

1	2	3	4	5	6	7	8	9	10
C	B	B	A	A	C	A	C	A	C
11	12	13	14	15	16	17			
B	B	B	D	C	C	C			

2014-2018 کے تمام بورڈز کے پرچہ جات میں سے باب نمبر 13 کے مختصر سوالات اور جوابات

1. Define escribed circle. (4 times)(2016)

جانبی دائرہ کی تعریف کیجئے۔

Ans. A circle that touches two produced sides of a triangle internally and one side externally is called an escribed circle .

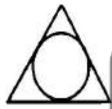
ایسا دائرہ جو تکون کے دو بڑھے ہوئے اضلاع کو اندر سے اور ایک ضلع کو باہر سے مس کرتا ہے جانبی دائرہ کہلاتا ہے۔

2. Define and draw the geometric figure of the inscribed circle? (10 times)(2018)

محصور دائرے کی تعریف کیجئے اور شکل بنائیے۔

Ans. A circle which is touch the all sides of triangles internally is called inscribed circle .

مثلث کے تینوں اضلاع کو اندرونی طور پر مس کرنے والا دائرہ محصور دائرہ کہلاتا ہے۔



3. The length of each side of a regular octagon is 3cm. Find its perimeter. (2 times)

ایک منتظم مثمن کے ہر ضلع کی لمبائی 3 سم ہے۔ اس کا احاطہ معلوم کیجئے۔

Ans. **Perimeter = 8 × 3** ⇒ 8 × 3 = 24

4. The length of each side fo a regualar octagon is 4cm . Find it perimeter.

Ans. Perimeter of octagon = 8 × Length of side

×84 = 32

ضلع کی لمبائی × 8 = احاطہ

= 8 × 4 = 32

4. The length of the side of a regular pentagon is 5cm . What is its perimeter? (2 times)(2018)

ایک منتظم مخمس کے ضلع کی لمبائی 5 سم ہے اس کا احاطہ کیا ہے؟

Ans.

Length of side = 5cm

Perimeter = 5 × length of side

= 5 × 5 = 25cm

5. The Measure of central Angle of Minor Arc is 60″ . Find the Measure of Angle subtended by the corresponding major arc.

کسی دائرہ میں قوس صغیرہ سے بننے والا مرکزی زاویہ 60″ کا ہو تو متعلقہ قوس کبیرہ کے محصور زاویہ کی مقدار معلوم کریں۔

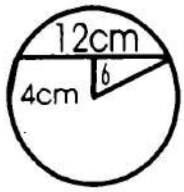
Ans. Then angle of the major arc is 120 °.

6. If the length of the chord $\overline{AB} = 12$ cm and its distance from the centre is 4cm , then find the diameter of the circle .

اگر وتر $\overline{AB}$ کی لمبائی 12 سم ہو اور اس کا مرکز سے فاصلہ 4 سم ہو تو اس کے دائرے کا قطر معلوم کریں۔

Ans. By pythagoras theorem

$$C^2 = 4^2 + 6^2 \quad , \quad C^2 = 16 + 36 \quad , \quad C^2 = 52 \quad , \quad C = 7.21$$



7. From which two Greek words, the word geometry is derived?

لفظ جیومیٹری کن دو یونانی الفاظ سے اخذ کیا گیا ہے؟

Ans. "Geo" means earth and "metron" means measurment.

جیو کا مطلب زمین اور میٹرون مطلب پیمائش

8. The length of the side of a a regular pentagon is a 7 cm. find its perameter.

ایک منظم مخمس کے ضلع کی لمبائی 7 سم ہے اس کا احاطہ معلوم کیجئے۔

Ans. Length of one side = 7 cm   Perimeter = 7× 4 = 28 cm

9. Define Polygon. (2 times)(2018)

کثیر الاضلاع کی تعریف کیجئے:

Ans. A closed figure having three or more sides is called polygon.

ایک بند شکل جو تین یا زیادہ اضلاع پر مشتمل ہو کثیر الاضلاع کہلاتی ہے۔

10. What is a regular Polygon? (2 time) (2016)

ریگولر کثیر الاضلاع کیا ہے؟

Ans. A polygon is regular polygon that is equiangular and equilateral.

ایک کثیر الاضلاع ریگولر کہلاتی ہے اگر تمام زاویے اور اضلاع برابر ہوں۔

11. What is a perimeter. (3 times)(2018)

احاطہ کیا ہوتا ہے؟

Ans. A boundary of any closed figure is called perimeter.

ایک بند شکل کی حدود احاطہ کہلاتی ہے۔

12. Define geometry.

جیومیٹری کی تعریف کیجئے۔

Ans. The word "Geometry" has been derived from two grrek words "Geo" means earth and "Metron" means measurment. Therefore geometry means measurment of earth.

لفظ جیومیٹری دو لاطینی الفاظ سے مل کر بنا ہے جیو (زمین) اور میٹرون (پیمائش) اسطرح اس سے مراد زمین کی پیمائش ہے۔

13. What is radius of a circle? (4 time) (2018)

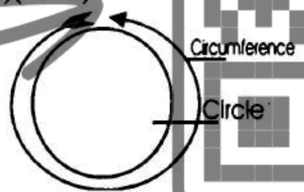
دائرے کا رداس کیا ہے؟

Ans. The distance from center to any point of circle is called radius.

دائرے کے مرکز سے کسی نقطہ تک کا فاصلہ رداس کہلاتا ہے۔

14 Differentiate between a circle and a circumference by diagrams. (4 times)(2018)

ایک دائرہ اور اس کے محیط میں بذریعہ شکل فرق بیان کیجئے۔



The boundary of circle is called circumference.
The locus of moviing point whose distance from fixed point is same is calle circle.

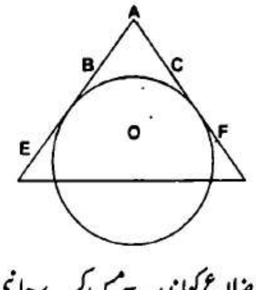
دائرے کے قوس کی کل لمبائی محیط کہلاتی ہے کسی سطح میں متحرک نقطہ P کا وہ راستہ جو ایک معین نقطہ O سے یکساں فاصلے پر رہے دائرہ کہلاتا ہے۔

2016

15 Define Escribed circle and draw geometric figure.

جانبی دائرے کی تعریف کریں اور شکل بنائیں۔

Ans: The circle touching one side of the triangle externally and two produced sides internally is called escribed circle (e.circle)



ایسا دائرہ جو مثلث کے ایک ضلع کو باہر سے اور دو بڑھتے ہوئے اضلاع کو اندر سے مس کرے جانبی دائرہ کہلاتا ہے۔

2018

16. Define the vertices of a polygon.

کثیر الاضلاع کے راس کی تعریف کریں۔

Ans. A point where two straight lines of polygon meet is called vertex of a polygon.

ایک ایسا نقطہ جہاں کثیر الاضلاع کی دو سیدھی لائنیں ملیں کثیر الاضلاع کا راس کہلاتا ہے۔